Pressure Measurements

- **The Barometer**

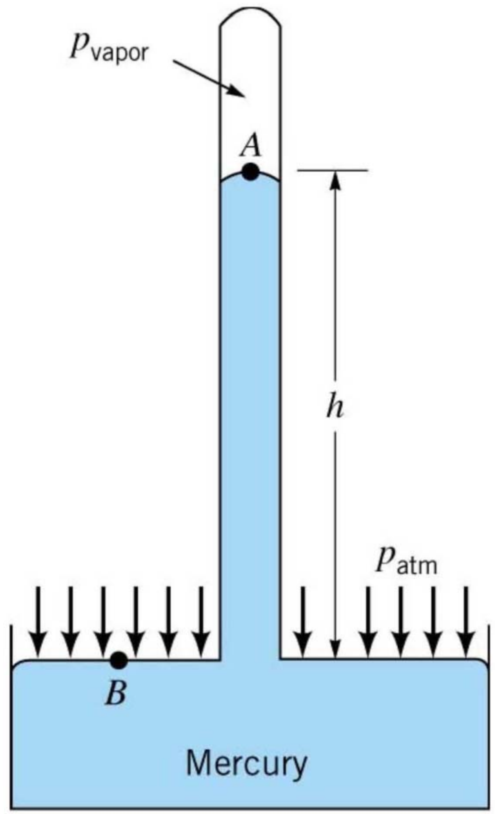
Mercury Barometer measures the atmospheric pressure in absolute units.

$$P_{atm} = P_{vap\ Hg} + h\,\gamma_{hg}$$

$$= 0 \quad + h\,\gamma_{hg}$$

$h = 76.0$ cm of Hg

N.B. The vapor pressure of mercury is negligible



Example:

Determine the atmospheric pressure when the mercury barometer records a height of 752 mm. The S.G. of mercury is 13.6.

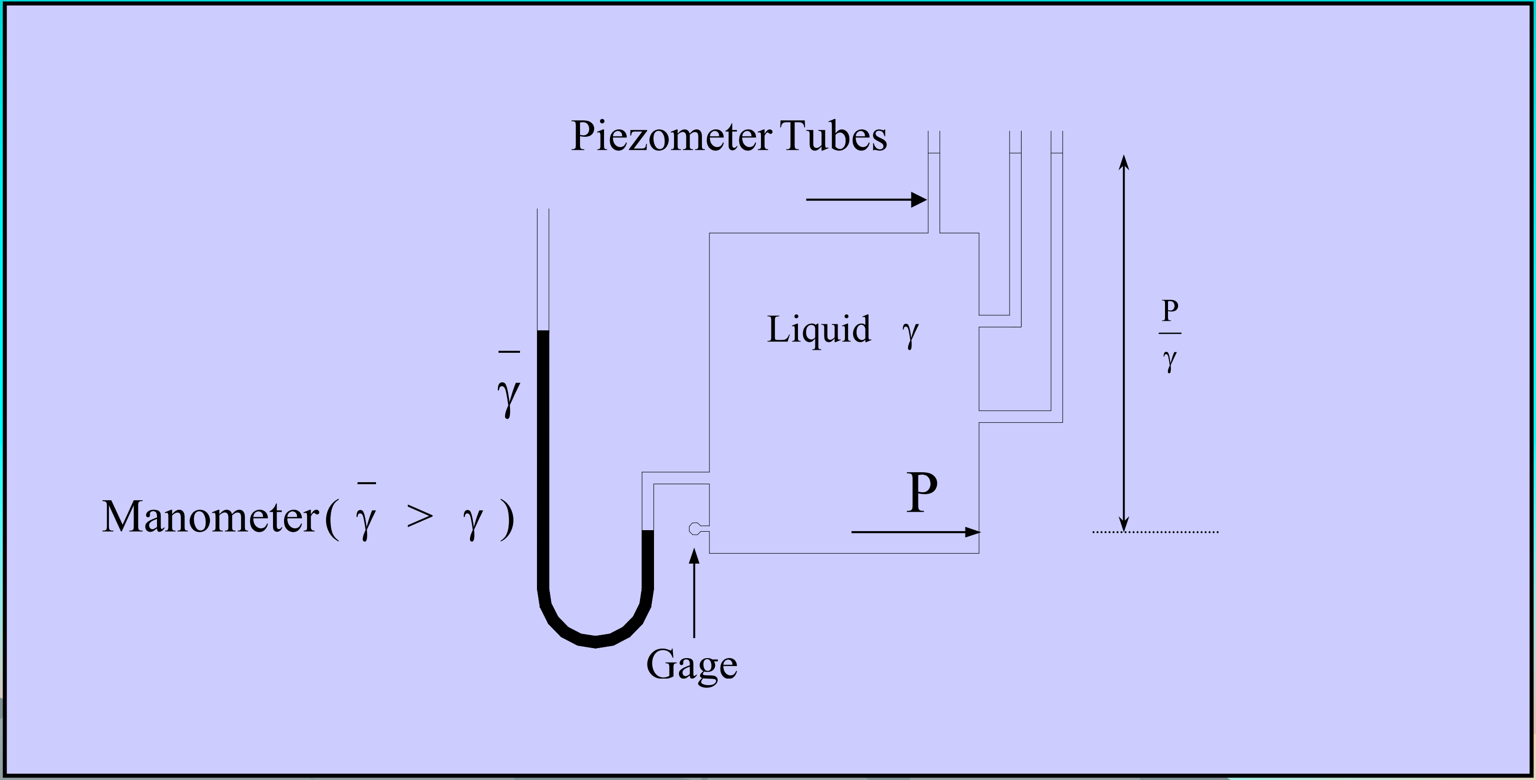
Solution:

$$P = \gamma_{hg}\, h = 13.6 \text{ x } 9.81 \text{ x } 0.752 = 100.3 \text{ kPa}$$

N.B. The vapor pressure of mercury is negligible

▪Piezometer tube

It is a simple type of manometers used to measure pressure of a liquid.



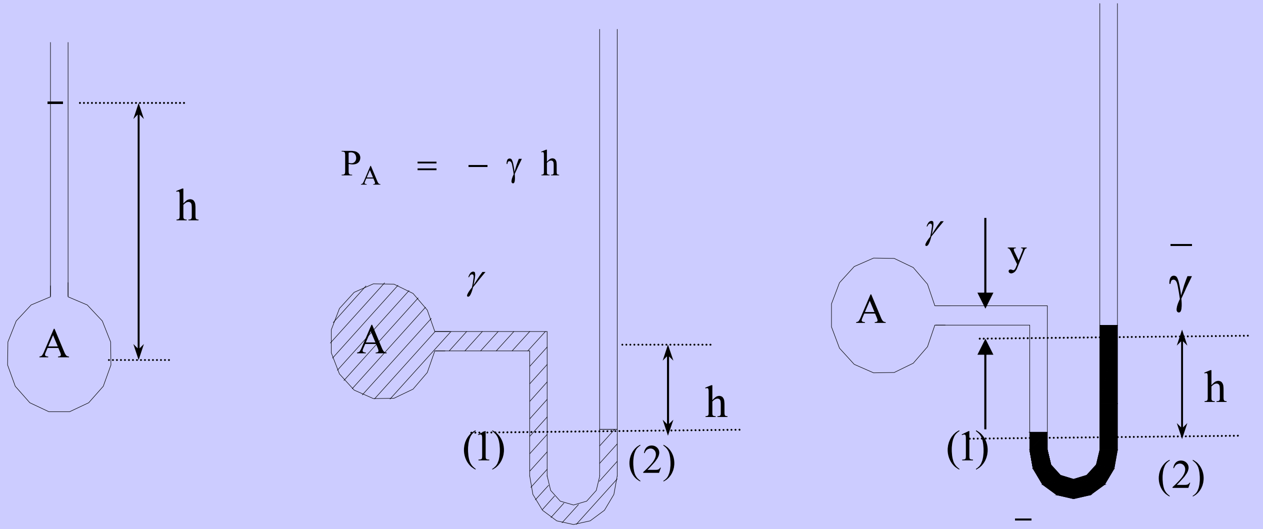
▪Manometers

Piezometer

Simple Manometers

$$\bar{\gamma}\, h - \gamma\, h - \gamma\, y - P_A = 0$$

$$P_A = -\gamma\, h$$



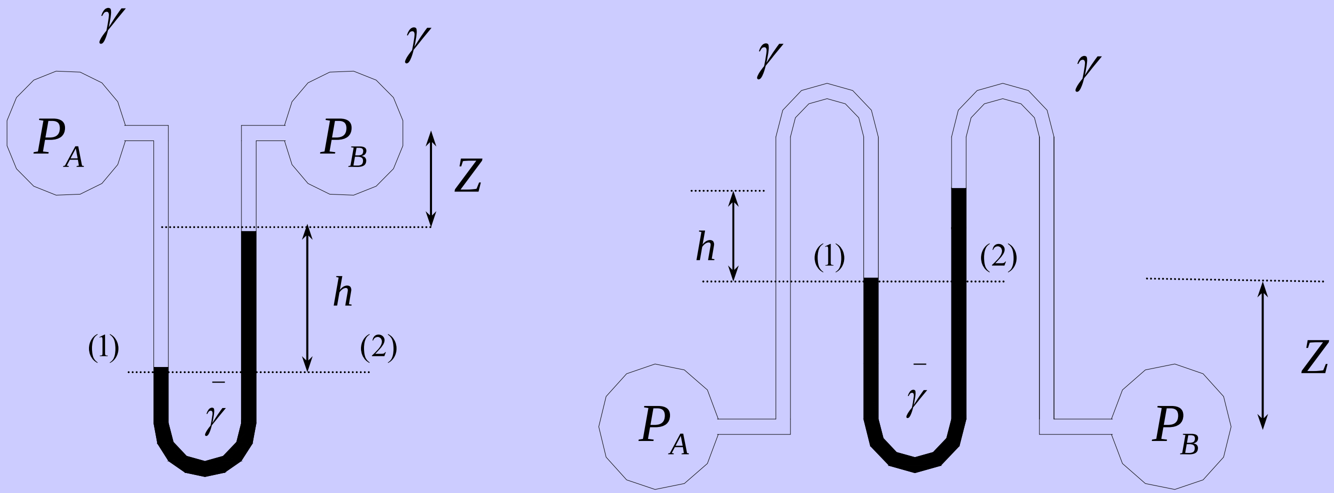
$$(\bar{\gamma} > \gamma)$$

Small +ve or –ve Pressure

Great +ve or –ve pressures

Note: The measuring (metering) fluid ($\bar{\gamma}$) should not interact with the other fluid γ

▪Differential Manometers



$(\bar{\gamma} > \gamma)$ $(\bar{\gamma} > \gamma)$

For points (1) and (2) on the same level

$$P_A + \gamma z + \gamma h = P_B + \gamma z + \bar{\gamma} h$$

$$P_A - P_B = \left(\bar{\gamma} - \gamma\right) h$$

$$\frac{P_A - P_B}{\gamma} = \left(\frac{\bar{\gamma}}{\gamma} - 1\right) h$$

$$P_A - \gamma z = -\gamma h + P_B + \bar{\gamma} h - \gamma z$$

$$P_A - P_B = \left(\bar{\gamma} - \gamma\right) h$$

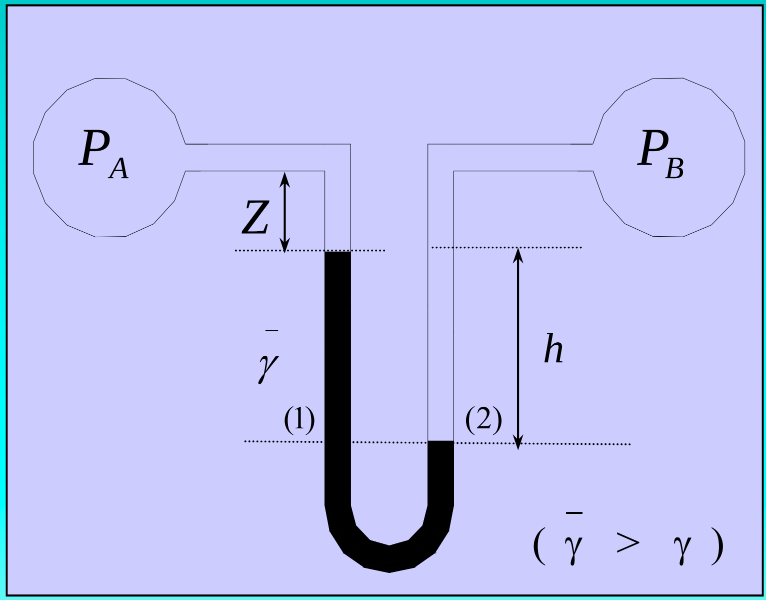
$$\frac{P_A - P_B}{\gamma} = \left(\frac{\bar{\gamma}}{\gamma} - 1\right) h$$

For small pressure differences a light liquid such as oil may be used.

$$P_A - \gamma z - \bar{\gamma} h = P_B - \gamma z - \gamma h$$

$$P_A - P_B = \bar{\gamma} h - \gamma h$$

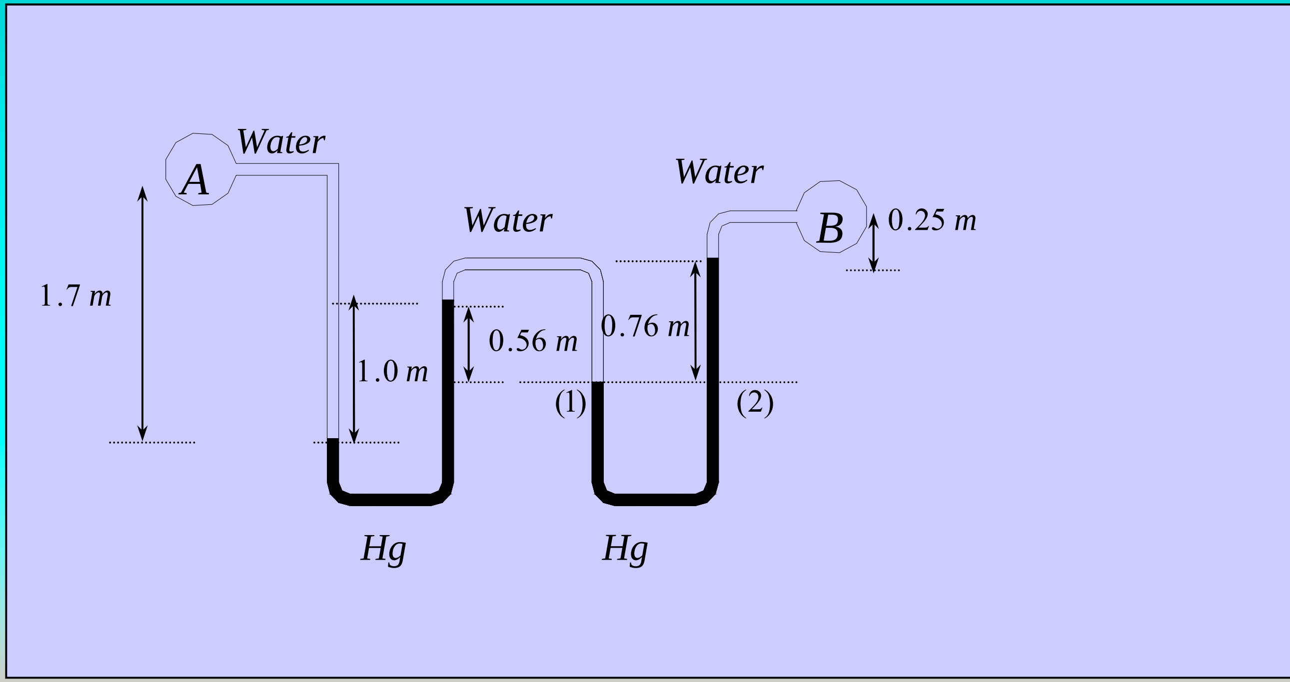
$$\frac{P_A - P_B}{\gamma} = \left(\frac{\bar{\gamma}}{\gamma} - 1 \right) h$$



N.B :No formulas for particular manometers should be memorized. Work out each case needed using first principles. [The starting point is to select two points in the same liquid at the same level – ***equipotential line***].

Example:

Calculate the pressure difference between A and B for the setup shown in fig.



Solution:

$$P_A + \gamma_w \times 1.7 - \gamma_{Hg} \times 1.0 + \gamma_w \times 0.56 = P_B + \gamma_w \times 0.25 + \gamma_{Hg} \times 0.76$$

$$P_A - P_B = \gamma_w \times 0.25 + \gamma_{Hg} \times 0.76 - \gamma_w \times 1.7 + \gamma_{Hg} \times 1.0 - \gamma_w \times 0.56$$

$$= \gamma_w \times (0.25 - 1.7 - 0.56) + \gamma_{Hg} \times (0.76 + 1.0)$$

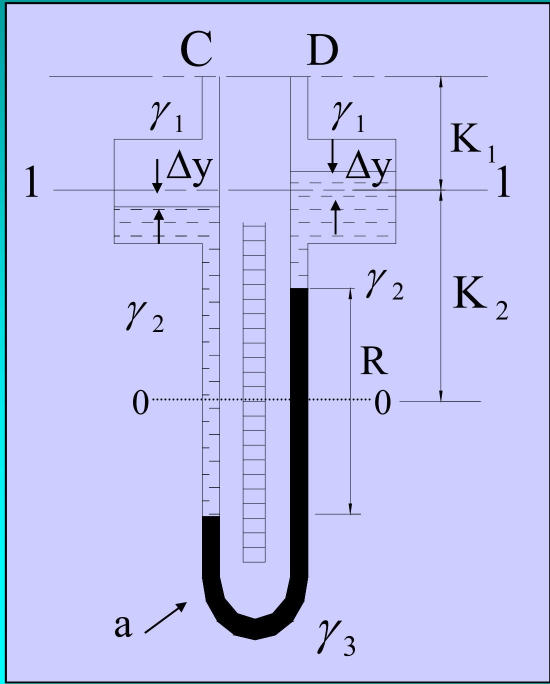
$$= \gamma_{Hg} \times 1.76 - \gamma_w \times 2.01$$

$$= 13.6 \times 9.8 \frac{KN}{m^3} \times 1.76 - \times 9.8 \frac{KN}{m^3} \times 2.01$$

$$= \underline{\underline{215.1\,Kpa}}$$

▪Micromanometers

$$\Delta y\, A = \frac{R}{2}\, a$$



A= the cross-sectional area of the reservoir
a = the cross-sectional area of the U-tube

$$P_C + (K_1 + \Delta y)\gamma_1 + \left(K_2 - \Delta y + \frac{R}{2}\right)\gamma_2 - R\gamma_3 - \left(K_2 - \frac{R}{2} + \Delta y\right)\gamma_2 - (K_1 - \Delta y)\gamma_1 = P_D$$

$$P_C - P_D = R\left[\gamma_3 - \gamma_2\left(1 - \frac{a}{A}\right) - \gamma_1 \frac{a}{A}\right]$$

Example:

In the micromanometer, the pressure difference is wanted, in pascals, when air is in the system, S2 = 1.0, S3 = 1.10, R = 5 mm, T = 20^{O} C, and the barometer reads 760 mm Hg.

Solution:

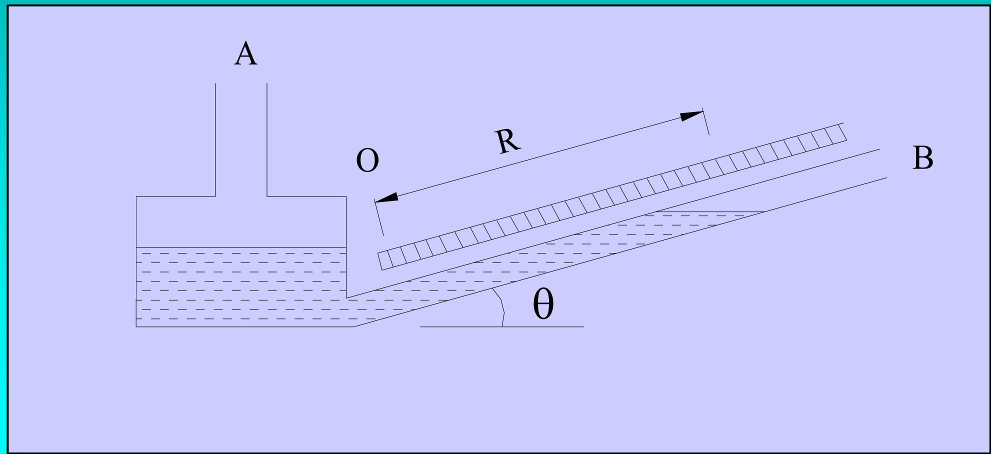
$$\rho_{air} = \frac{P}{R\,T} = \frac{0.76\,m \times 13.6 \times 9806\ N/m^3}{287\,(N.m/kg.K) \times (273 + 20\,K)} = 1.205\ kg/m^3$$

$$\gamma_1 \frac{a}{A} = (1.205\ kg/m^3)(9.806\ m/s^2)(0.01) = 0.118\ N/m^3$$

$$\gamma_3 - \gamma_2 \left(1 - \frac{a}{A}\right) = (9806\ kg/m^3)(1.10 - 0.99) = 1079\ N/m^3$$

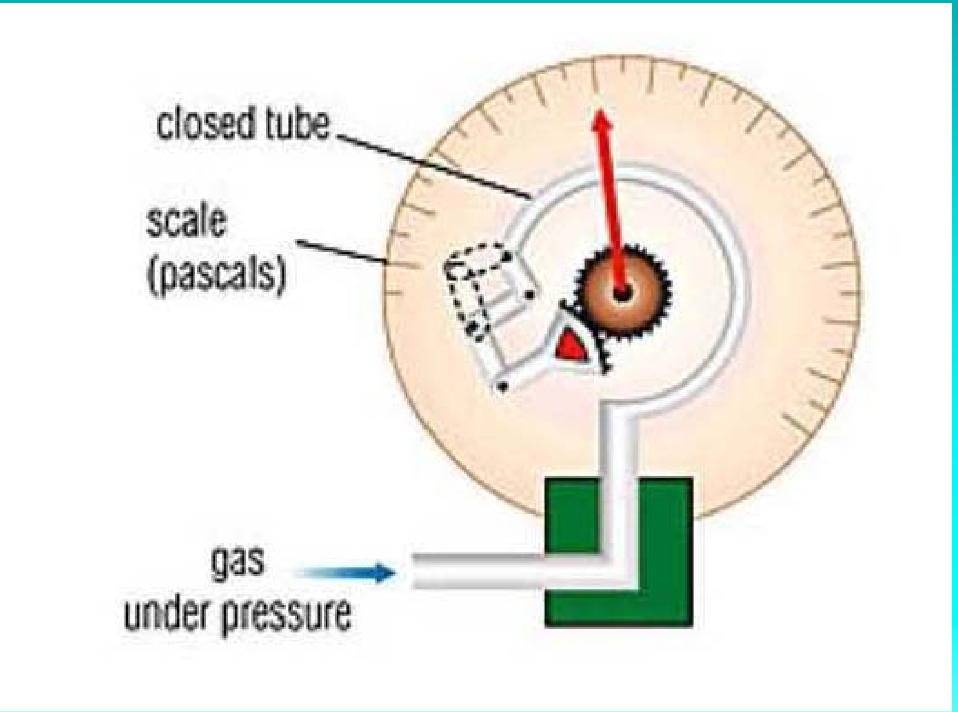
$$P_C - P_D = (0.005\ m)(1079\ N/m^3) = 5.39\ Pa$$

▪Inclined Manometers



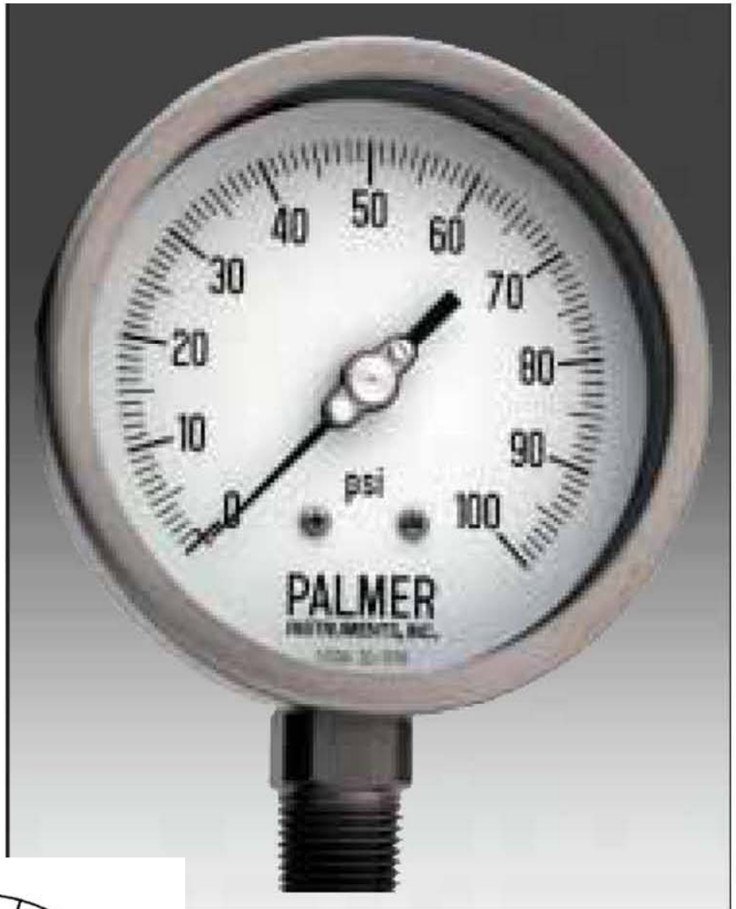
The inclined manometer is frequently used for measuring small differences in gas pressures

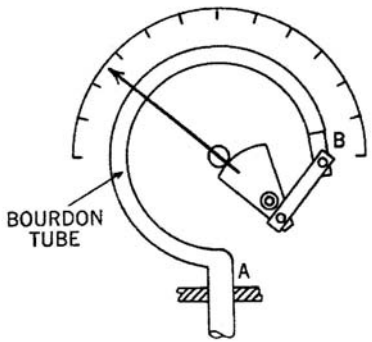
Mechanical Devices for Measuring Gage and Atmospheric Pressure



Note: The Bourdon gage measures the difference between the pressure inside and outside the tube. The aneroid measures the local atmospheric pressure.

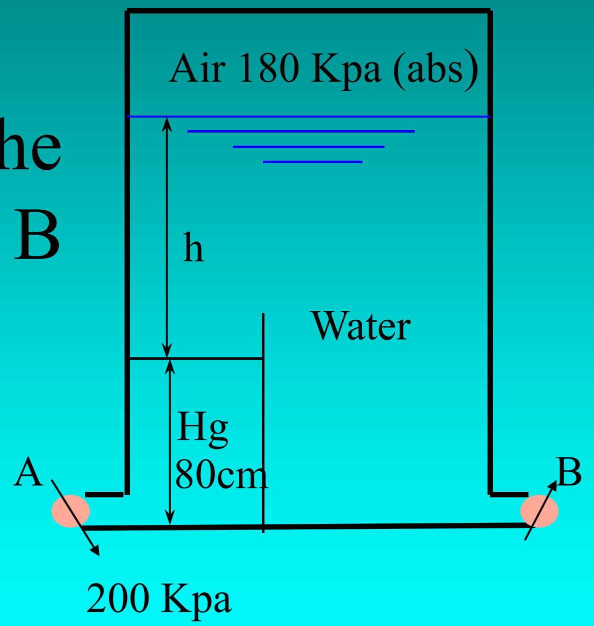

0.4
0.6
0.2
0.8
0
1
bar


0
10
20
30
40
50
60
70
80
90
100
psi
PALMER


BOURDON
TUBE
A
B

Example:

Gage A read 200 Kpa. What is the height h of water? What should gage B read?



Solution:

$$P_{air\,gage} = 180 - 101.3 = 78.7\,KPa$$

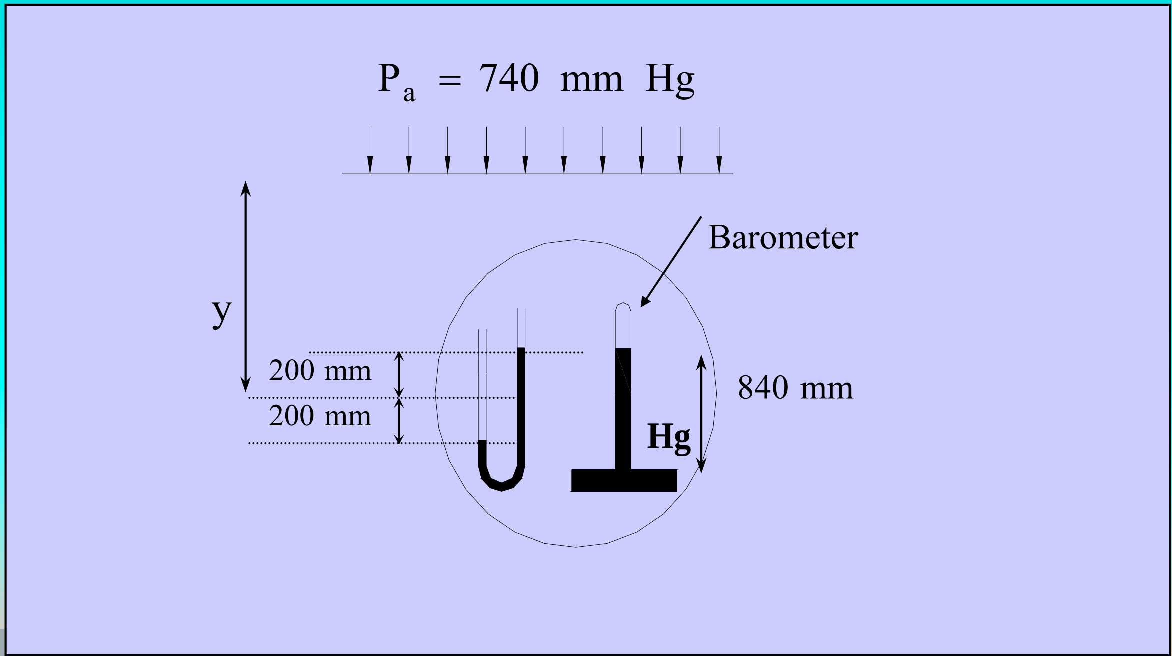
$$P_{air} + \gamma_w h + \gamma_{Hg} \times 0.8 = 200\,KPa = P_A$$

$$78.7 + 9.8h + (13.6 \times 9.8) \times 0.8 = 200\,KPa$$

$$h = \underline{\underline{1.5\,m}}$$

$$P_B = 78.7 + \gamma_w (1.5 + 0.8) = \underline{\underline{101.24\,KPa}}$$

The sketch shows a sectional view through a submarine. Calculate the depth of sub-mergence y. Assume γ of Seawater equal 10 KN/m3



Solution:

By using Absolute pressures For both limbs.

$$(0.74 \times \gamma_{Hg}) + (\gamma_w \times y) + (\gamma_w \times 0.2) = (0.84 \times \gamma_{Hg}) + (\gamma_{Hg} \times 0.4)$$

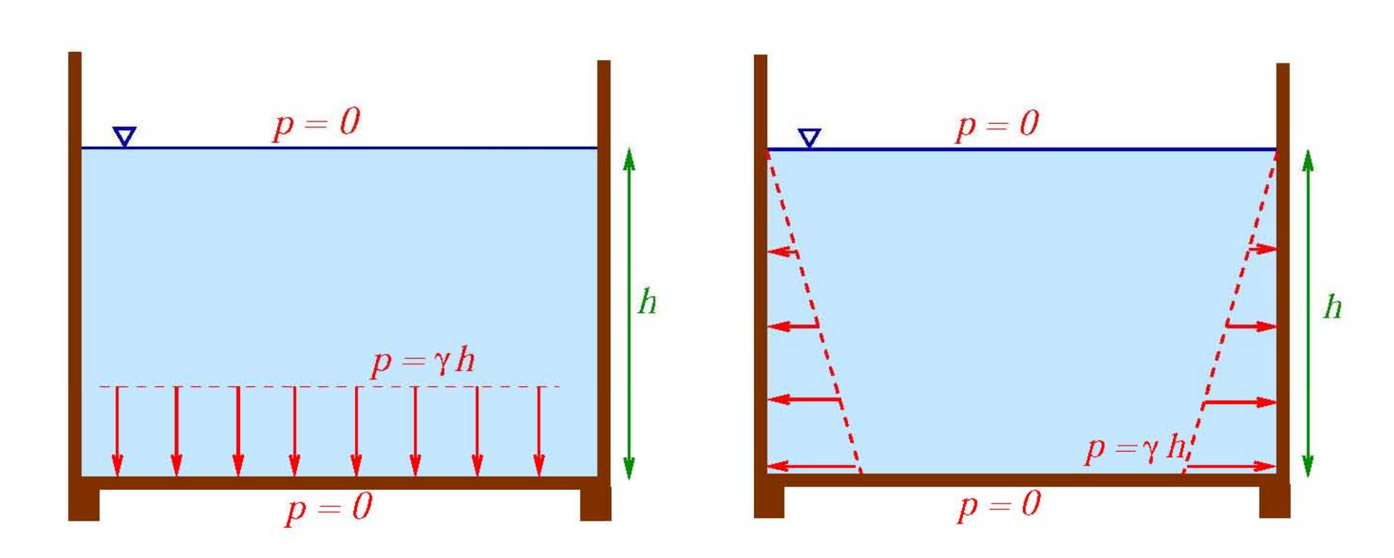
$$y = \frac{\gamma_{Hg}(0.84 + 0.4 - 0.74) - \gamma_w \times 0.2}{\gamma_w} = \underline{\underline{6.6\,m}}$$

Hydrostatic Forces on Submerged Surfaces

• The pressure on the bottom is uniform so the resultant force acts through the centroid.

• The pressure of the sides increases with decreasing depth. The force will not act through the centroid of the surface.

• The centroid is the geometric mean position center of the surface. The line of action (center of pressure) weights the area integral by the force applied through that area.

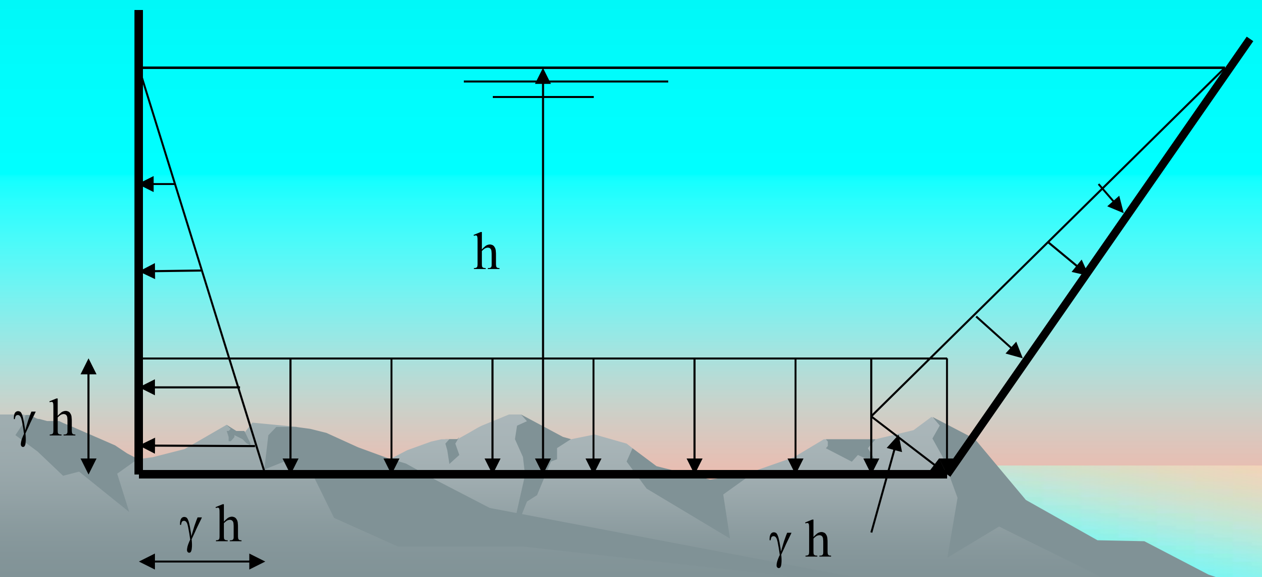


This section concentrates on the methods employed to compute the magnitude, location and direction of a resultant pressure force acting on:

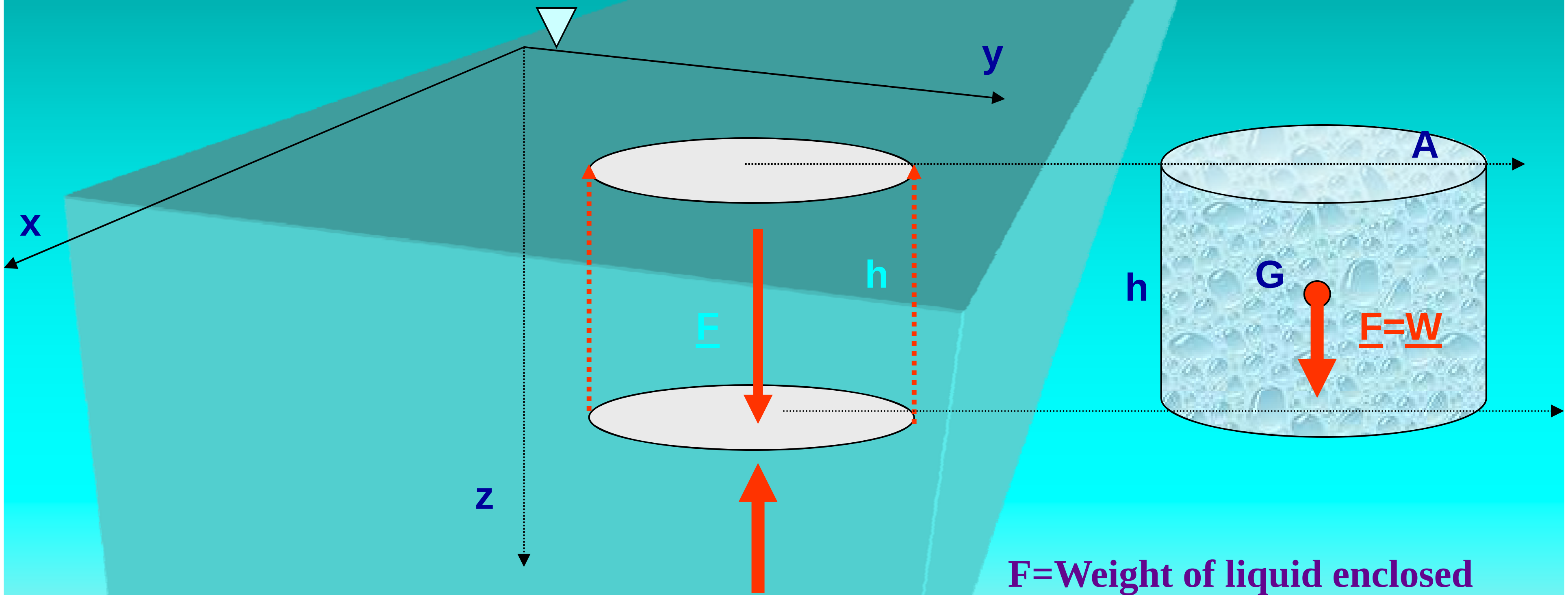
- A plane surface.
- A curved surface.

This figure shows the pressure distribution on a horizontal, vertical and inclined surfaces.

Free Liquid Surface ($P = P_{atm.}$)



▪Hydrostatic Forces on a Horizontal Plane Surface

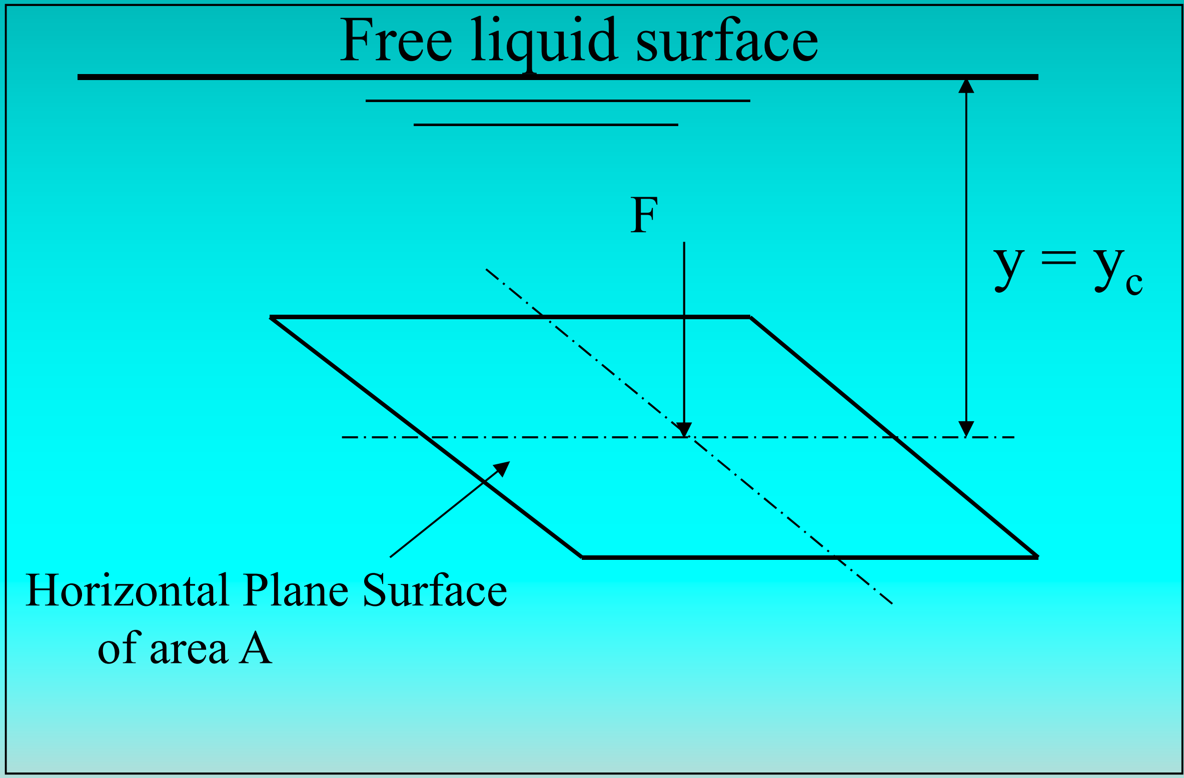


F=Weight of liquid enclosed

F= g h A

And acts at the C.G. of the enclosed liquid

▪Hydrostatic Forces on a Horizontal Plane Surface



Force on a horizontal submerged plane surface

The total pressure force on the horizontal surface is:

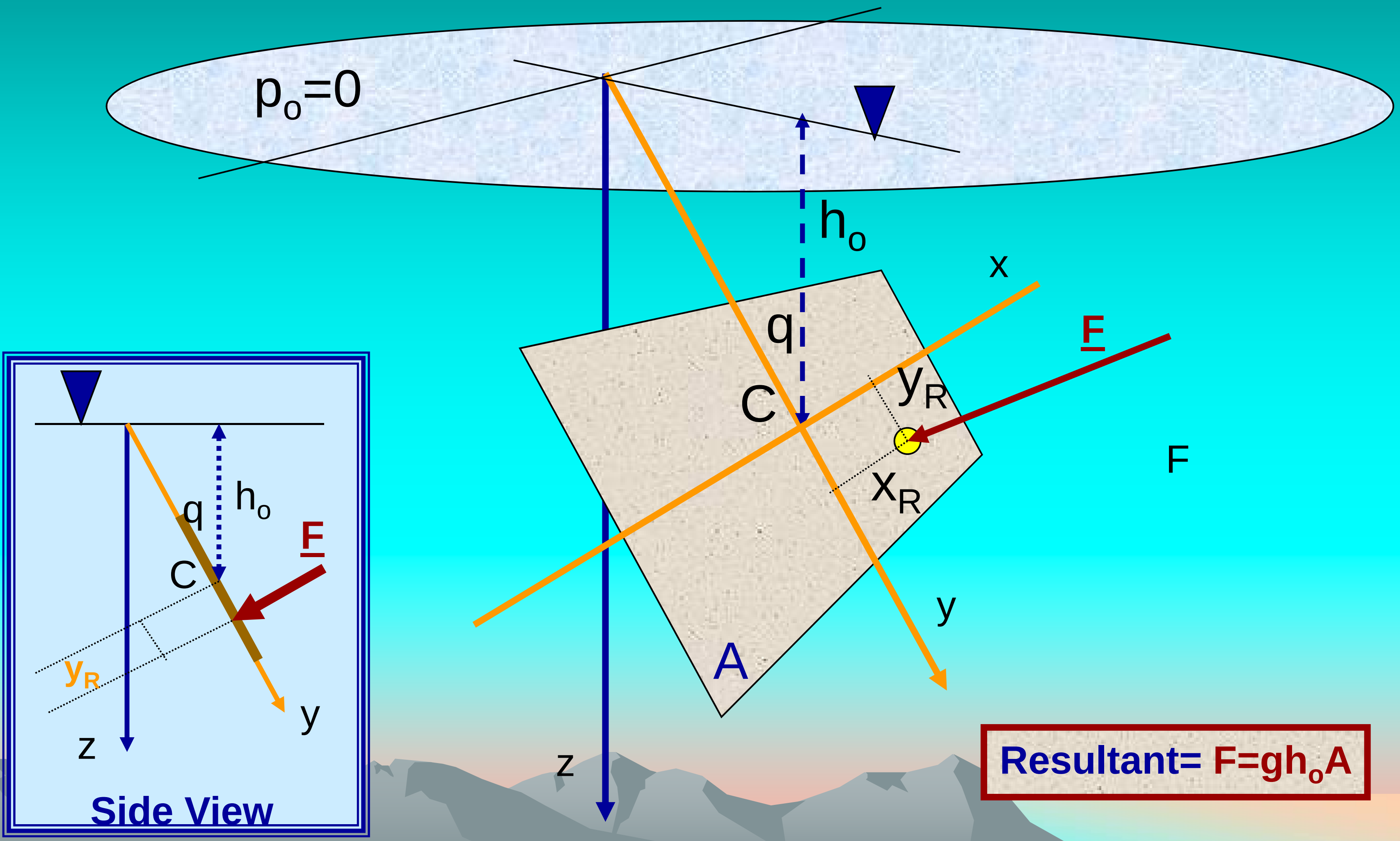
$$F = P\,A = \gamma\, y\, A$$

Where
A = the total area of the surface.
Y= the depth below the free surface of the liquid.

For the given configuration $y = y_c$ depth of the center of gravity (centroid) of the submerged surface below the free surface of the liquid

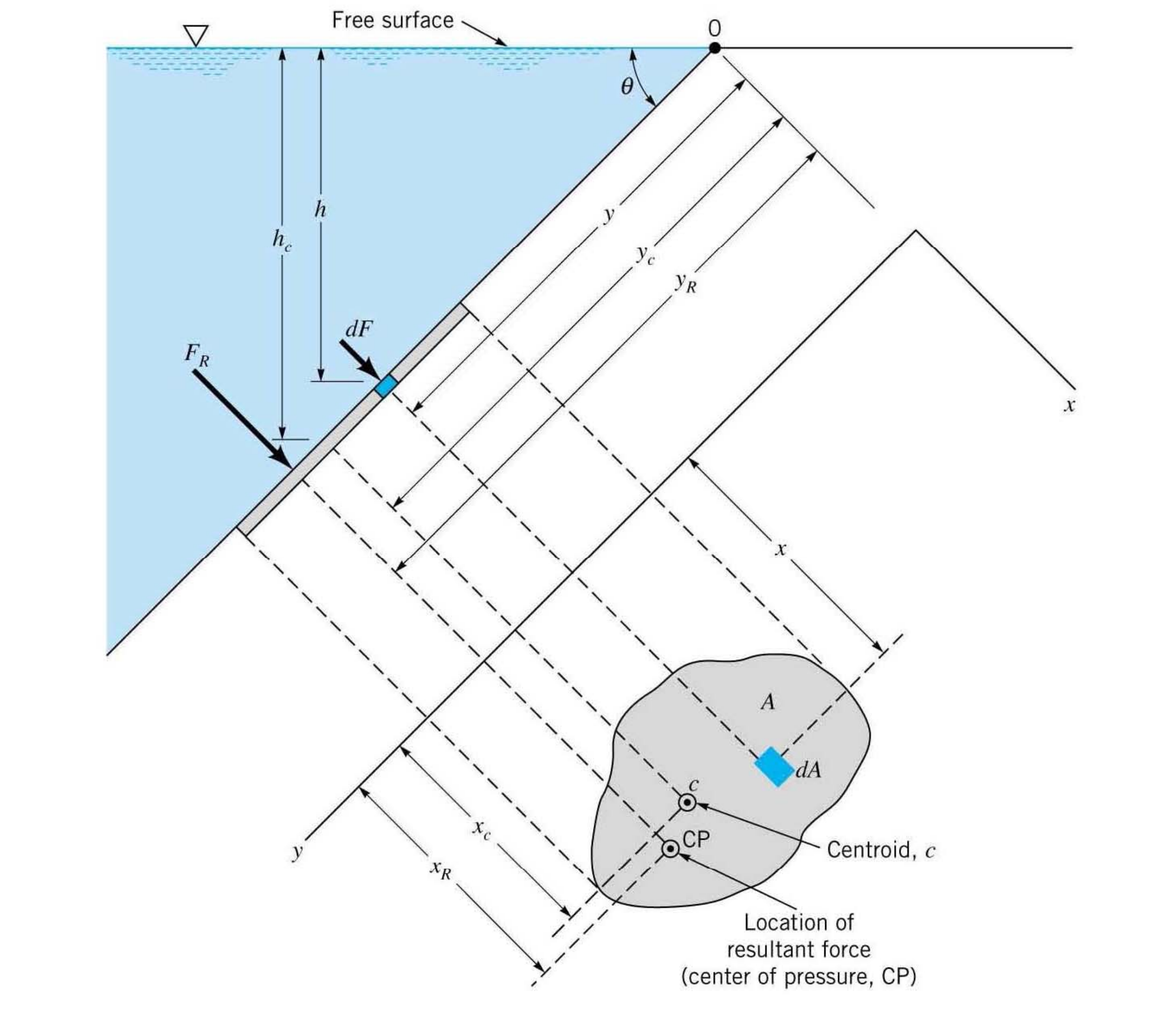
$$F = \gamma\ y_c\ A$$

▪Hydrostatic Forces on Submerged Plane Surfaces



Resultant= F=gh_oA

▪Hydrostatic Forces on Submerged Plane Surfaces



The force dF on the area dA is given by:

$$dF = P\,.\,dA = \gamma\, h\, dA$$

$$= \gamma\, y \sin\alpha\, dA$$

$$= \gamma \sin\alpha\; y\, dA$$

$$F = \int dF = \gamma \sin\alpha \int^{A} y\, dA$$

$\int y.dA$ is the first moment of the area about axis O – O/ which equals :

$$\int^{A} y\;dA = A\;\bar{y}$$

Then $F = \gamma \sin \alpha \; A \; \bar{y} = \gamma \; A \; (\bar{y} \sin \alpha)$

$$F = \gamma \; A \; \bar{h}$$

To find the point of action: take moments about axis $O - O^{/}$

The moment dM of the force dF about axis $O - O^{/}$ is:

$$\begin{aligned} dM &= dF \; y \\ &= \gamma \; h \; dA \; y \\ &= \gamma \; y \sin \alpha \; dA \; y \\ &= \gamma \sin \alpha \; y^2 \; dA \end{aligned}$$

$I_{C.G.}$ is the moment of inertia about C.G. of the surface

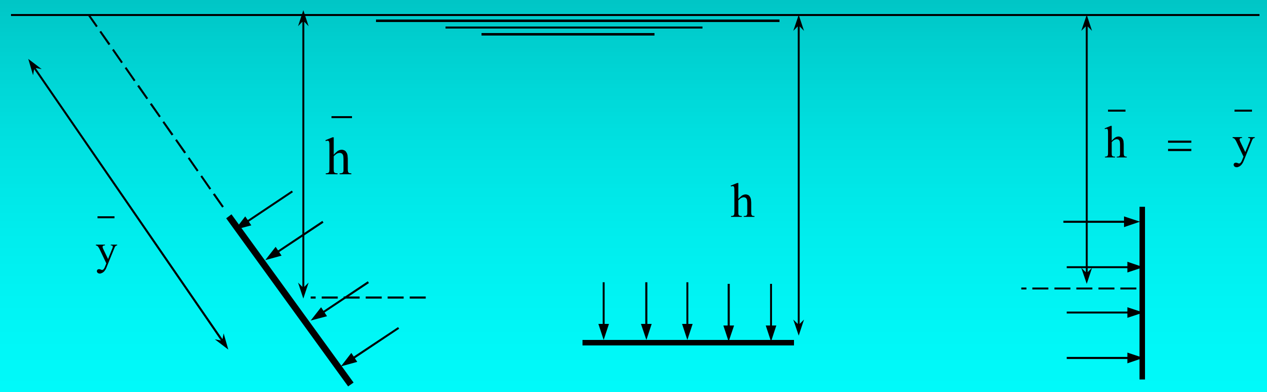
$$y_p = \bar{y} + \frac{I_{C.G.}}{A\,\bar{y}}$$

The center of pressure is always deeper than the center of gravity by Δ

$$\Delta = \frac{I_{C.G.}}{A\,\bar{y}}$$

Note :

Values of the second moment of inertia for different geometrical shapes are given in an attached annex



$$F = \gamma A \bar{h}$$

$$P = \gamma h$$

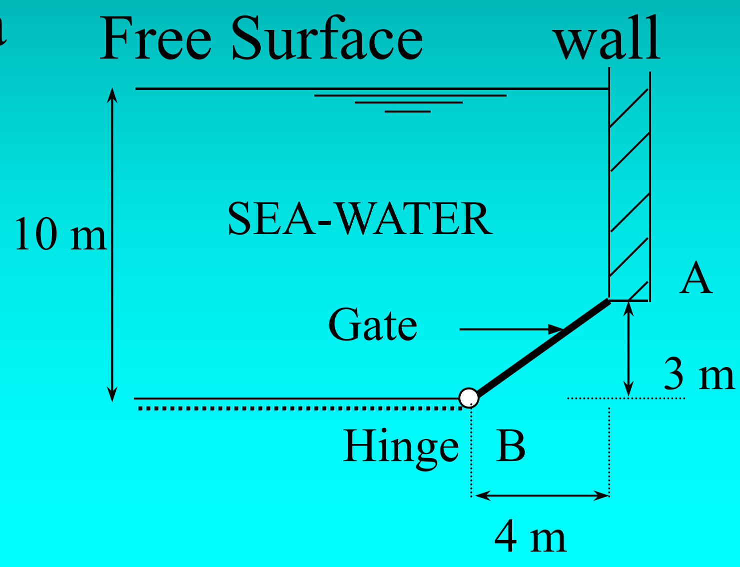
$$F = P A$$

$$F = \gamma A \bar{h}$$

$$y_p = \bar{y} + \frac{I_{C.G.}}{A \bar{y}}$$

Example:

A rectangular gate 2 m wide is hinged at point B and rests against a smooth Wall at point A as shown.



Calculate:

(a) The pressure force on the gate due To the seawater (s.g. = 1.034).

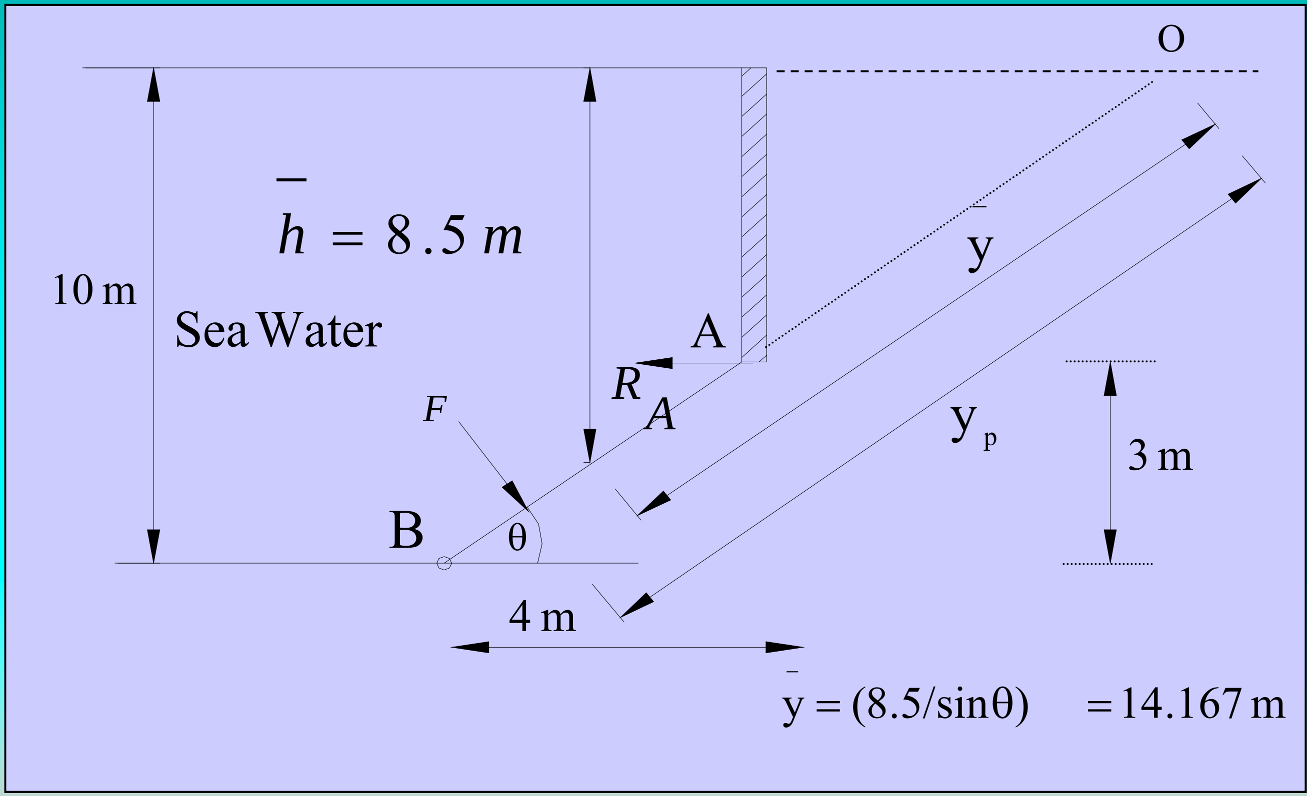
(b) The force exerted by the wall at A.

(c) The force at hinge B.

(d) The location of the center of pressure

N.B. Neglect weight of gate".

Solution:

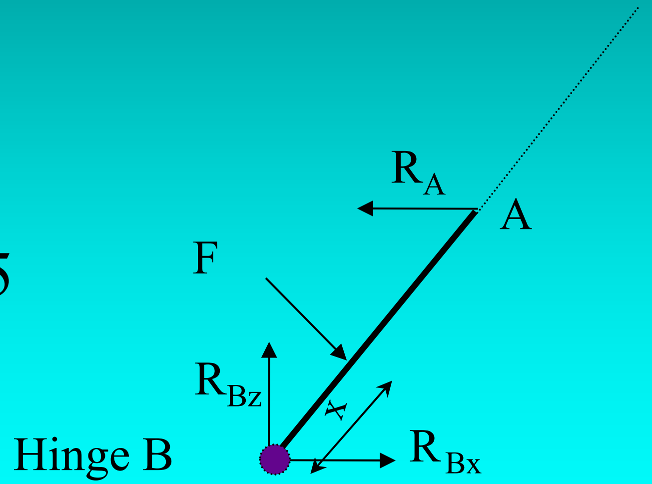

O
$\bar{h} = 8.5\ m$
10 m
Sea Water
A
R
A
F
B
θ
$\bar{y}$
y_p
3 m
4 m
$\bar{y}$ = (8.5/sinθ) = 14.167 m

(a) To find the pressure force on the gate:

$$F = \gamma A \bar{h}$$

$$= 1.034 \times 9.8 \frac{KN}{m^3} \times (5 \times 2) \times 8.5$$

$$= \underline{\underline{861.3\,KN}}$$



X = (10/0.6) – 14.314 = 2.353 m

(b) To find the force at A Moment about the hinge B = 0

$$F \times 2.353 - R_A \times 3 = 0 \quad \Rightarrow \quad R_A = \underline{\underline{675.5\,KN}}$$

(c) To find the force at B

$$\sum F_x = 0 \quad \Rightarrow \quad R_{Bx} + F\sin\theta - R_A = 0$$

$$R_{Bx} = \underline{\underline{158.7\ KN}}$$

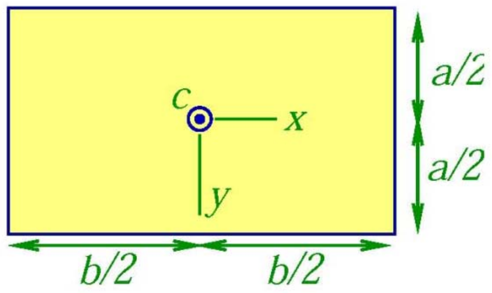
$$\sum F_z = 0 \quad \Rightarrow \quad R_{Bz} - F\cos\theta = 0$$

$$R_{Bz} = \underline{\underline{689.04\ KN}}$$

(d) To find the location of the center of pressure

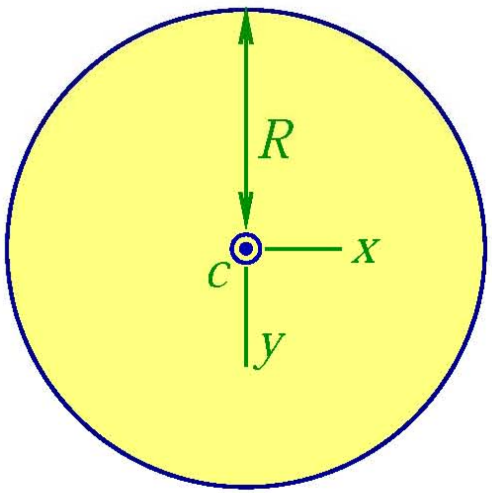
$$y_{cp} = \bar{y} + \frac{I_{C.G.}}{A\bar{y}} = \left(\frac{8.5}{\sin\theta}\right) + \frac{(2\times5^3/12)}{10\times14.167}$$

$$= \underline{\underline{14.314\ m}}$$



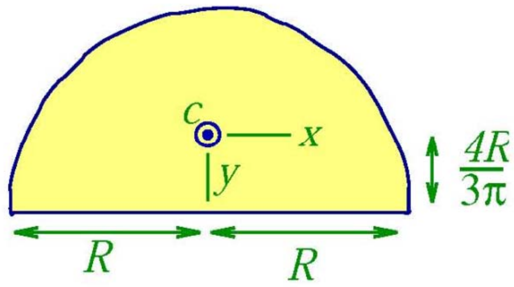
Rectangle

$$A = ba$$
$$I_{xc} = ba^3/12$$
$$I_{yc} = ab^3/12$$
$$I_{xyc} = 0$$



Circle

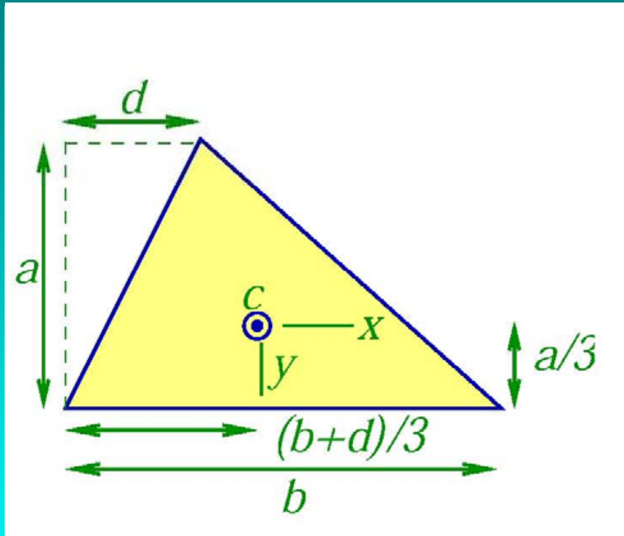
$$A = \pi R^2$$
$$I_{xc} = I_{yc} = \pi R^4/4$$
$$I_{xyc} = 0$$



Half-circle

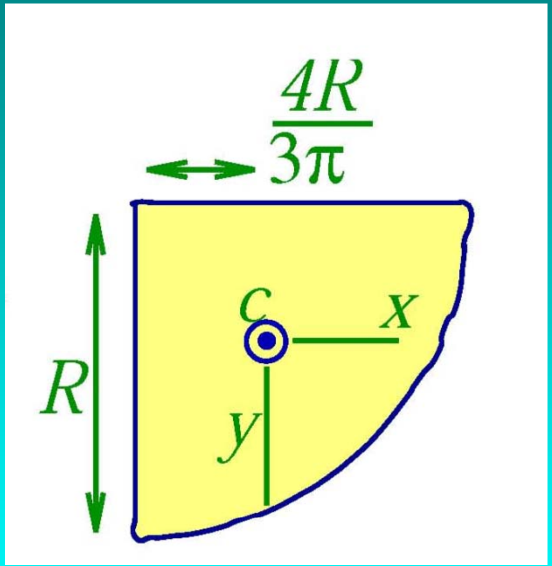
$$A = \pi R^2/2$$
$$I_{xc} = 0.1098R^4$$
$$I_{yc} = 0.3927R^4$$
$$I_{xyc} = 0$$

I_{xyc} is only non-zero if the shape does not have a bilateral symmetry.



Triangle

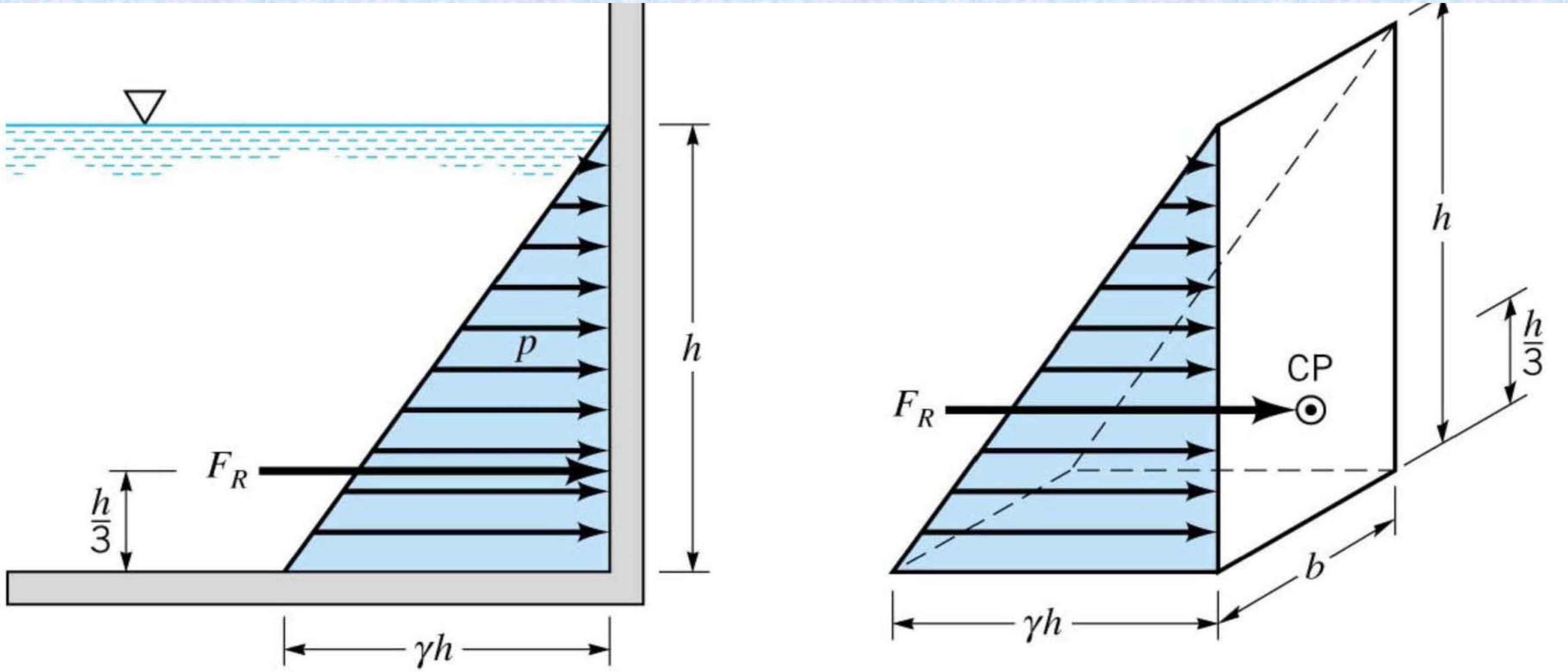
$$A = ba/2$$
$$I_{xc} = ba^3/36$$
$$I_{xyc} = ba^2(b-2d)/72$$



Quarter Circle

$$A = \pi R^2/4$$
$$I_{xc} = I_{yc} = 0.05488R^4$$
$$I_{xyc} = -0.01647R^4$$

I_{xyc} is only non-zero if the shape does not have a bilateral symmetry.



This is a intuitive recipe for determining the force on submerged surfaces. Useful for surfaces that are rectangular in shape.

- Gauge pressure is zero at top and γh at bottom.
- Pressure variation with h is linear.
- Average pressure (P) = γ h/2
- Resultant force F_r = (P)*A = γ(h*A/2)
- Volume of pressure prism (= γ h A/2) .
- The center of pressure passes through the centroid of the pressure prism.