

Faculty of engineering  
3rd year

11  
-1/0  
5

# structural analysis

No (1)

عزیز الدین

The term will contain:-

chapter one moment dist<sup>n</sup> method

Chapter two approximate method

mid  
term

Chapter three plates

Chapter four shells



## Moment distribution method

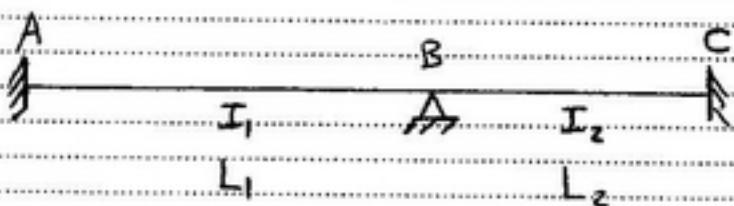
طريقة لتوزيع العزوم Indeterminate, مسندة

الطريقة مخصصة لـ Frames & beams

يقدر هذه الطريقة توزيع العزوم عند نقطة التلاقح بين الأجزاء حسب Inertia لكل جزء.

Steps of solution :-

1- Calculate relative stiffness (R.S)



AB : BC

$$\frac{I_1}{L_1} : \frac{I_2}{L_2}$$

2- Calculate of distn factor (D.F)

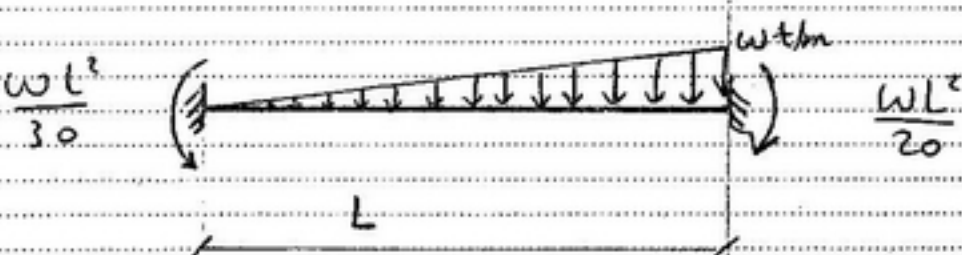
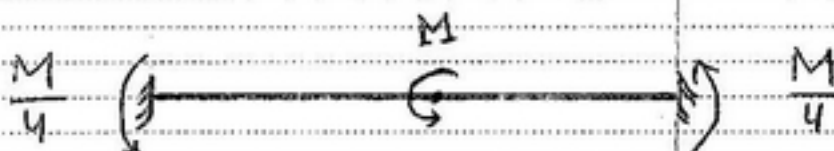
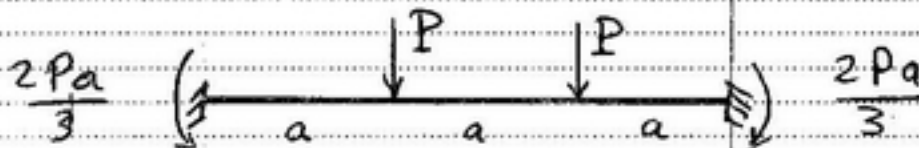
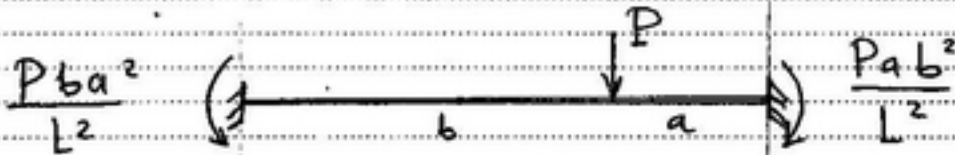
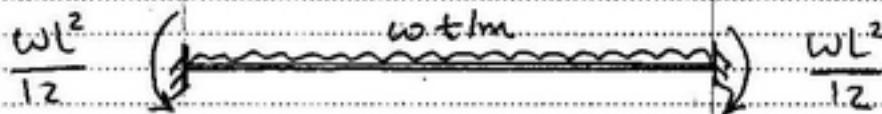
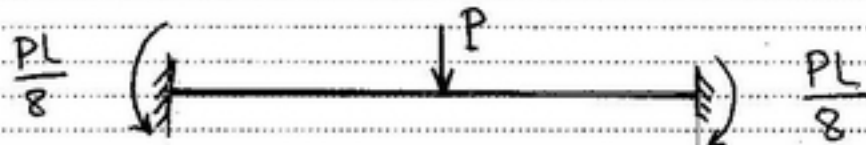
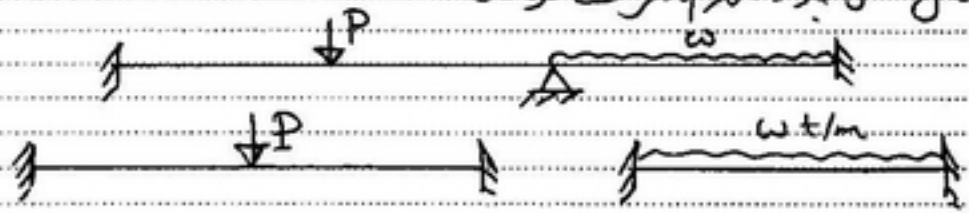
$$\frac{\left(\frac{I_1}{L_1}\right)}{\left(\frac{I_1}{L_1}\right) + \left(\frac{I_2}{L_2}\right)} : \frac{\left(\frac{I_2}{L_2}\right)}{\left(\frac{I_1}{L_1}\right) + \left(\frac{I_2}{L_2}\right)}$$

$$F_1 : F_2$$

(9)

### 3. Calculate of fixed End moment:- (F.E.M.)

یہ جدول کو جزوہ الحکرات اوصاف



(+ve) or (-ve) as per sign convention

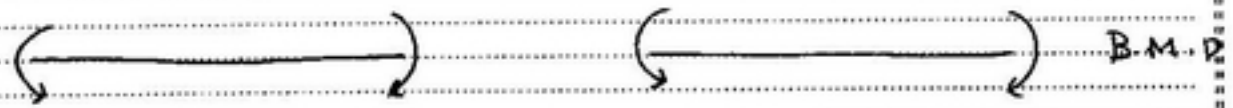
(+ve) or (-ve) as per sign convention







\* انما عسى ان يجدوا له مخرج اعز من الحقيقة على الكبريات

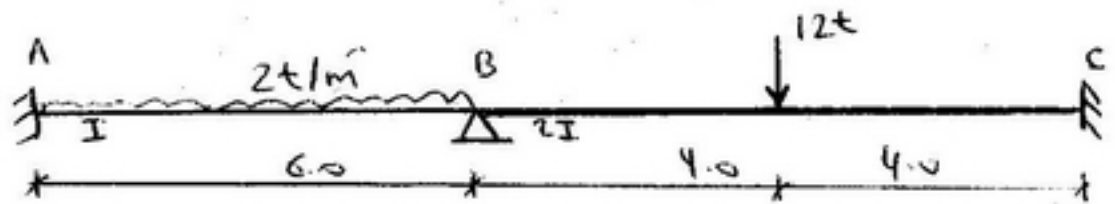


final moment

فسيه ال f.B.D ← السج 1 f.f.D  
free body diagram

free body diagram

## Example



by using moment distribn method

draw :- N.f.D, s.f.D, B.M.D

and so on draw elastic curve  
& Reinforcement.

— Sol —

R.S

$$K_{AB} : K_{BC}$$

$$\frac{1}{6} : \frac{2}{8}$$

$$\frac{1}{6} : \frac{1}{4}$$

$$\frac{1}{3} : \frac{1}{2}$$

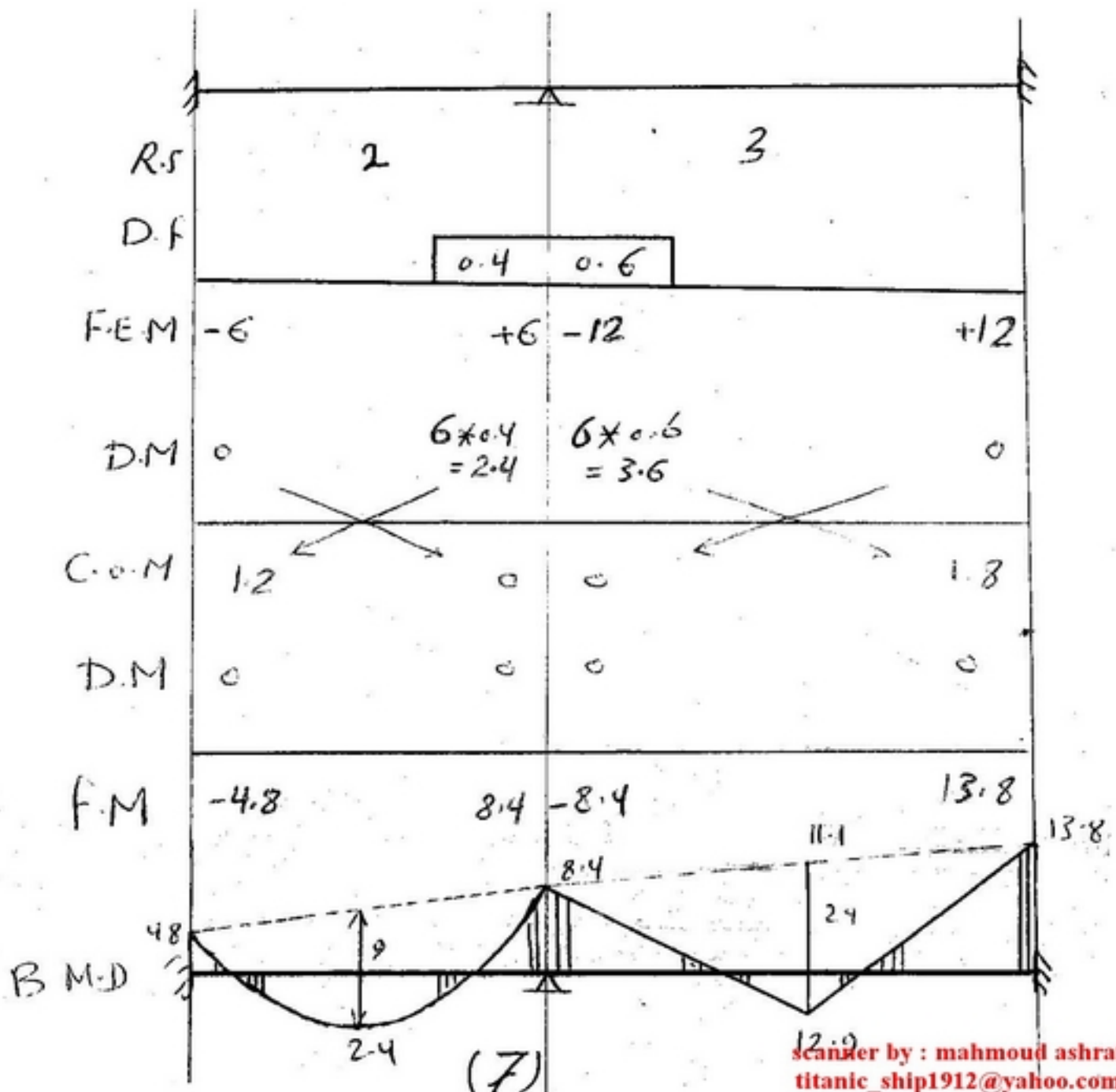
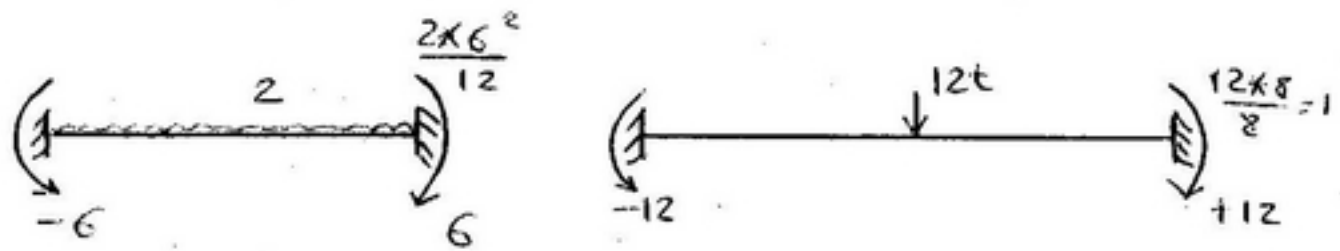
$$2 : 3$$

D.f

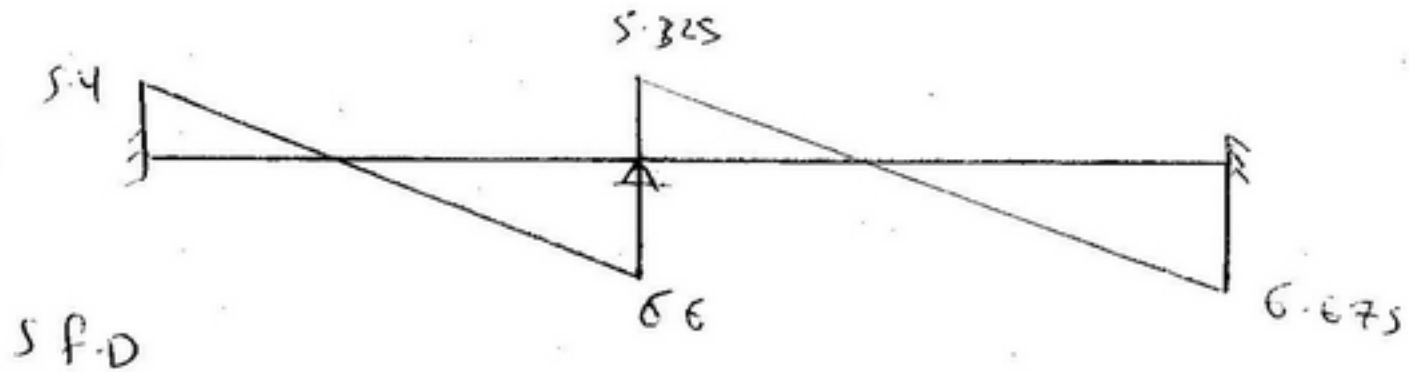
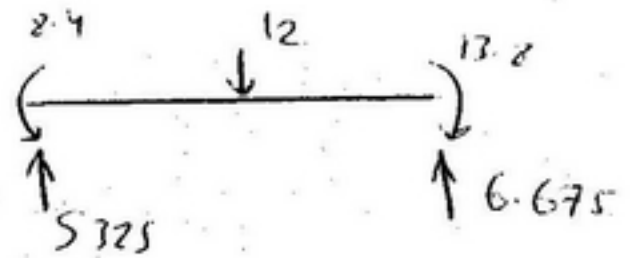
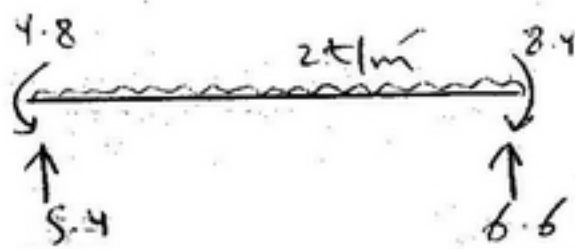
$$\frac{2}{5} : \frac{3}{5}$$

$$(A) 0.4 : 0.6$$

# Fixed End moment







Elastic Curve



Reinforcement

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**3rd year**

2  
w-1-b

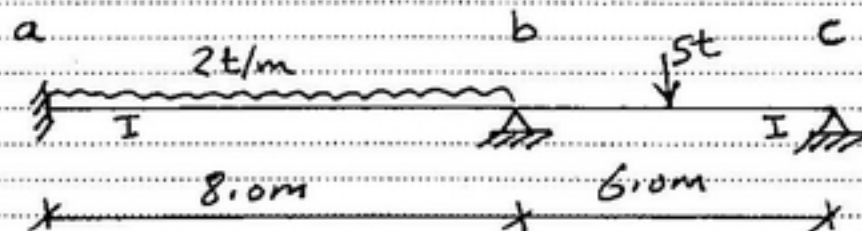
# **structural analysis**

**No ( 2 )**

Continue to moment dist'n

حالات التثبيت

II حالة مايلو المف (hinged support)



(i) لإختلاف الأبعاد

R.S

AB : BC

$$\frac{I_1}{L_1} : \left(\frac{3}{4}\right) * \frac{I_2}{L_2}$$

$$\frac{1}{8} : \frac{3}{4} * \frac{1}{6}$$

$$\frac{1}{8} : \frac{1}{8}$$

$$1 : 1$$

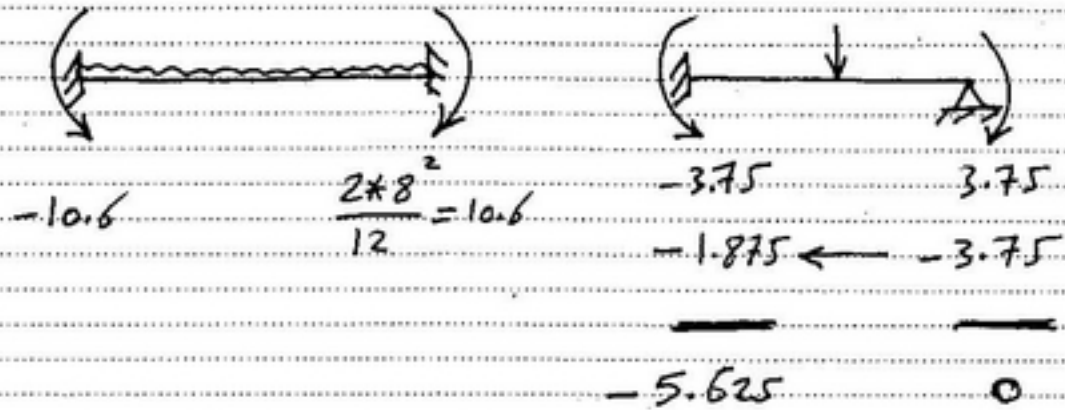
D-f

$$0.5 : 0.5$$

(1)



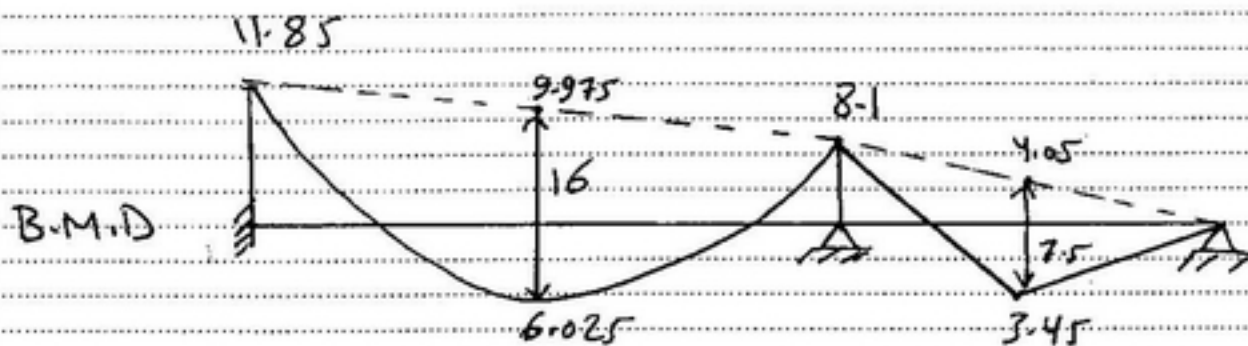
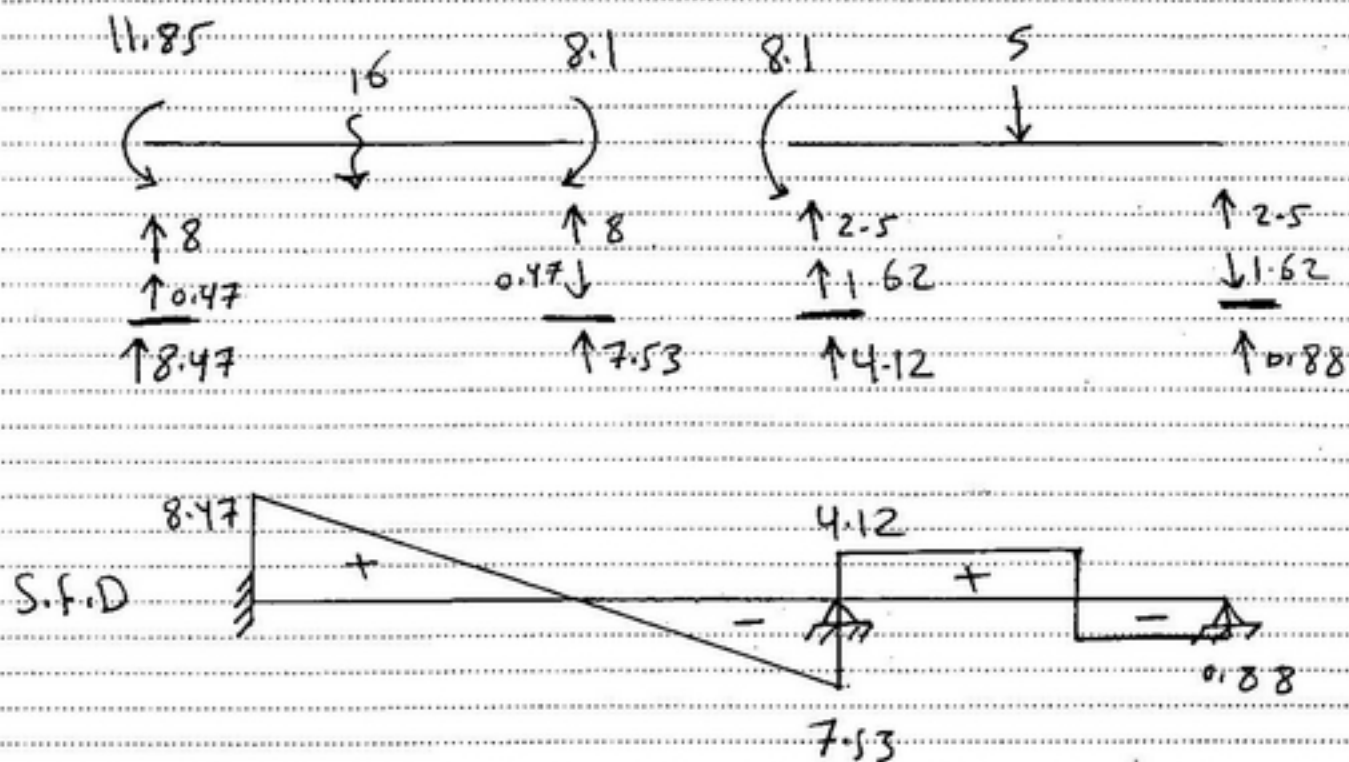
(ii) لإيجاد الانكسار



يتم وضع كل نصف لـ (F.E.M) عند (C) للحصول على عزم = 0

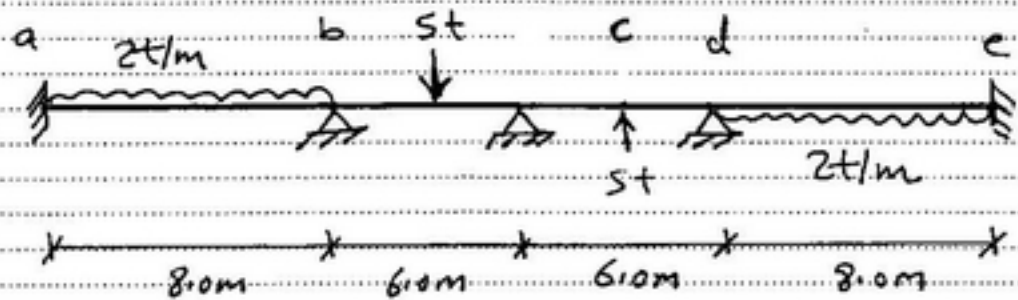
عند (C) ويتم تحويل نصف النصف من النصف الآخر عند B بـ C.O.M

D.F		0.5	0.5	
F.E.M	-10.6	10.6	-5.6	0
D.M	0	-2.5	-2.5	0
C.O.M	-1.25	0	0	0
D.M	0	0	0	0
f.m	-11.85	8.1	-8.1	0

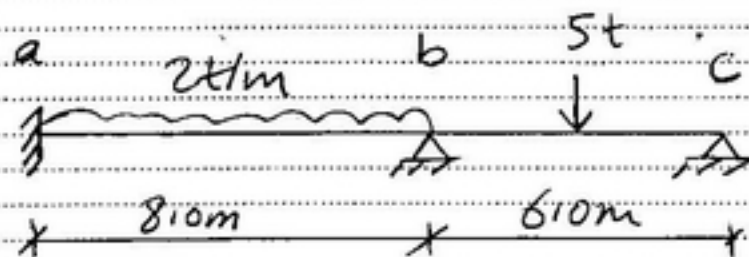




وجود قائل على Support



عندما يكون خط القائل على (support) في نقطة hinge  
 ← عند support في نقطة hinge

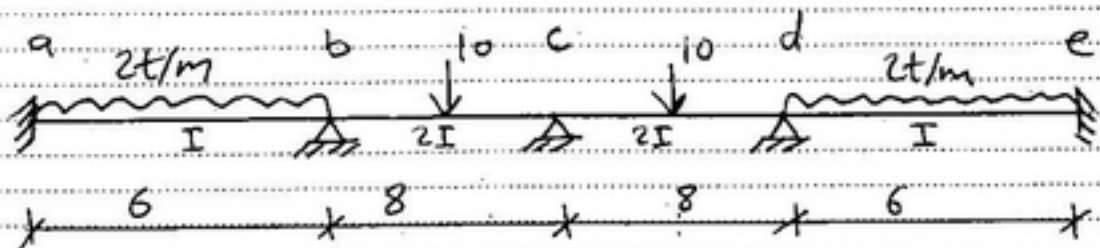


في نفس الحالة إذا كان (a) hinge

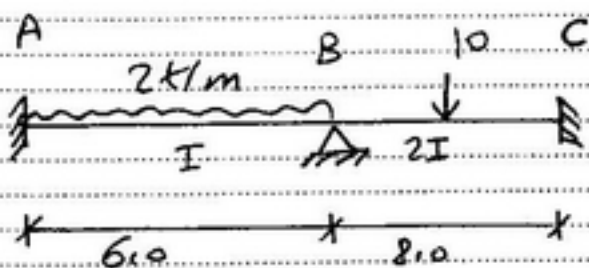
المادة



□ حالة ما يكون خط التماس غير support



يقع أيسر التماس فقط، مع support fixed



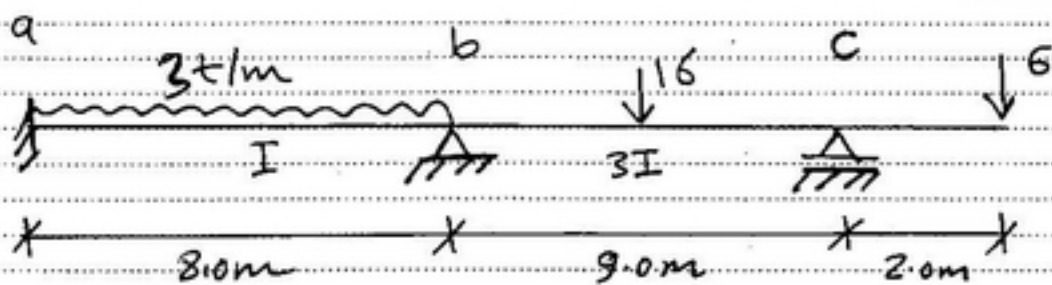
حل من التماس في الطرف (اليسار)

الخط

٤) حالة اربعة وجود (Cantilever) عند الطرف

نفس طريقة hinge الطرف ولكن يتم تسوية عزم

hinge بقية العزم لوجود كل Cantilever



— Solution —

R.5

AB

BC

$$\frac{1}{8} = \left( \frac{3}{4} \right) \times \frac{3}{9}$$

$$\frac{1}{8} = \frac{1}{4}$$

$$1 = 2$$

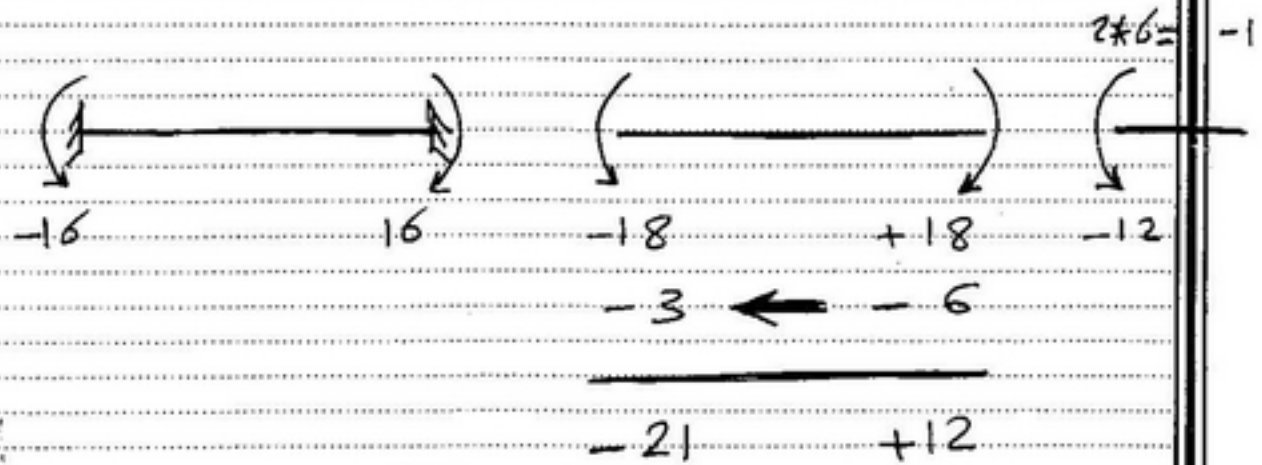
D.f

$$\frac{1}{3} = \frac{2}{3}$$

(6)

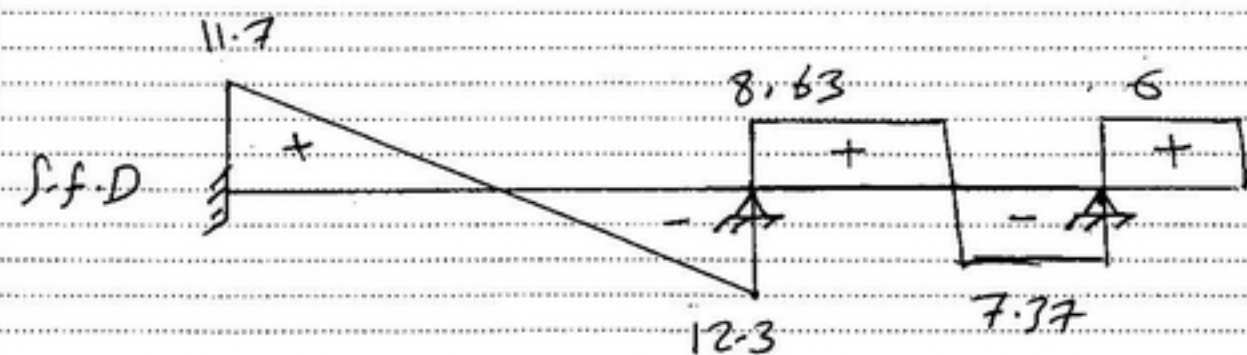
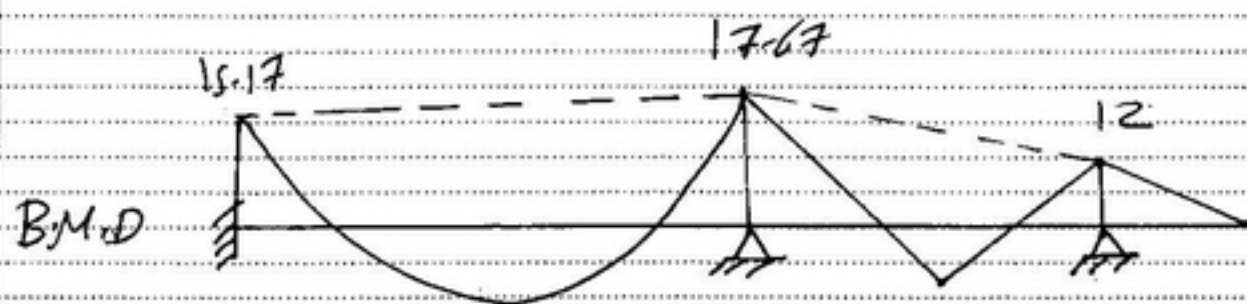
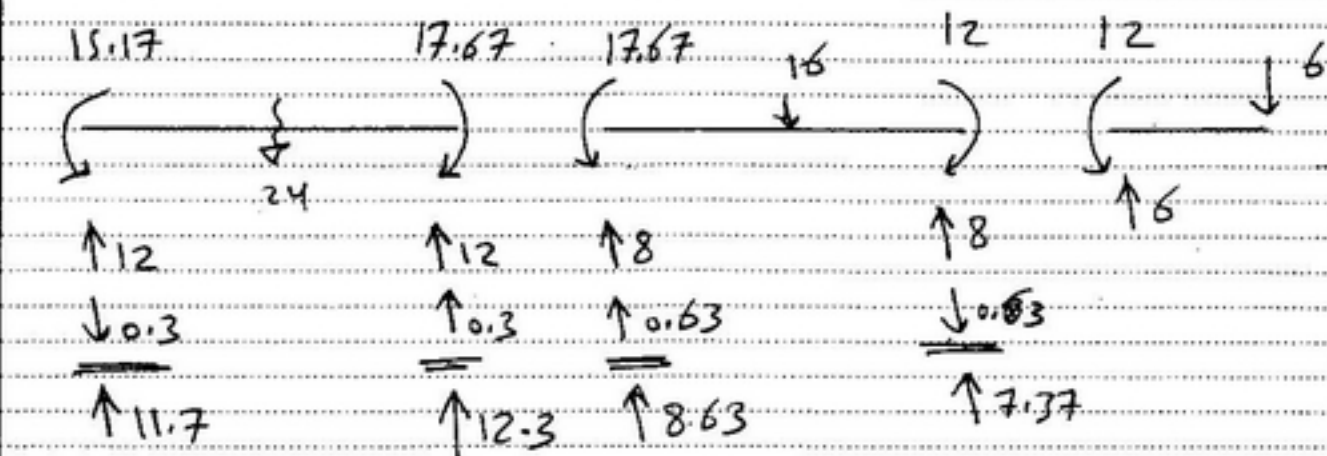


# F.E.M



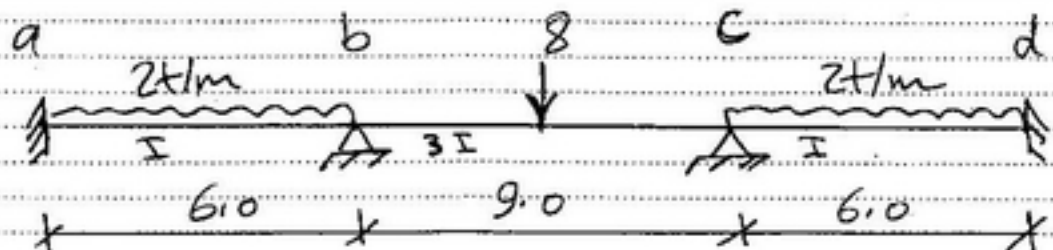
نہیں ہے (C) کے لیے اس لیے  
Cantilever ہے اس لیے

D.F		$\frac{1}{3}$	$\frac{2}{3}$	
F.E.M	$-16$	$+16$	$-21$	$+12$
D.M	$0$	$\frac{5}{3}$	$\frac{10}{3}$	$0$
C.O.M	$2\frac{2}{3}$	$0$	$0$	$0$
D.M	$0$	$0$	$0$	$0$
f.m	$-15.17$	$17.67$	$-17.67$	$12$





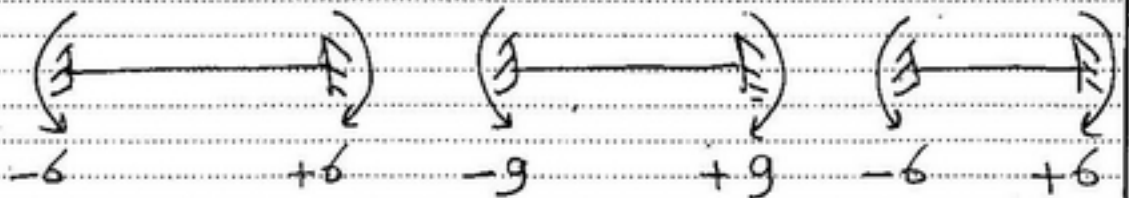
□ حالة مسانين في نفس بنية a



القطاعات متساوية  $R_{ab} = R_{bc}$

$$\frac{I_1}{L_1} = \left(\frac{1}{2}\right) \frac{I_2}{L_2}$$

← حالة واحدة ← F.E.M ← كما في المثال



R.S

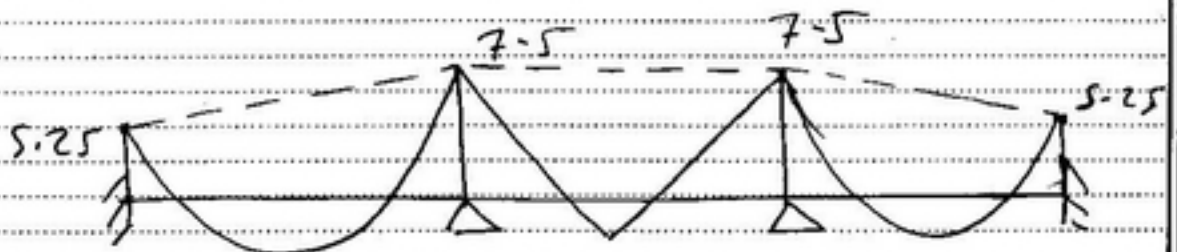
$$\frac{1}{6} : \left(\frac{1}{2}\right) \times \frac{3}{9}$$

$$1 : 1$$

D.f

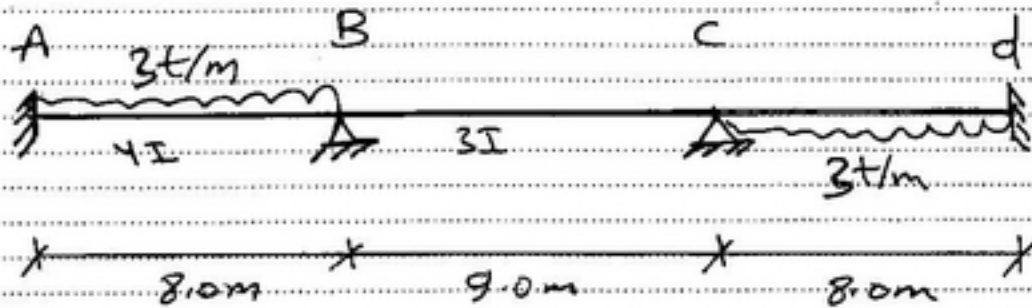
$$0.5 : 0.5$$

		0.5	0.5	
F.E.M	-6	+6	-9	
D.M	0	1.5	1.5	
C.O.M	0.75	0	0	
D.M	0	0	0	
F.M	5.25	7.5	-7.5	





طالع رقم 7 حالة وجود خط لاف كس



D-f

R.5

$$\left(\frac{I_1}{L}\right) \div \left(\frac{3}{2}\right) \times \frac{I_2}{L_2}$$

$$\frac{4}{8} \div \frac{3}{2} \times \frac{3}{9}$$

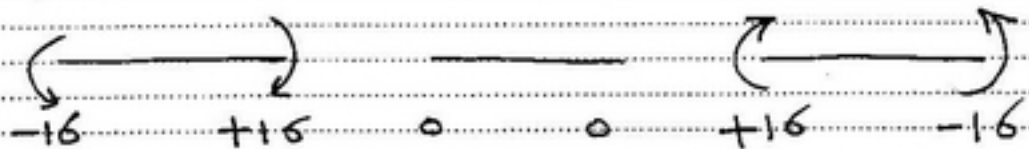
$$\frac{1}{2} \div \frac{1}{2}$$

$$1 \div 1$$

D-f

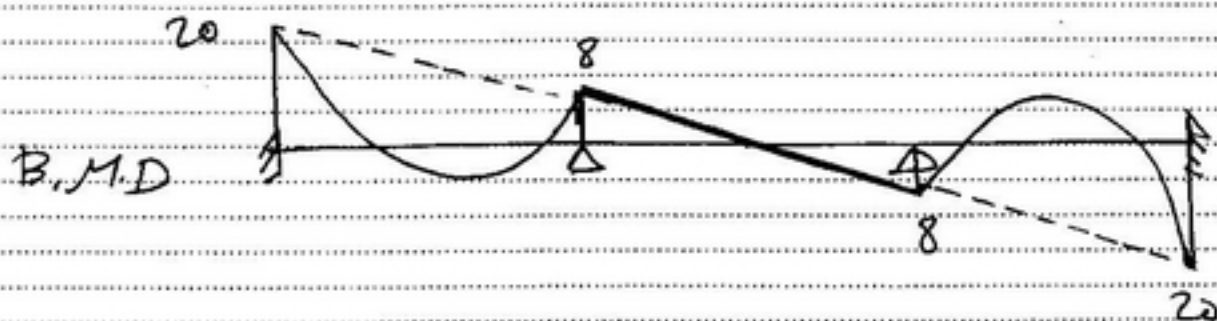
$$0.5 \div 0.5$$

F-GM



7

D.F		0.5	0.5
F.E.M	-16	+16	0
D.M	0	-8	-8
C.M	4	0	0
D.M	0	0	0
f.M	-20	+8	-8





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# structural analysis

No ( )

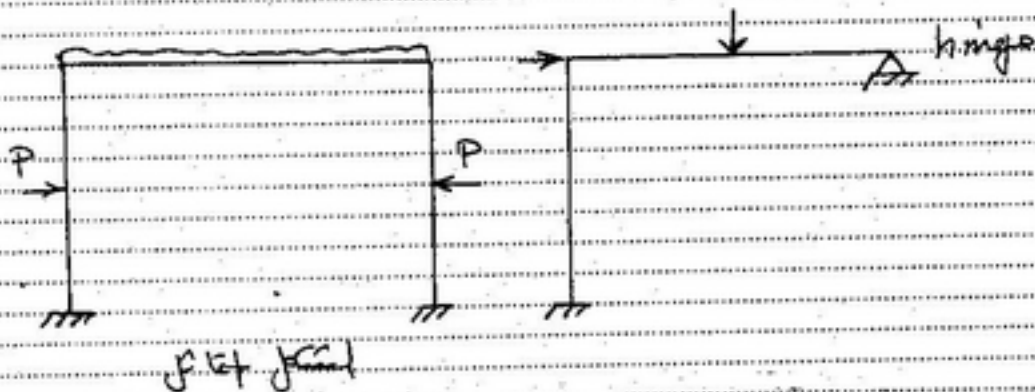
بسیار از آن ها

## Frame

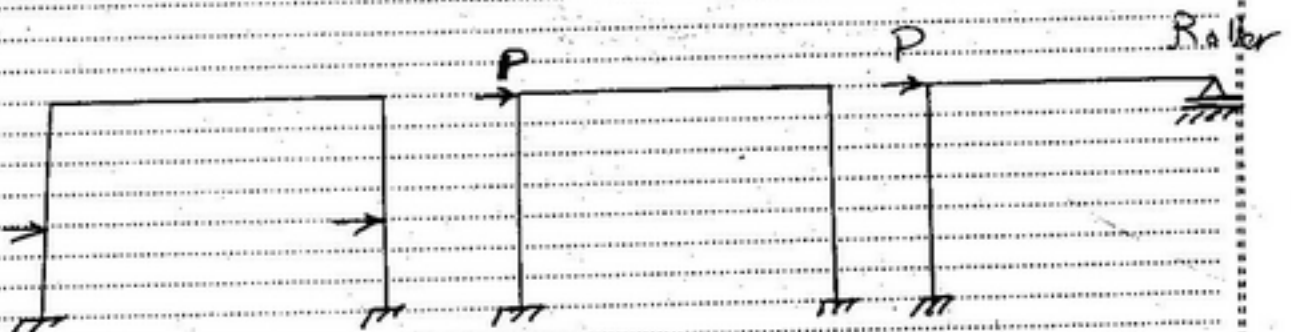
types of frames :-

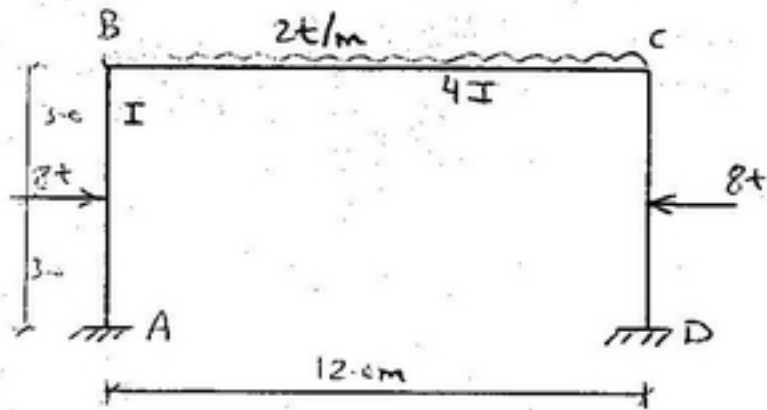
① frame without sway

در این نوع چارچوب، بارها را به صورت عمودی اعمال می‌کنیم.



② frame with sideway



Σ Beispiele

for the following frame

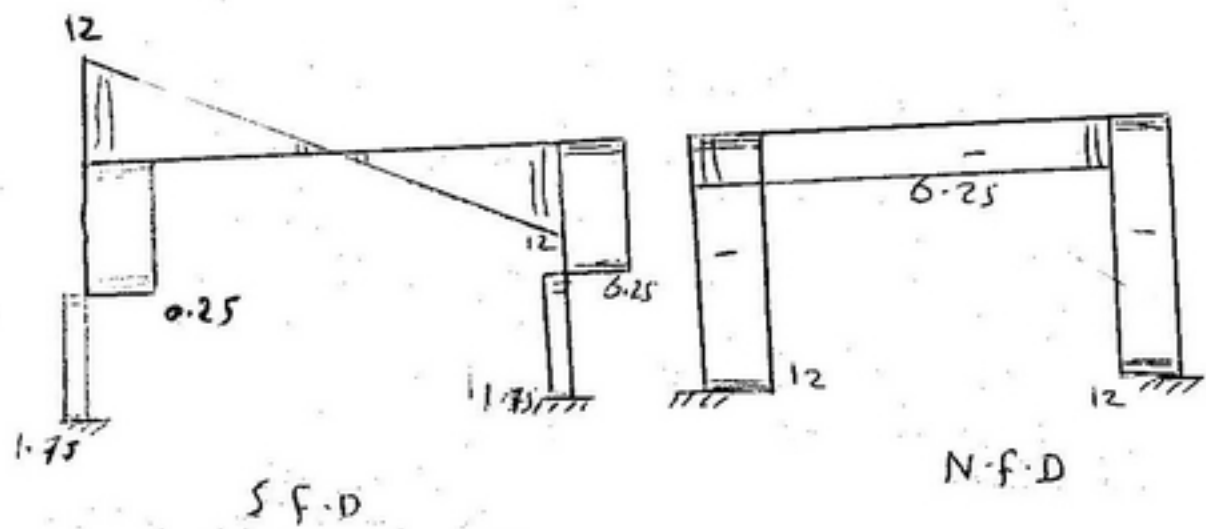
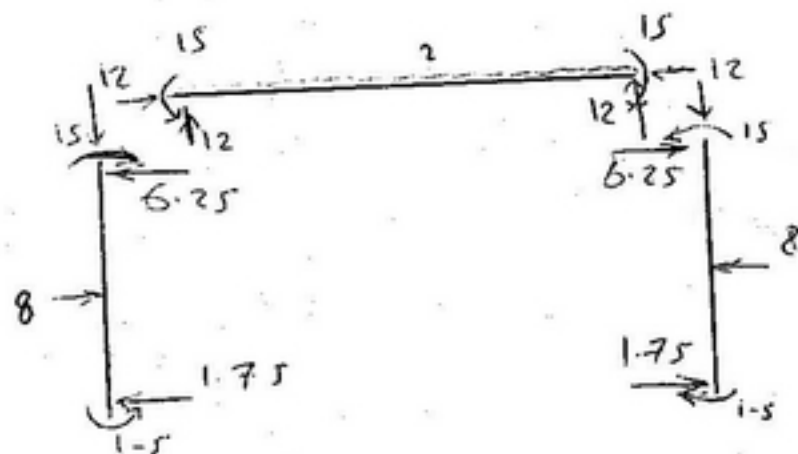
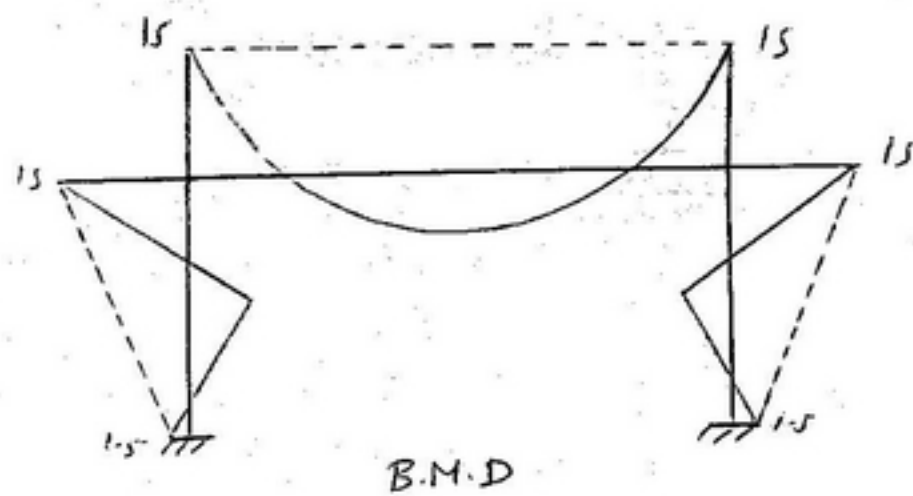
draw B.M.D, S.F.D, N.F.D

—sol—

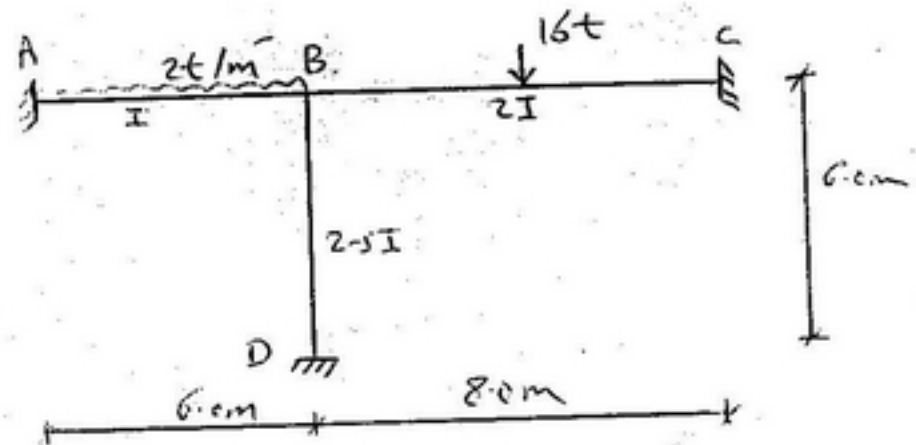
Synonymy

Diagram of a continuous beam with three spans. The first span is 6m, the second is 6m, and the third is 6m. A uniformly distributed load of 1 kN/m is applied over the entire length. The beam is supported by a fixed support at the left end, a roller support in the middle of the second span, and a fixed support at the right end. The diagram shows the beam with downward arrows indicating the load and upward arrows indicating the supports.





# Example



— Sol —

$$BA : BC : BD$$

$$\frac{1}{6} : \frac{2}{8} : \frac{2.5}{6}$$

$$\frac{1}{3} : \frac{1}{2} : \frac{1}{1.2}$$

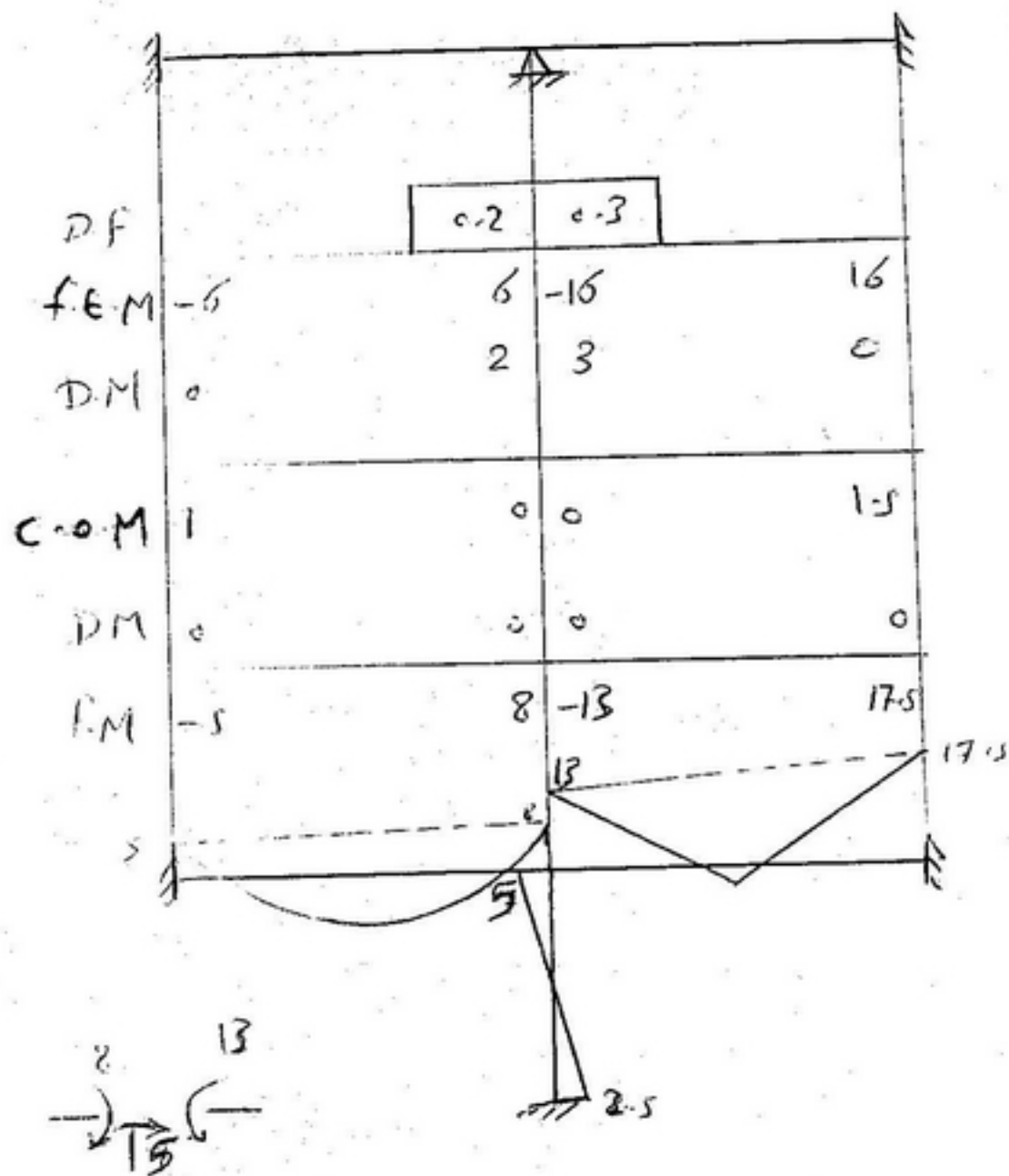
$$0.2 : 0.3 : 0.5$$

D.F

\* لا ننسى توزيع (D.F)  
بين المفاصل (members)  
لأنه مرتبط بين المفاصل (B)



طالما هذا العنصر لا يوجد عليه أحمال  
عليه الله صحت النظرية  
أحد من مبادئ الهندسة



5 → تم حذف دایره و به زیره اندازم  
2.5 → (C.O.M)

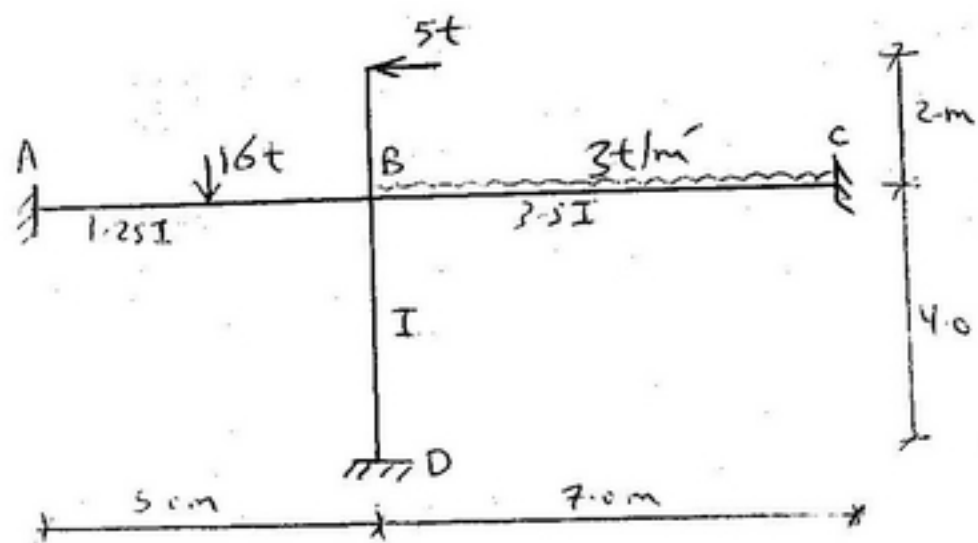


hinge - بر محل

Roller - D.M. 0

طالع درگاه الحرف ایشان به فزاد BD





— SOL —

D.f

AB : BC : BD

$$\frac{1.25}{5} : \frac{3.5}{7} : \frac{1}{4}$$

$$\frac{1}{4} : \frac{1}{2} : \frac{1}{4}$$

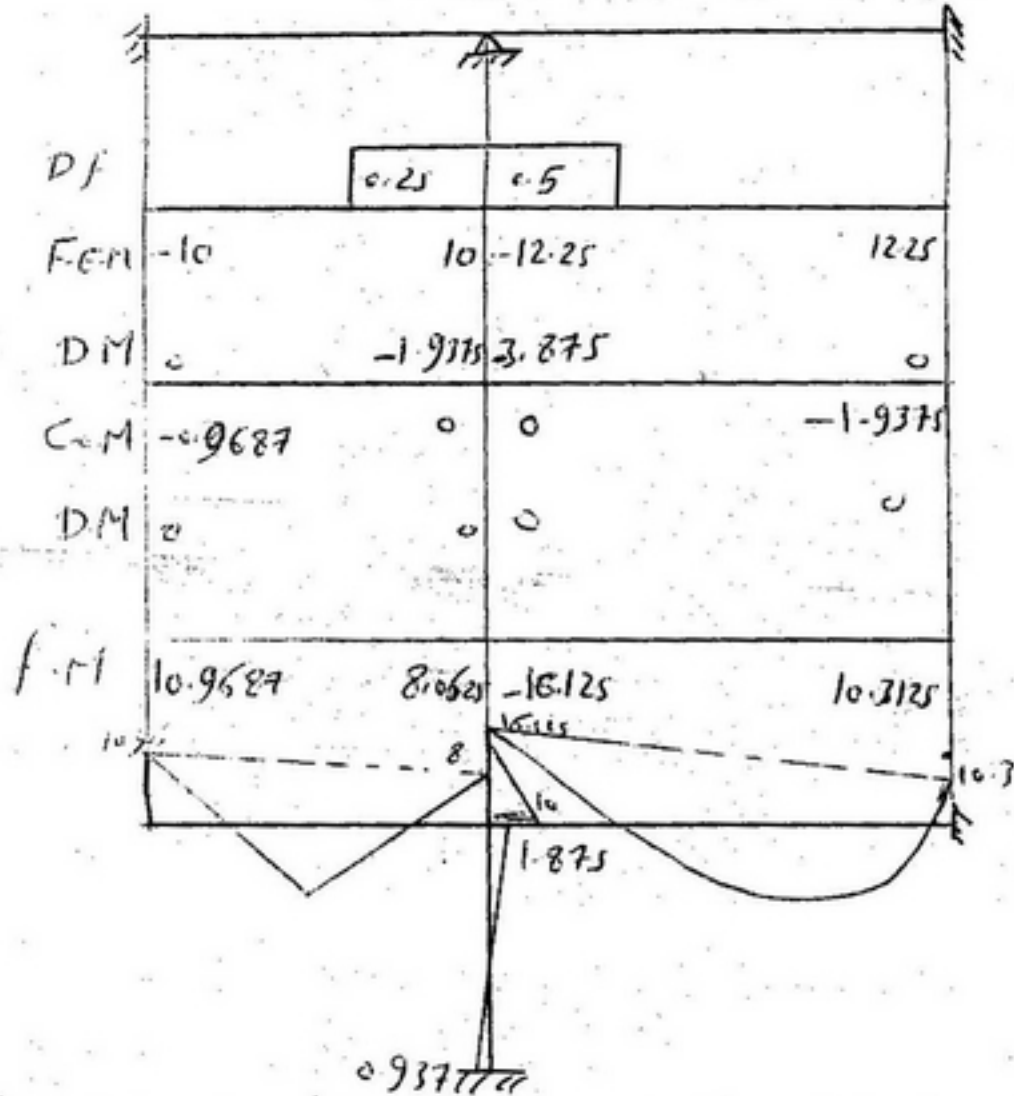
$$0.25 : 0.5 : 0.25$$

F.E.M

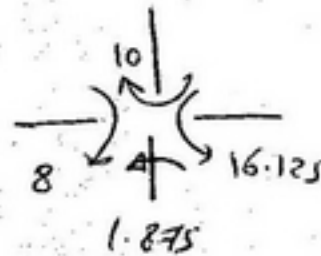
$$\begin{array}{c} -10 \qquad 10 \\ \curvearrowleft \text{---} \text{---} \curvearrowright \end{array}$$

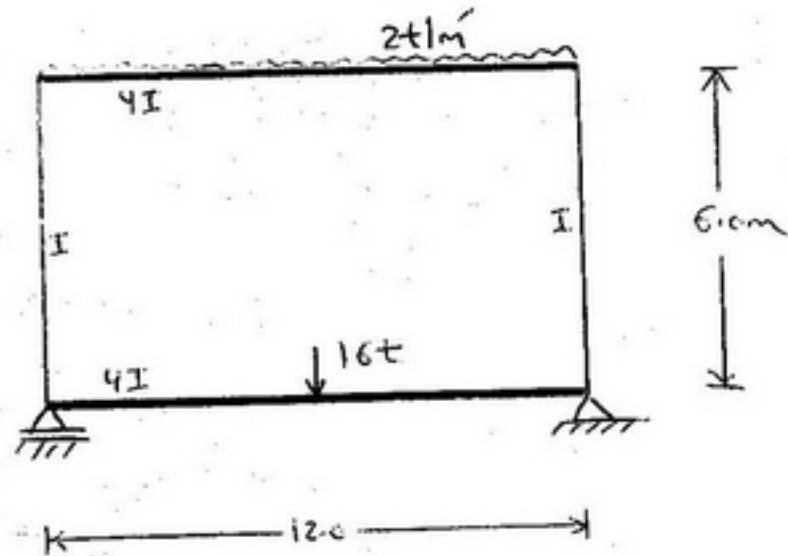
$$\begin{array}{c} -12.25 \qquad 12.25 \\ \curvearrowleft \text{---} \text{---} \curvearrowright \end{array}$$

لا حظ ان كل طرف من الطرفين  
يرتفع عن الوسط  
 $10 + 10 - 12.25$   
 $= 7.75$



B.M.D





for the following closed str-

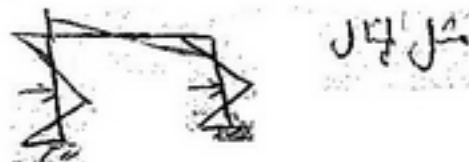
draw B.M.D, N.f.D, s.f.D

— sol —

← لاپظ سید منتخبت، اس یو صرظ نائل

← - اس یو صرظ نائل ککر (لا) -  
 (  $\begin{matrix} -24 & 24 \\ \hline \end{matrix}$  )  
 $\begin{matrix} -24 & 24 \\ \hline \end{matrix}$

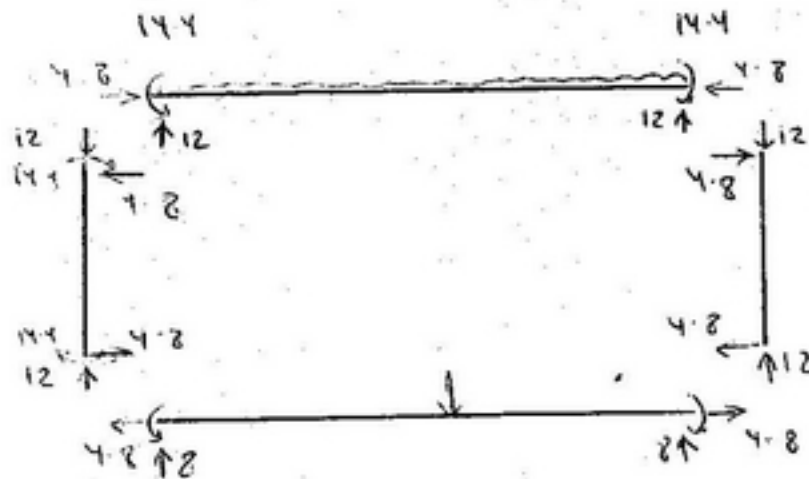
F.E.M نل برسیو متبادر غرضیو، پرتیو



نائل



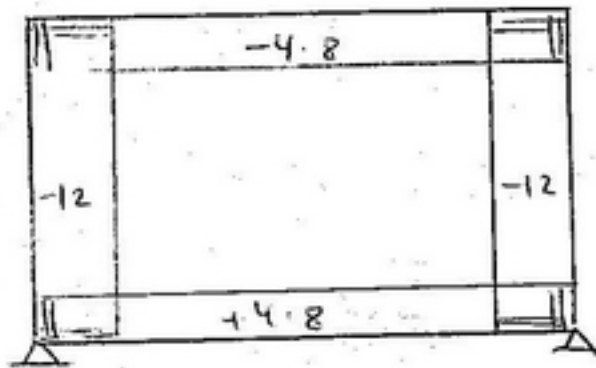




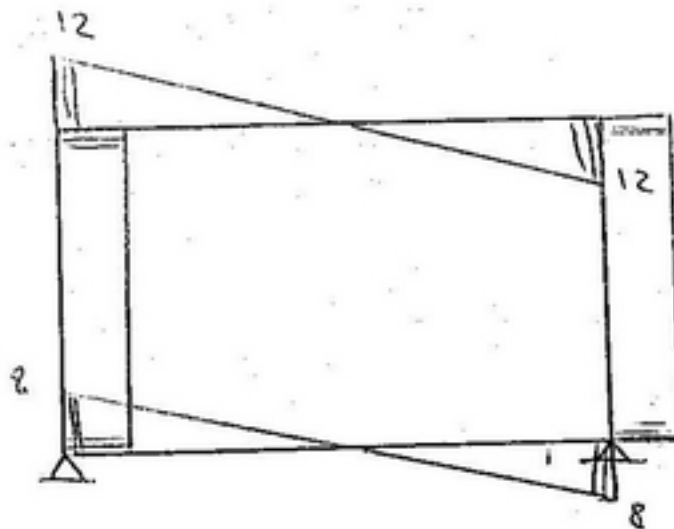
Reaction  
 $= 8 + 12 = 20$

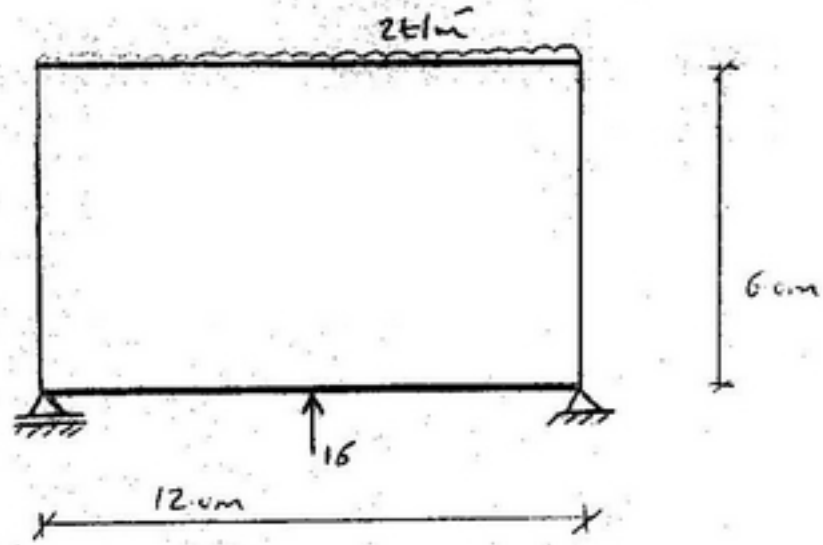


NfD

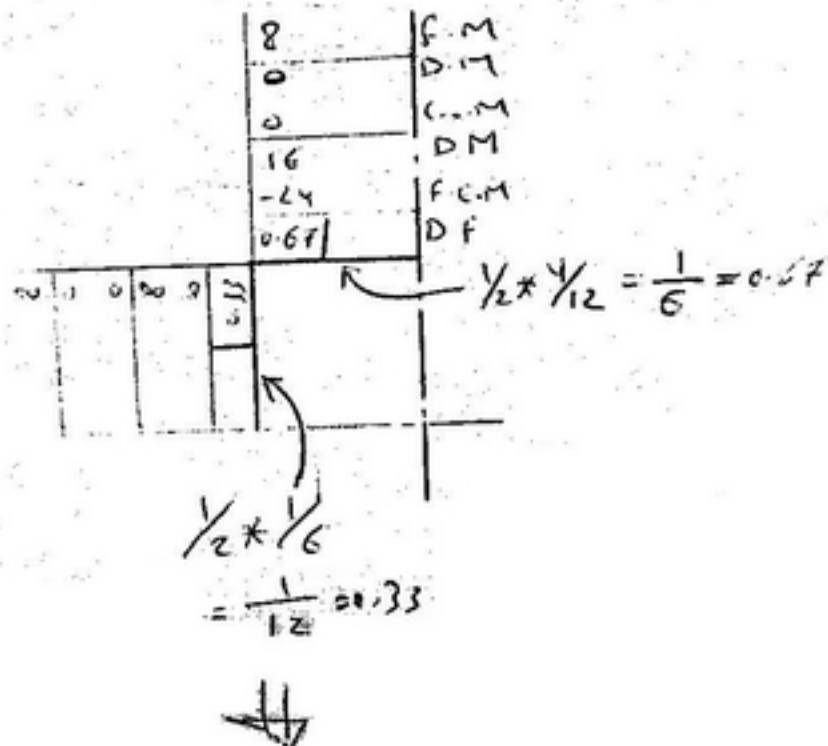


SfD





فناشیه ستاتیک الی تیر





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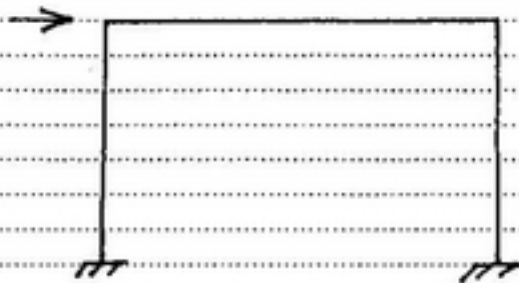
4  
1.8  
3

# structural analysis

No (4)

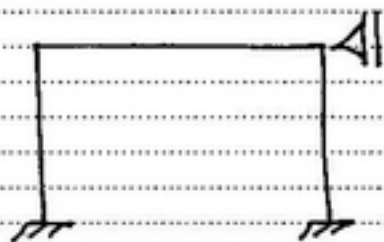
سواء الجانبي

## Frame with sidesway



(i)

يتم منع حركة في اتجاه sway  
(without) حركة في اتجاه



(M<sub>0</sub>)

حركة في اتجاه

R<sub>0</sub>

حركة في اتجاه

R<sub>0</sub>

(ii)

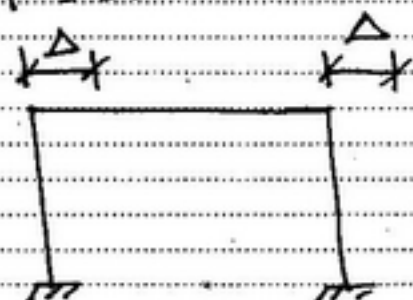
Δ = Δ<sub>sway</sub> ← حركة في اتجاه

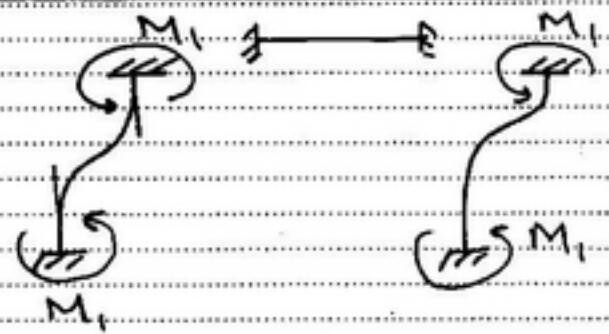
$$F.E.M \Rightarrow \frac{6EI}{L^2} \Delta$$

$$M = \frac{6EI}{L^2}$$

EI حركة في اتجاه

$$F.E.M = \leftarrow$$



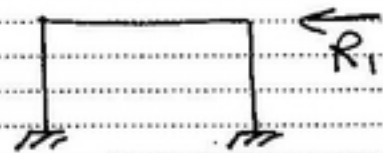


حالتی نادی



$M_1$

مقاومت  $R_1$

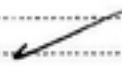


$R_1$

$$R_0 + \Delta \cdot R_1 = 0 \rightarrow$$

$$\Delta = \checkmark$$

$$m_f = m_0 + \Delta \cdot m_1$$

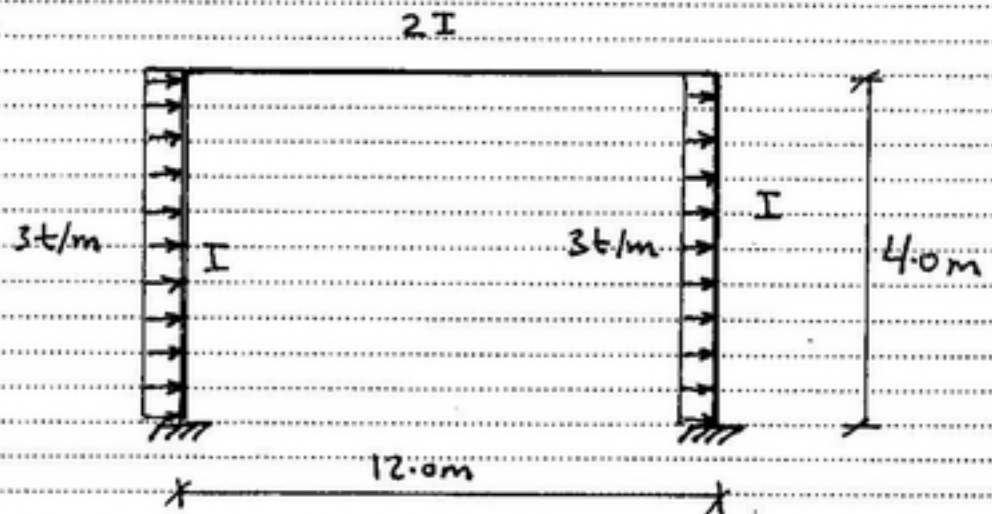




# Cont. to moment distribution

## Frame with sway

final 2005

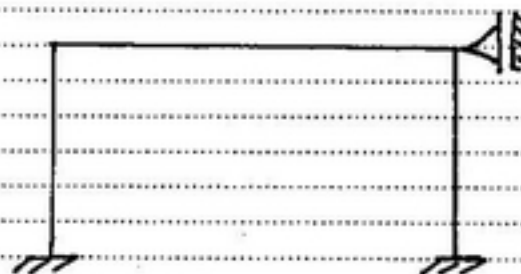


خطوات حل

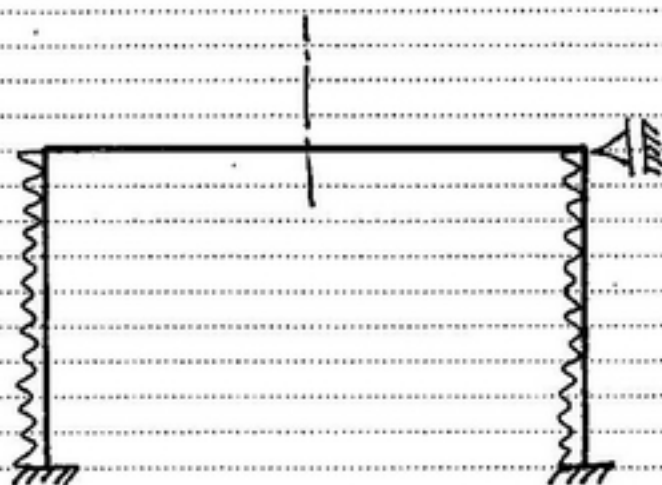
(1) يتم وضع support في أركان المثلثات

without sway ← frame 1

و هو frame 2



عاجل على



$$\frac{I}{L} = \frac{1}{4}$$

$$\frac{3}{2} \frac{I}{L} = \frac{3}{2} \times \frac{2}{12} = \frac{1}{4}$$



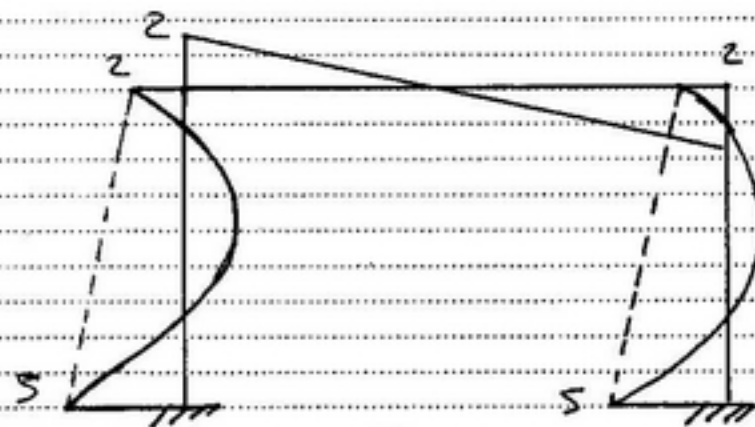
FEM	-4	+4	0
-----	----	----	---

D.M	0	-2	-2
-----	---	----	----

GOM	-1	0	0
-----	----	---	---

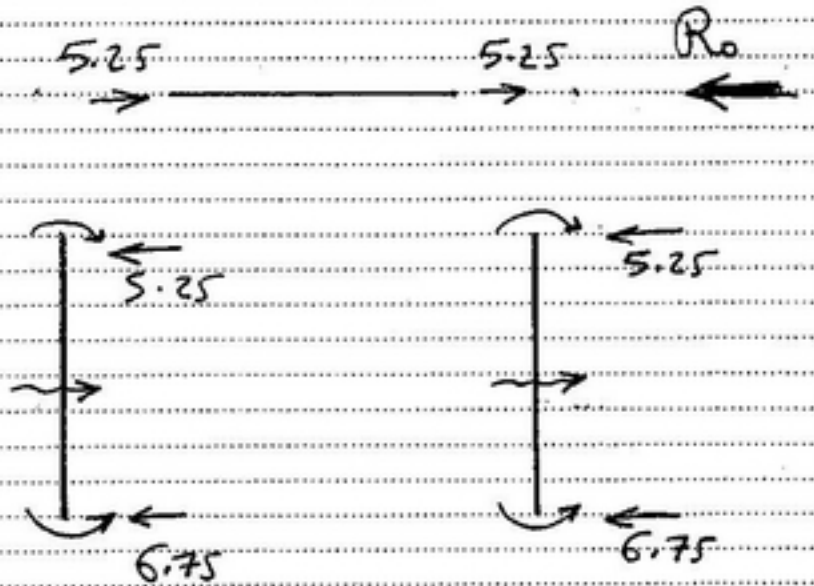
D.M	0	0	0
-----	---	---	---

f.M	-5	+2	-2
-----	----	----	----



$M_0$





$$R_0 = 5.25 \times 2 = 10.5 \text{ ton} \leftarrow$$

الخطوة الثانية

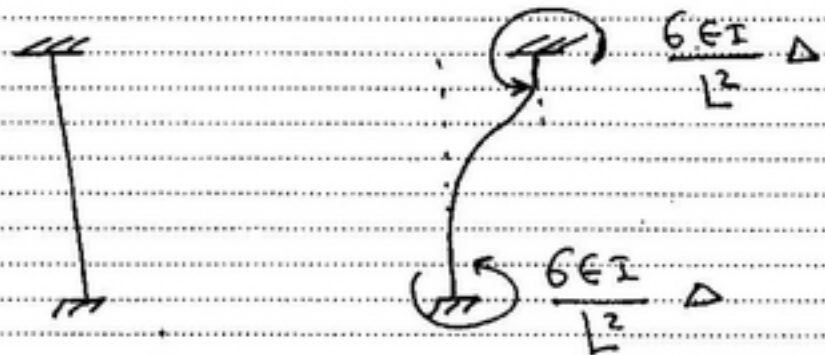
يتم إزالة الأحمال وتحرير sway  $\Delta$  من المفاصل  
وعلى أساس نقل لحالاته

F.E.M  $\Rightarrow$   $\Delta$  ثابت

$$M = \frac{6EI}{L^2} \Delta$$



\* تأثير  $\Delta$  عند تكونه عمودياً على العنصر

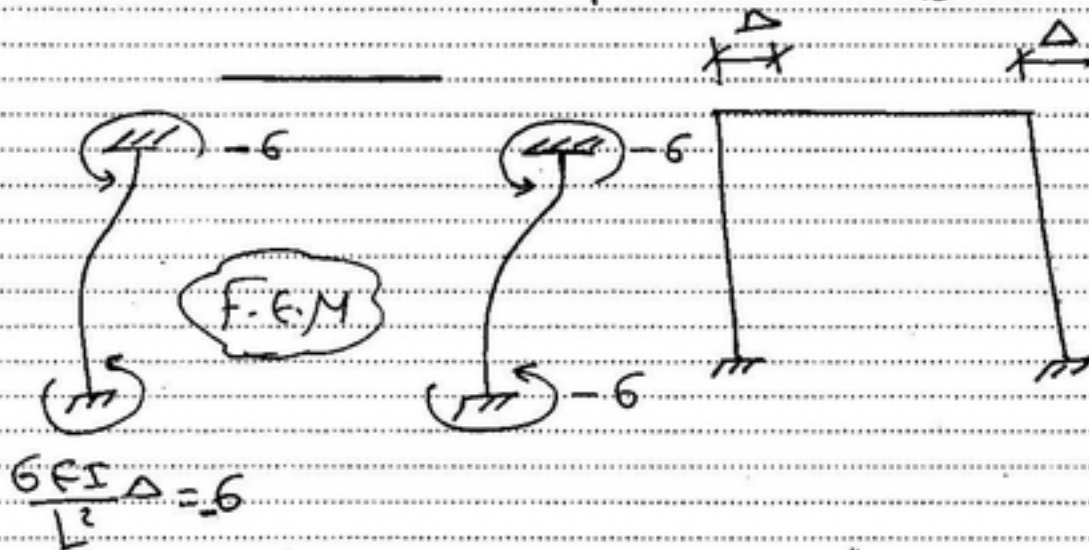


$$\Delta = 1 \text{ cm}$$

لنمضِ  $L^2 = EI$  للتبسيط فقط

أرأي، قسم

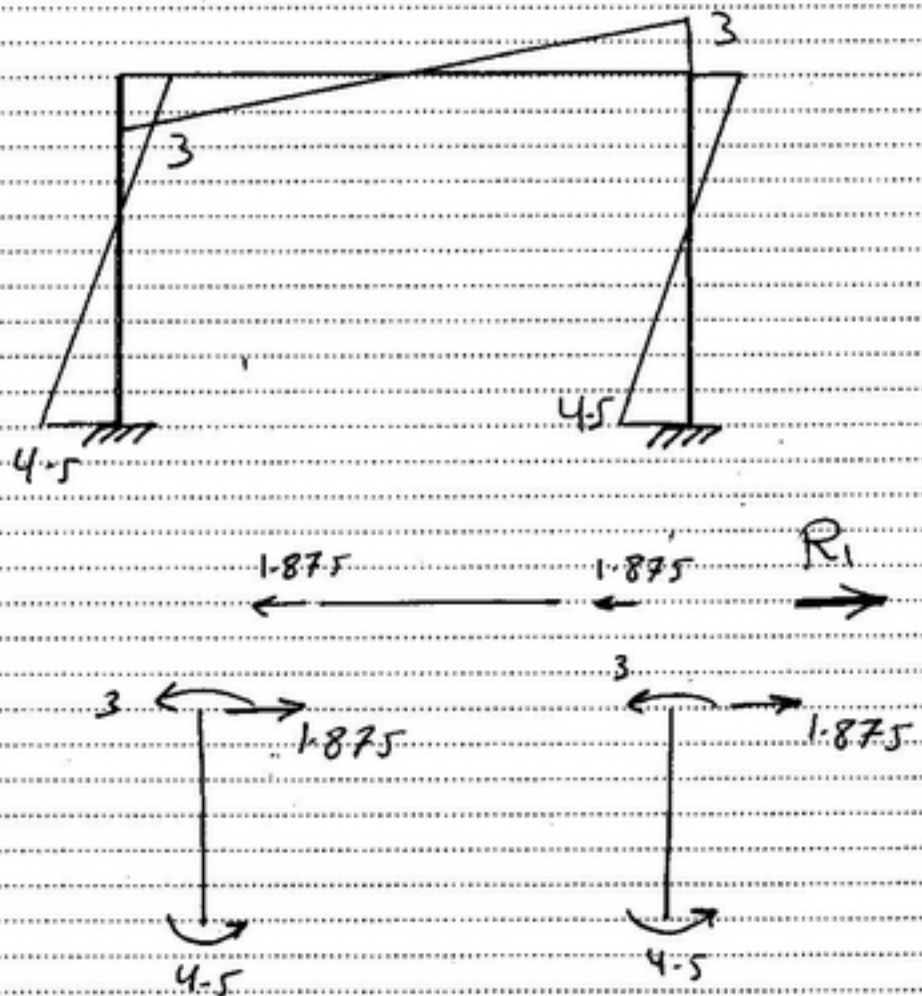
\* لا تأثير  $\Delta$  من العنصر الموازي لها



من الملاحظ أن العنصر الموازي لها

لا يتأثر بتأثيره

		0.5	0.5
			A
F.E.M	-6	-6	0
D.M	0	3	3
C.O.M	1.5	0	0
D.M	0	0	0
f.M	-4.5	-3	+3





$$R_1 = 1.875 \times 2 = 3.75 \rightarrow$$

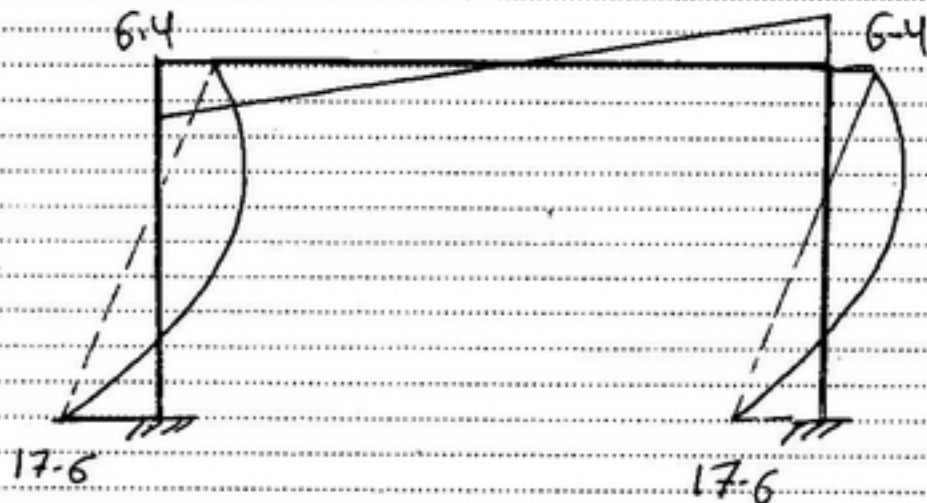
م قابل

$$R_0 + \Delta \cdot R_1 = 0 \rightarrow$$

$$10.5 - 3.75 \Delta = 0 \rightarrow$$

$$\Delta = 2.8$$

$$m_f = m_0 + 2.8 m_1$$



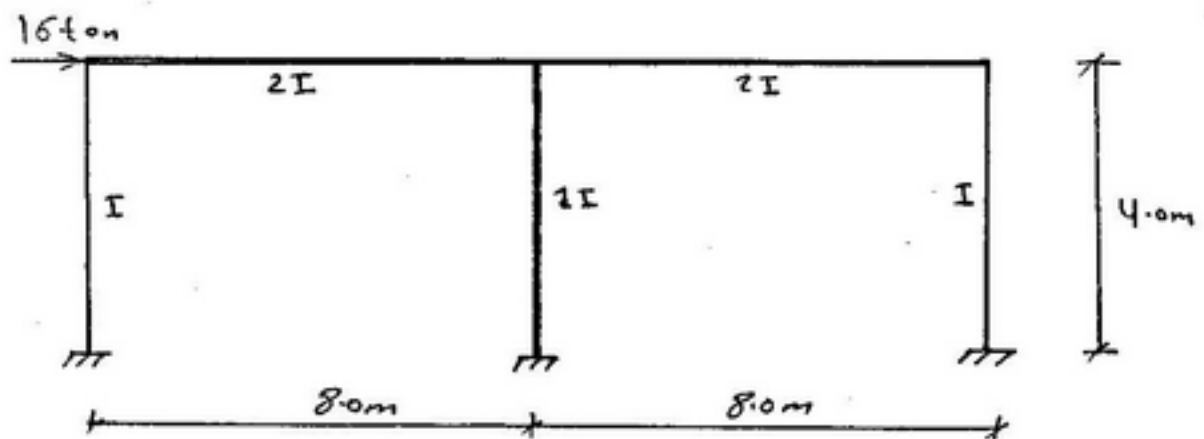
$m_f$



For the following frame solve it twice:-

2006

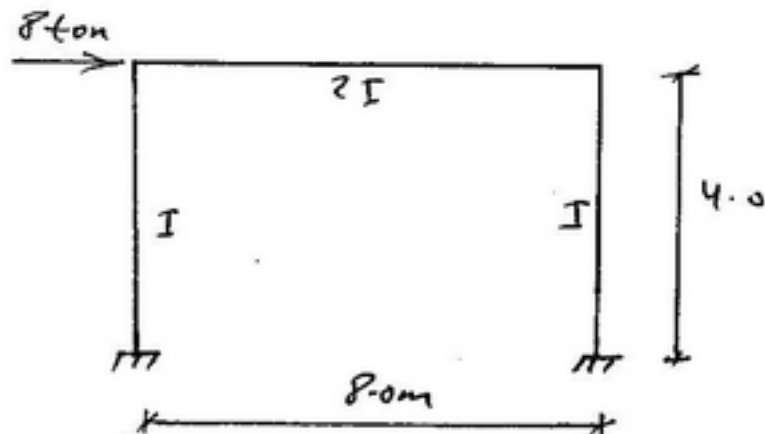
(i) using moment dist<sup>n</sup> method.



— Sol —

Using moment dist<sup>n</sup> method

هناك عدم تماثل في الأحمال، أفقي وعمودي، بالتالي يجب أن نستخدم طريقة التماثل العكسي (antisymmetry) في الحل.



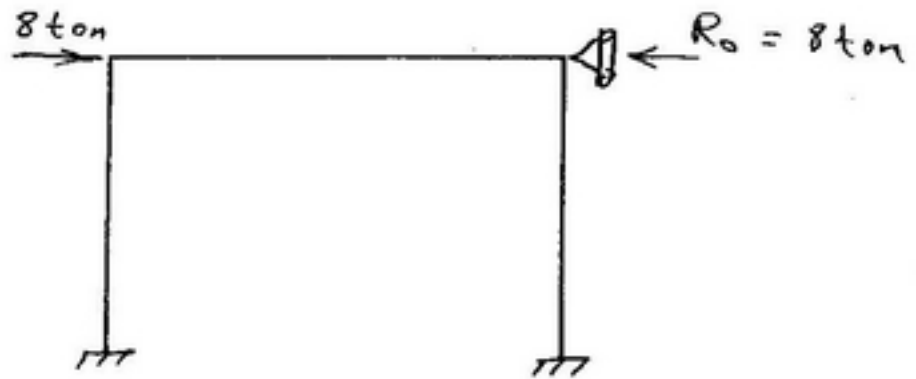
هناك عدم تماثل في الأحمال، أفقي وعمودي، بالتالي يجب أن نستخدم طريقة التماثل العكسي (antisymmetry) في الحل.

## Stage (I)

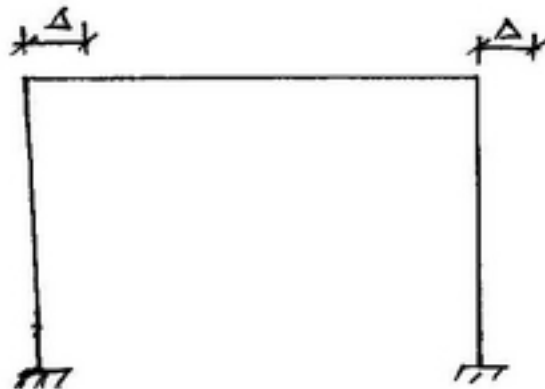
Fixed end moment  
on all member  
equal Zero.

∴ final moment = 0.

$$\boxed{M_0} = \text{Zero.}$$



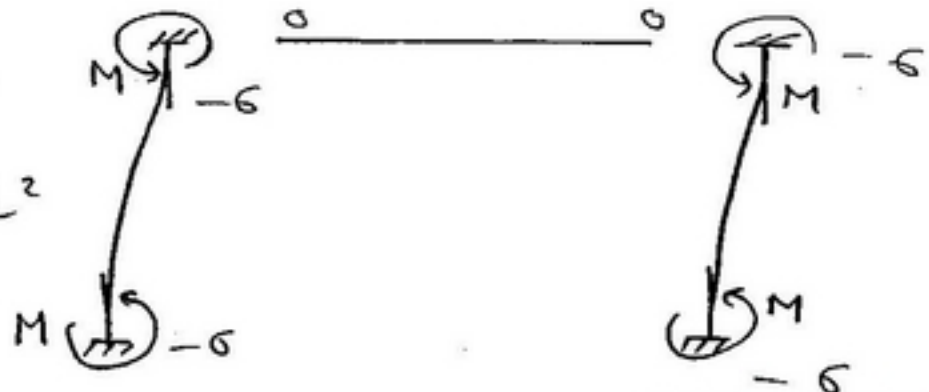
## Stage (II)



$$M = \frac{6EI\Delta}{L^2}$$

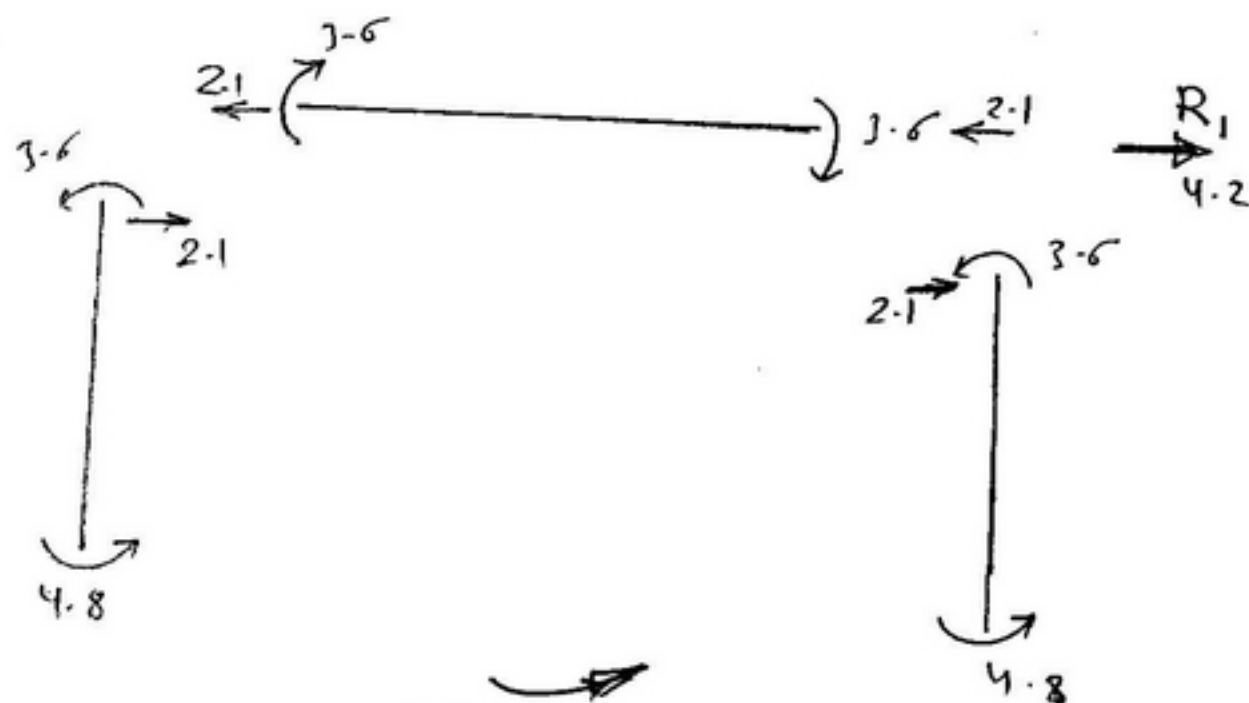
assume  $EI\Delta = L^2$

$$M = 6$$



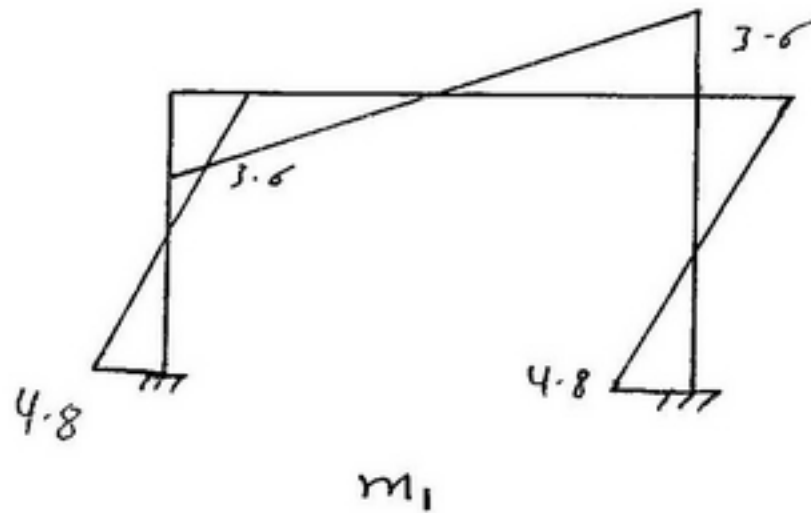
anti symmetry

R.s		$\frac{1}{4} = 2$	$\frac{3}{2} \times \frac{3}{8} = 3$	
D.f		0.4	0.6	
F.E.M	-6	-6	0	
D.M	0	2.4	3.6	
C.o.M	1.2	0	0	
D.M	0	0	0	
F.M	-4.8	-3.6	3.6	



يتم نقل العزوم  
بنفسه أيضاً





$$R_o = 8 \text{ ton} \leftarrow$$

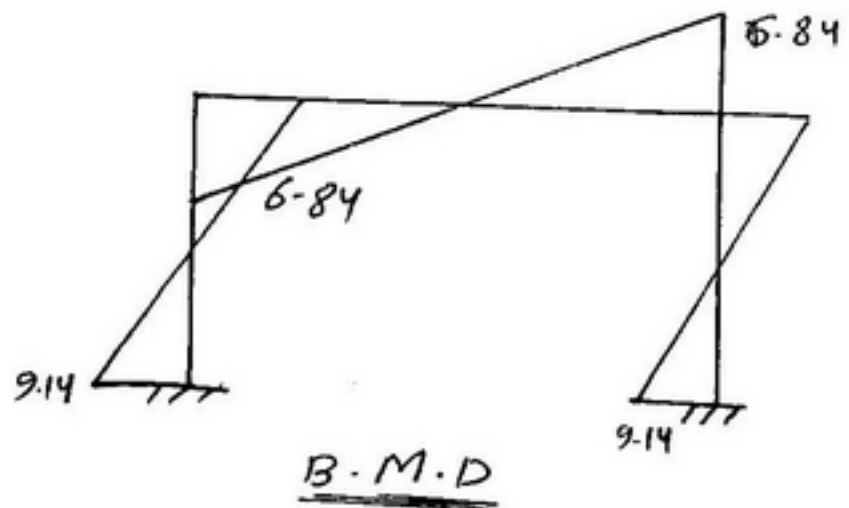
$$R_1 = 4.2 \rightarrow$$

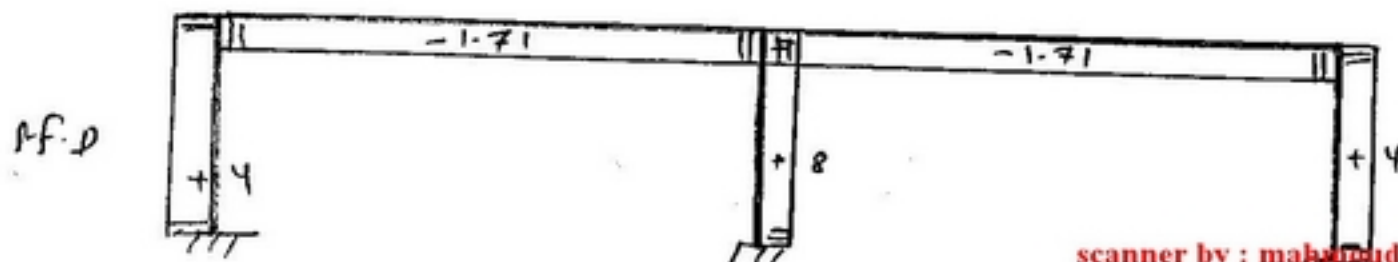
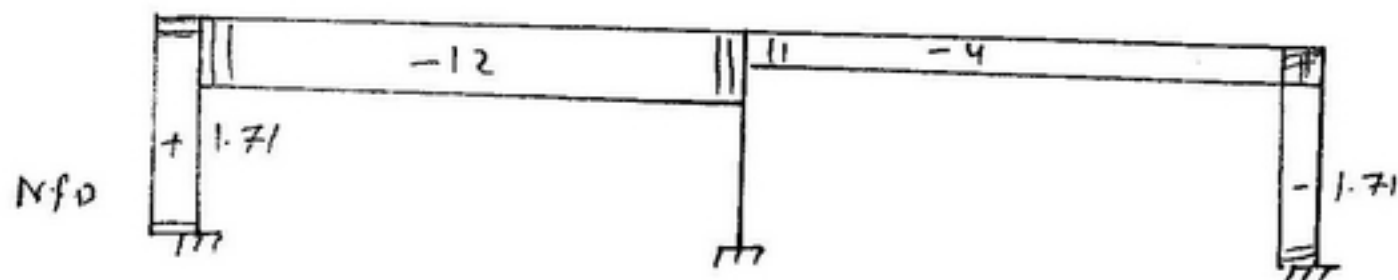
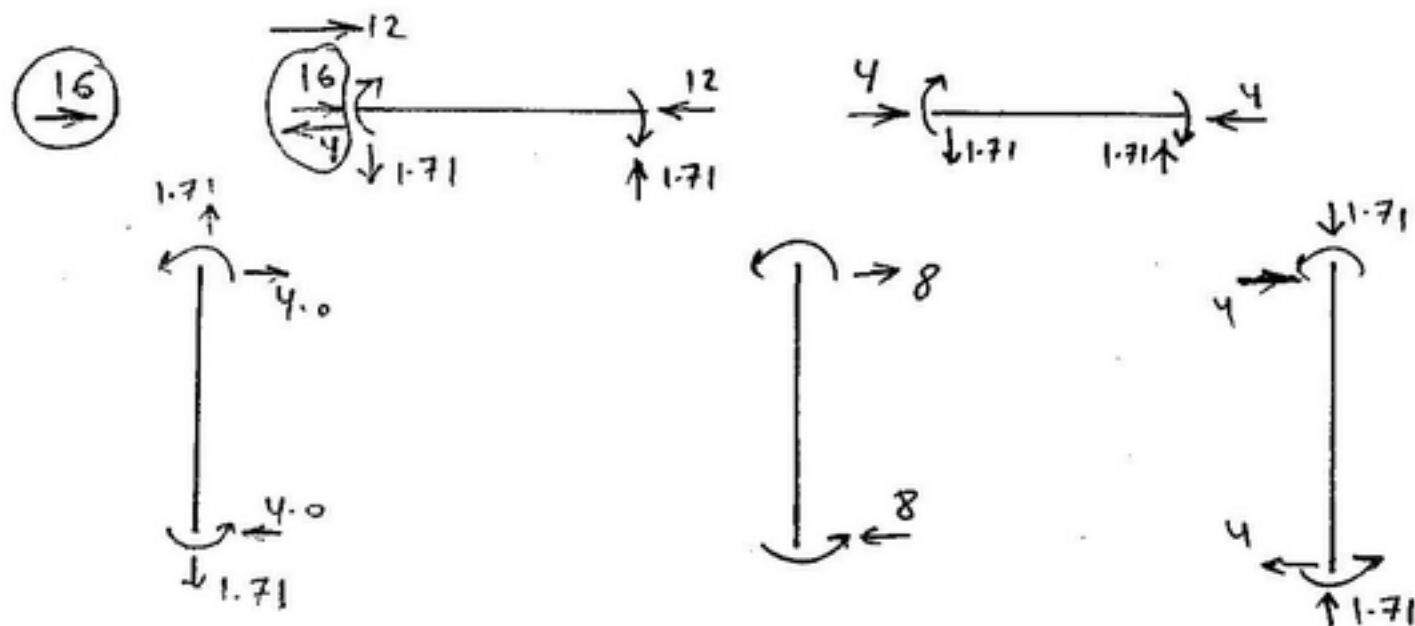
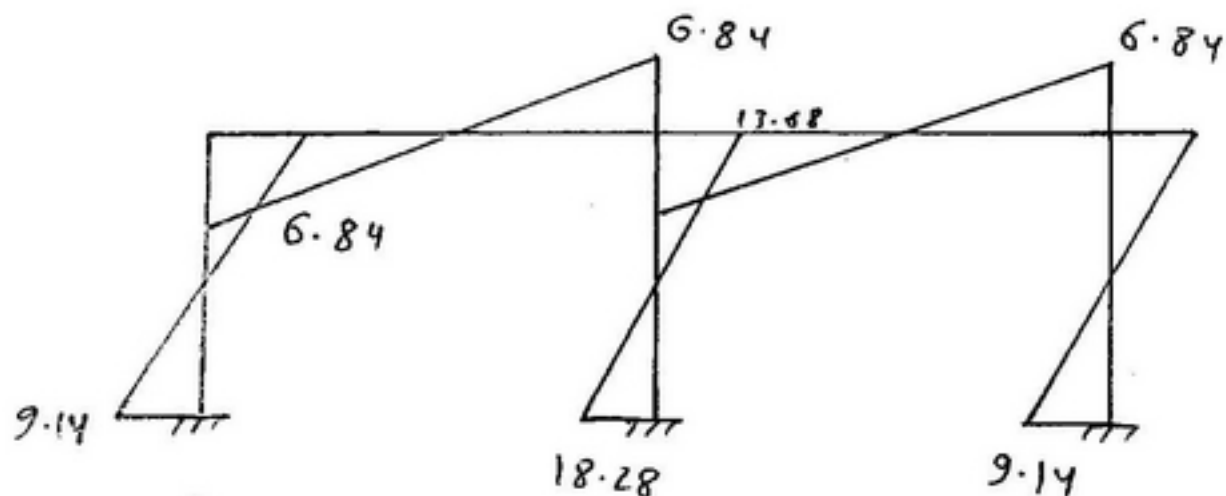
$$R_o + \Delta R_1 = 0$$

$$\Delta = 8/4.2 = 1.9$$

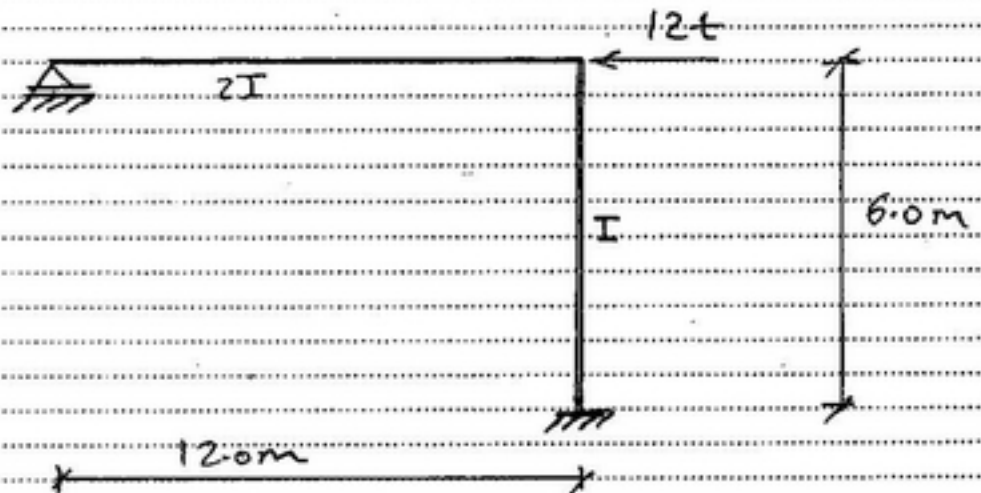
$$M_f = M_o + \Delta M_1$$

$$= \Delta M_1$$





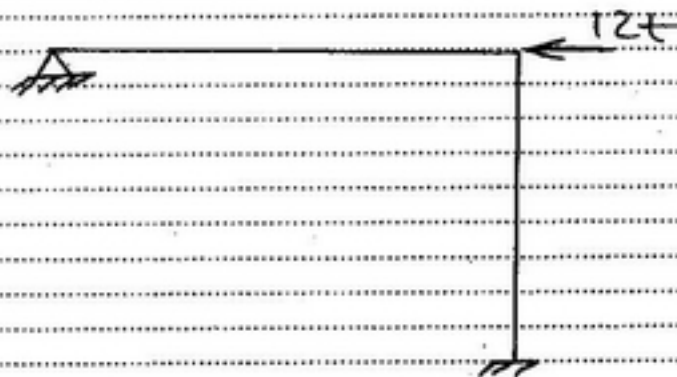
mid & final 2007



draw N.F.D, S.F.D & B.M.D

~~~~~ sol ~~~~~

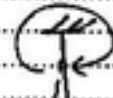
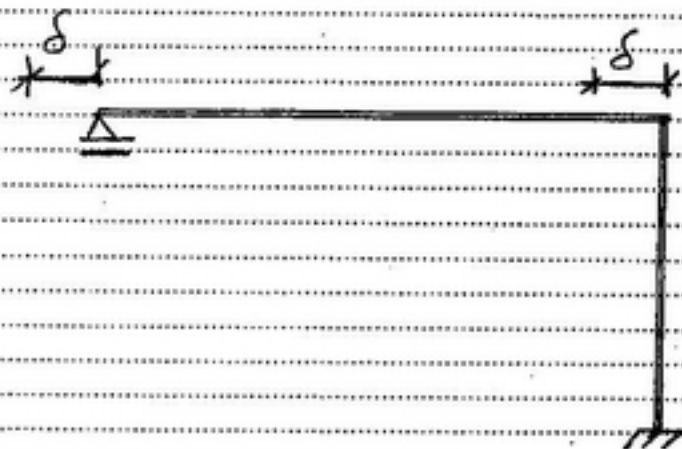
Step ①



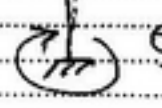
$M_0 = \text{Zero}$   
 عند نقطة التماس  
 $R_0 = 12t \rightarrow$   
 رد فعل التماس  
 للأكبر مقدار



step ②



$$\frac{6EI}{L^2}$$

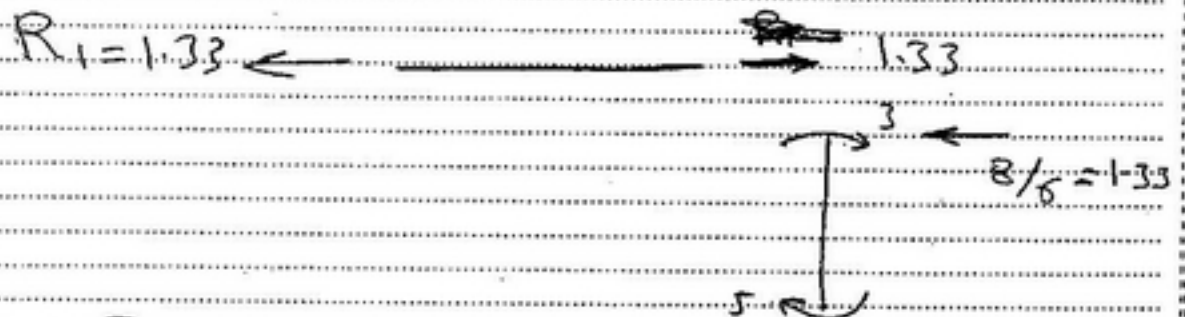
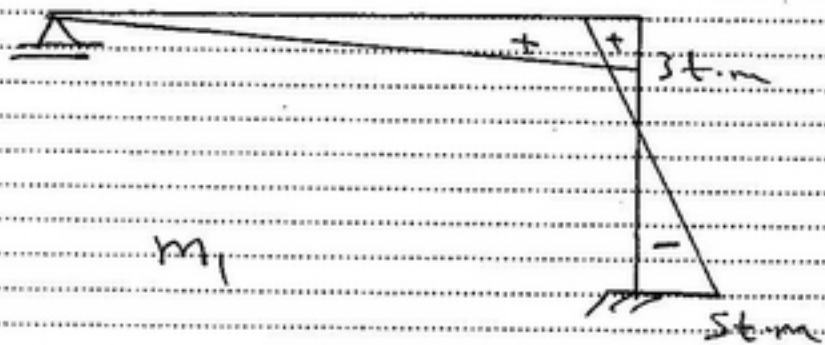


$$\frac{6EI}{L^2}$$

الرمح ده ليس له تأثير  
في الـ F.E.M  
لذلك  
let  $EI = 42$

$$\therefore \text{F.E.M} = \frac{6 \times 42}{6^2} = +7$$

|       |   |                                                    |                   |    |
|-------|---|----------------------------------------------------|-------------------|----|
| R.S   |   | $\frac{3}{4} \times \frac{1}{2} = \frac{1}{8} = 3$ | $\frac{1}{6} = 4$ |    |
| D.f   |   | $\frac{3}{7}$                                      | $\frac{4}{7}$     |    |
| F.E.M | 0 | 0                                                  | +7                | +7 |
| D.M   | 0 | -3                                                 | -4                | 0  |
| C.O.M | 0 | 0                                                  | 0                 | +2 |
| D.M   | 0 | 0                                                  | 0                 | 0  |
| R.M   | 0 | -3                                                 | +3                | +5 |



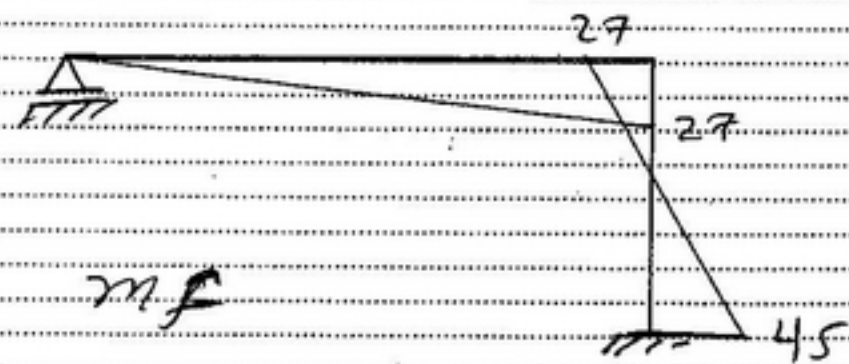
$$\therefore R_1 + X_1 R_1 = 0.0$$

$$12 - 1.33 X_1 = 0.0$$

$$X_1 = 9.0$$

$$M_F = m/6 + 9 M_1$$

$$= 9 M_1$$





Faculty of engineering  
3rd year

115  
١١٥

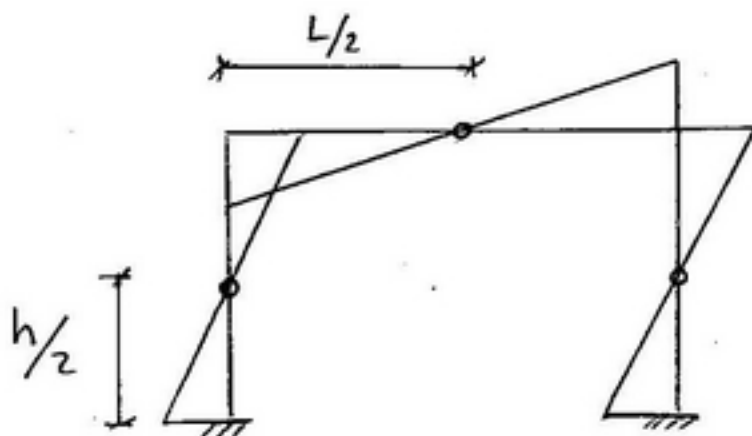
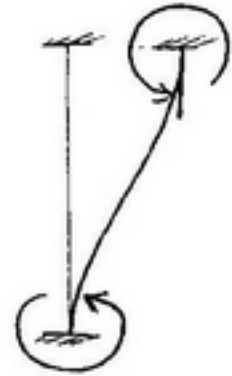
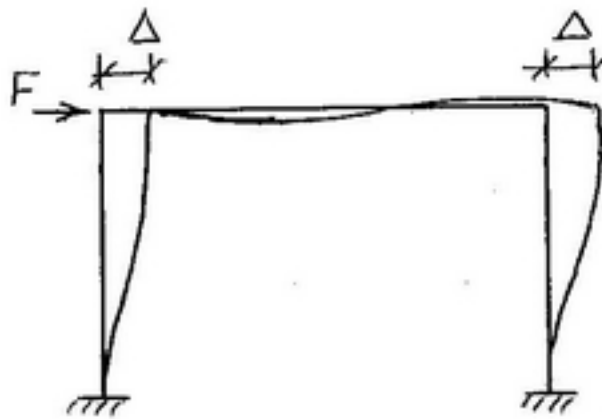
# structural analysis

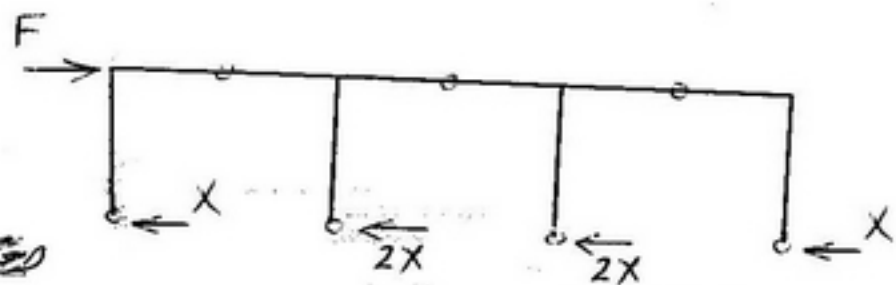
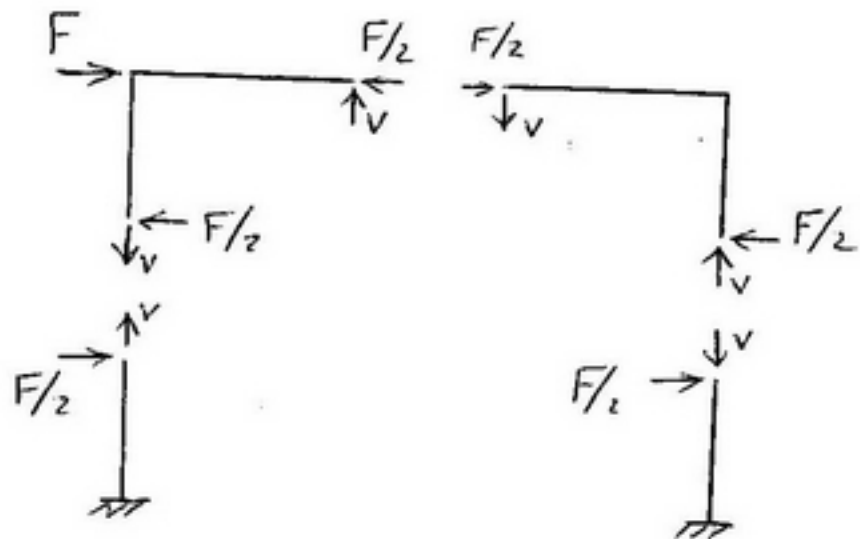
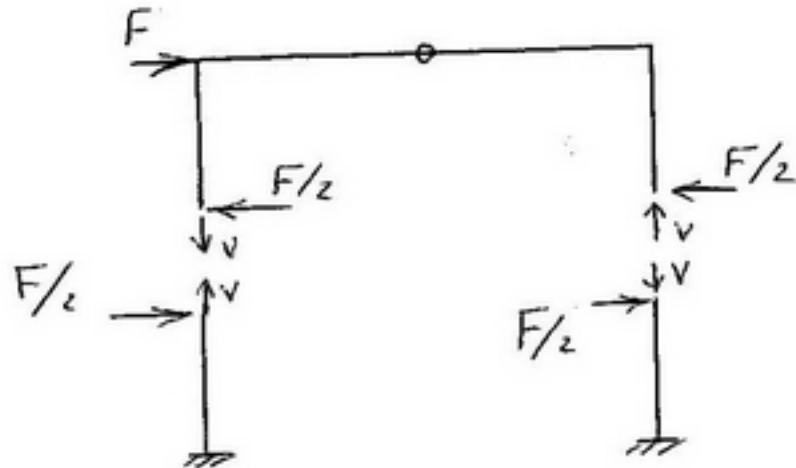
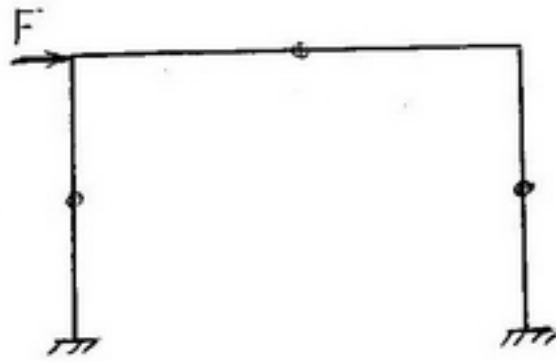
No ( )



بسم الله الرحمن الرحيم  
approximate method

طريقة تقريبية يتم تحديد أماكن انقطاع العزم (عزم = صفر) وإختبارها Intermediate  
 وبالتالي تكون في منتصفاً حراً استاتيكيًا



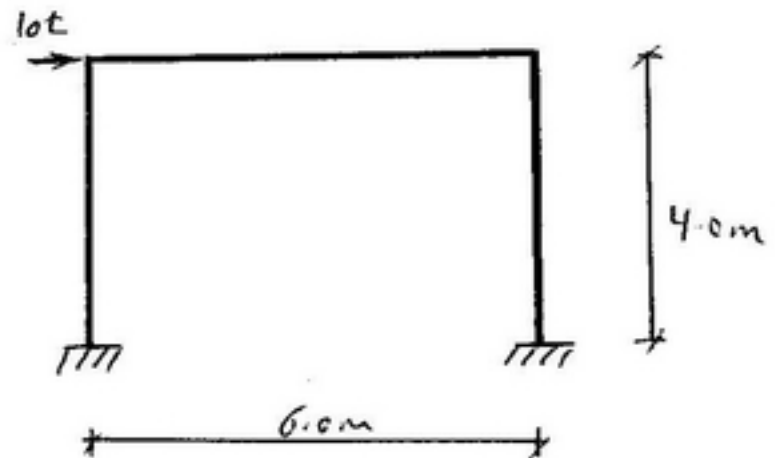


بنادر الیمنه  
المنه

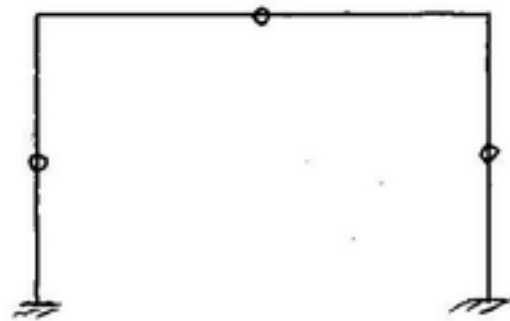
# Example ①

draw B.M.D, S.F.D  
N.F.D

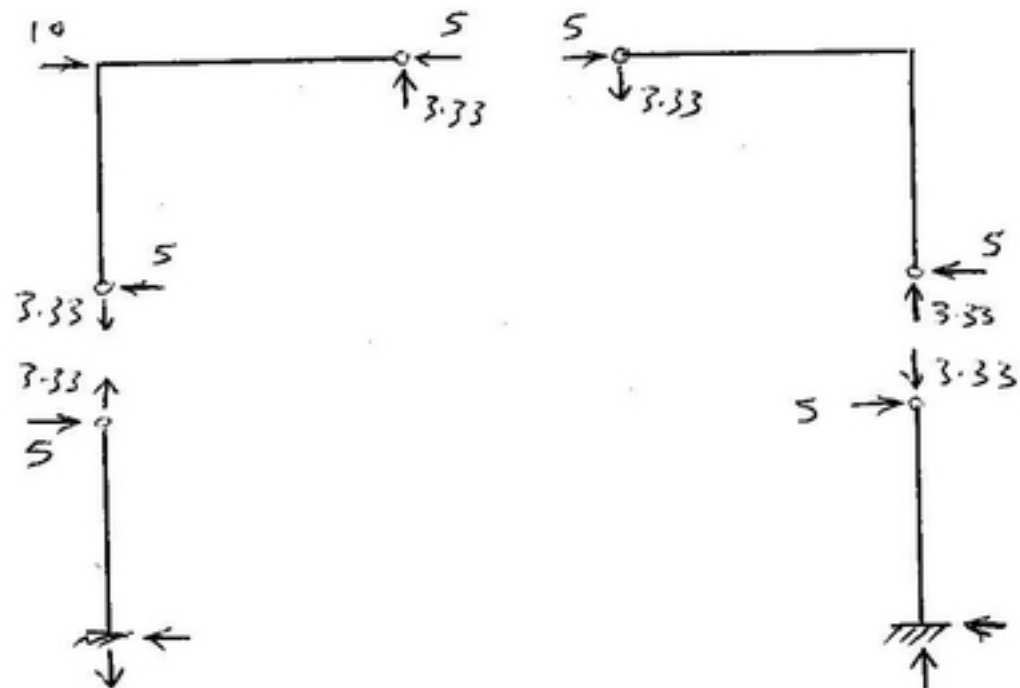
for the following  
shape.



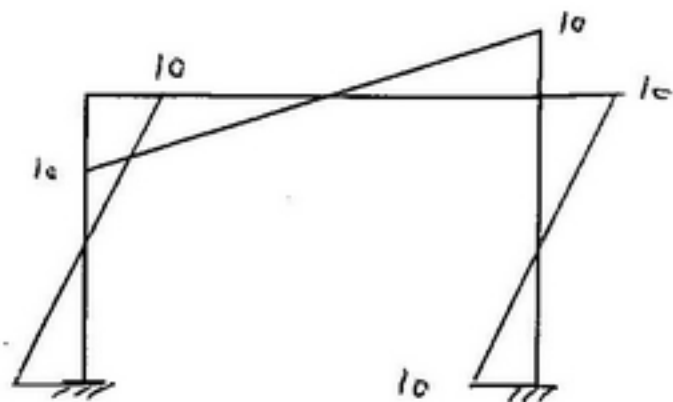
———— SOL ————



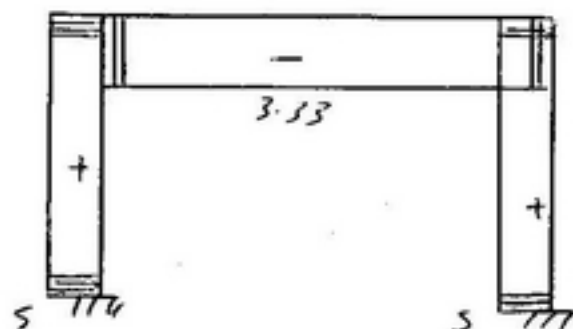




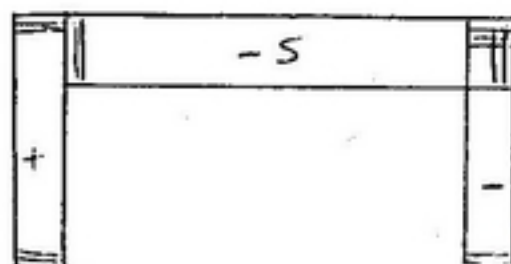
B.M.D



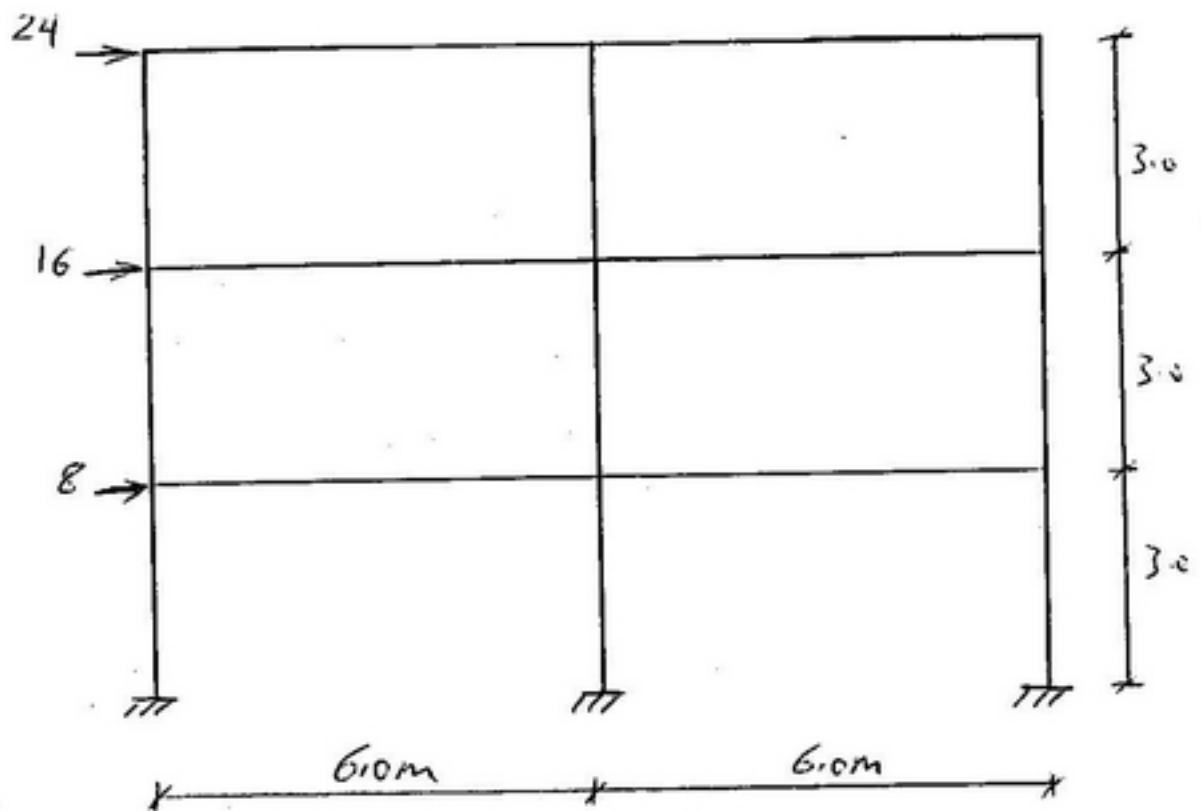
S.f.D



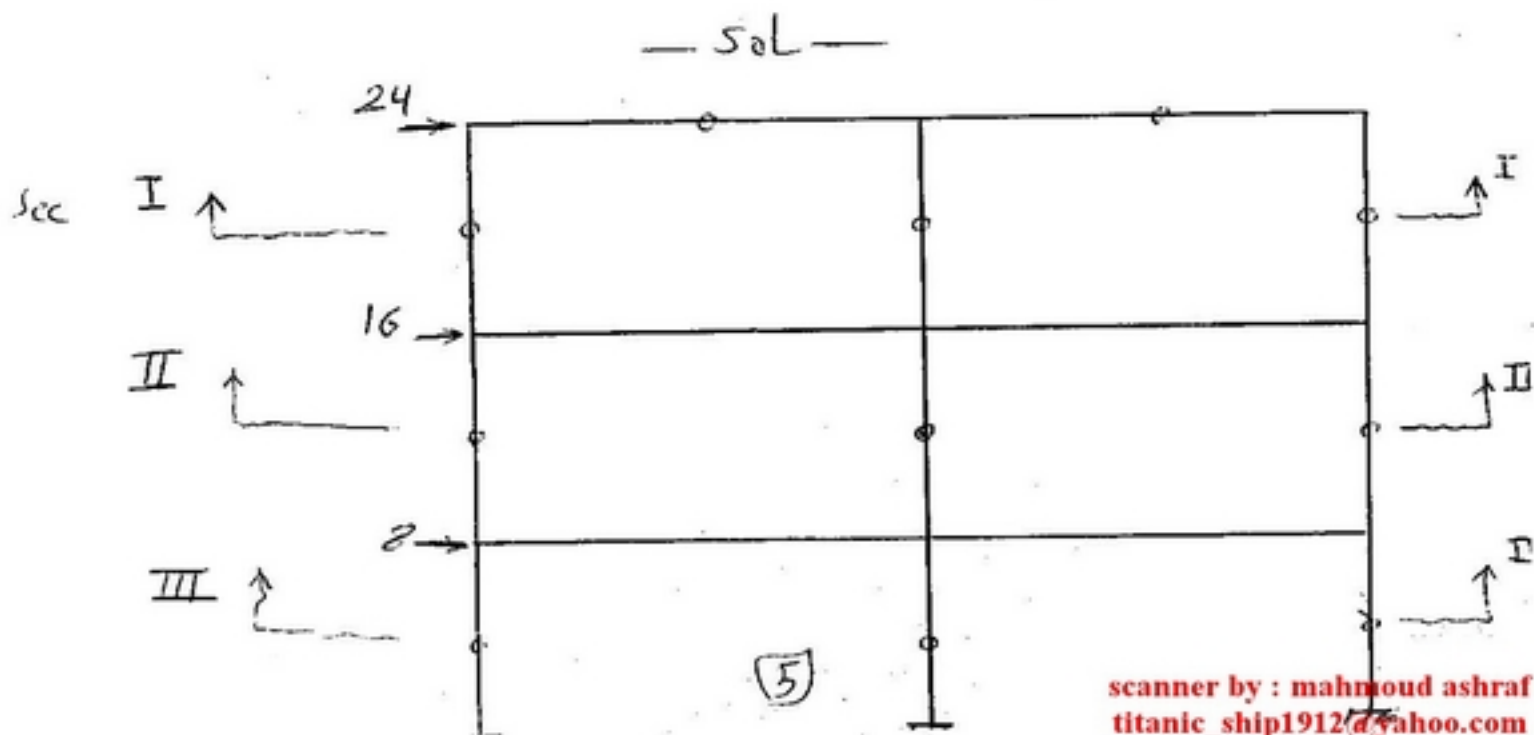
N.f.D



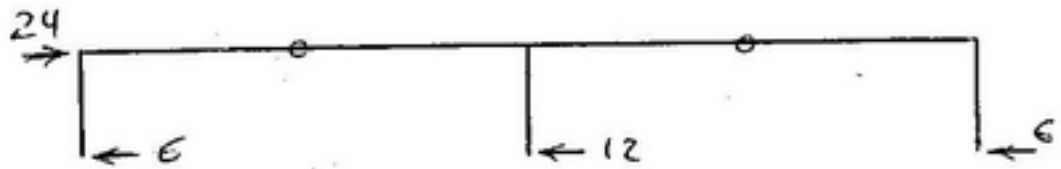
## Example



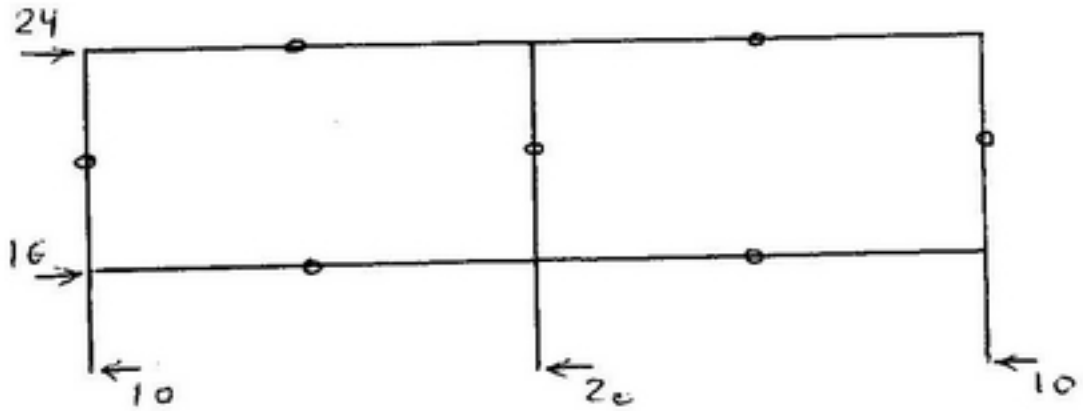
draw N.F.D, S.F.D, B.M.D



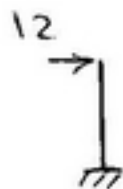
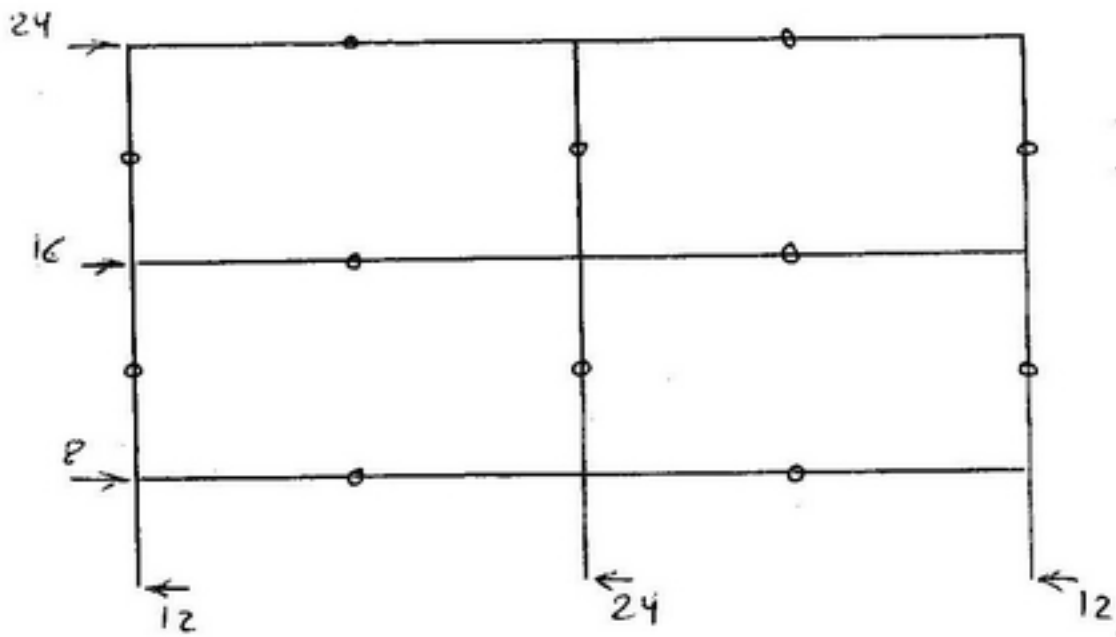
Sec I - I



Sec II - II

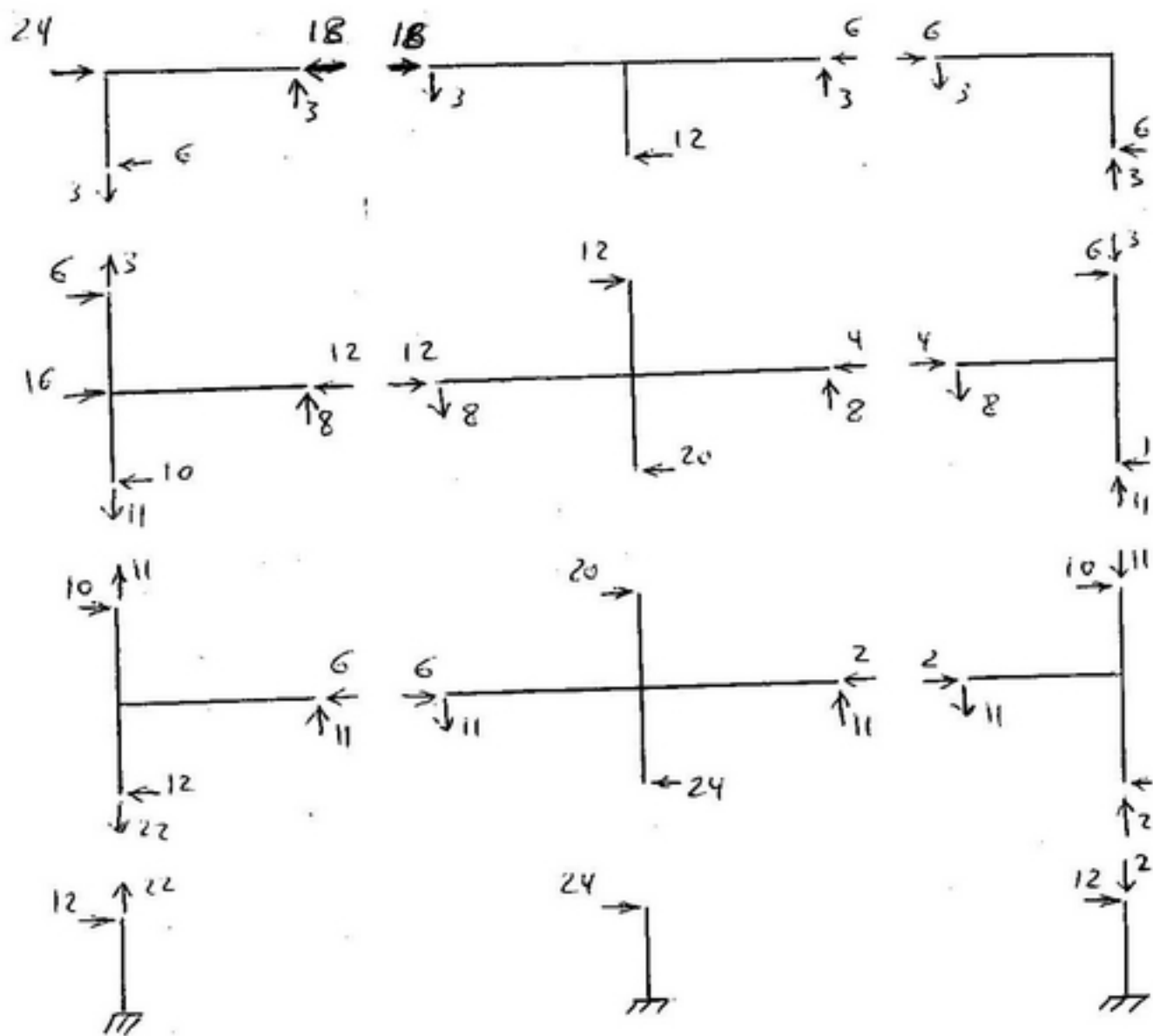


Sec III - III

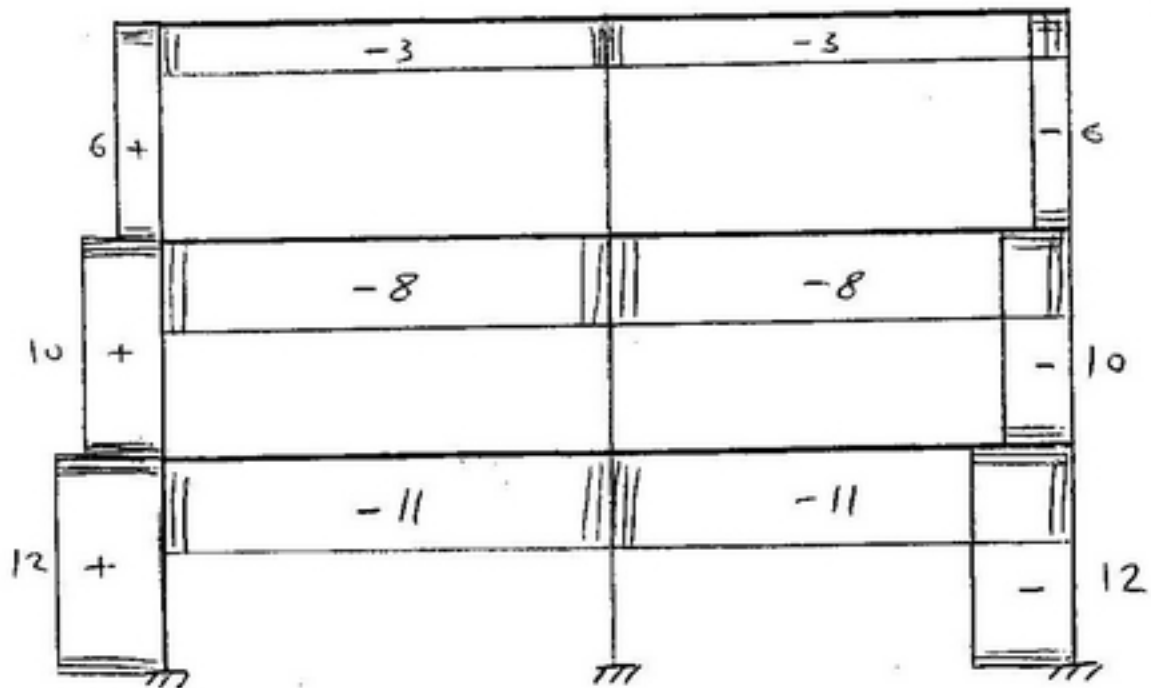
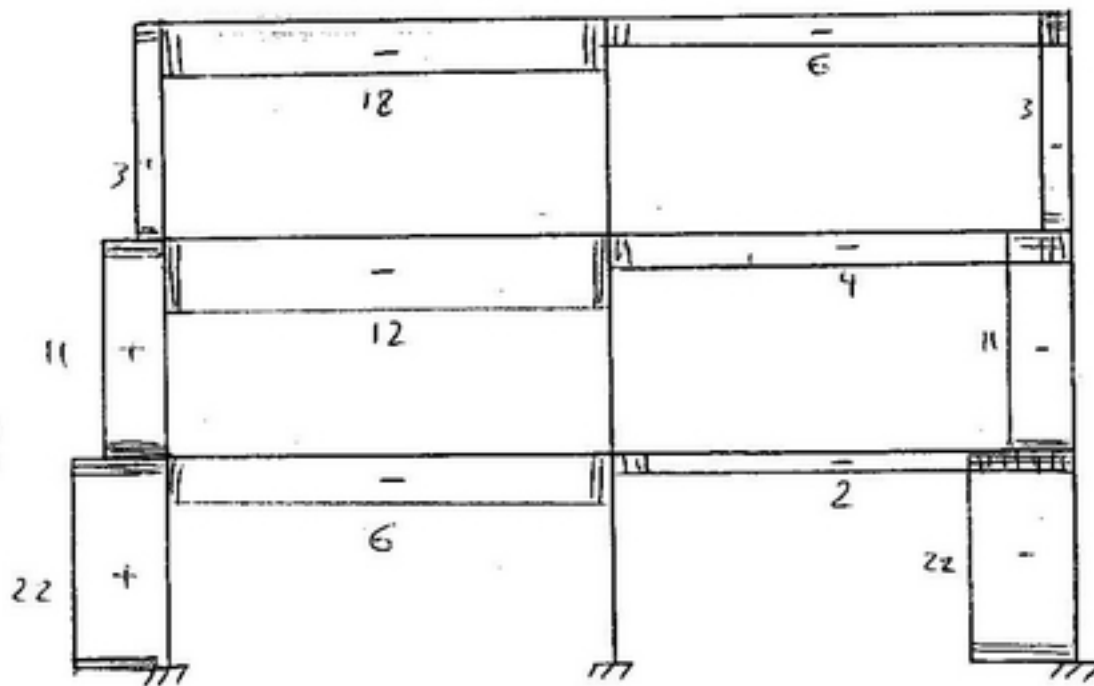


(6)

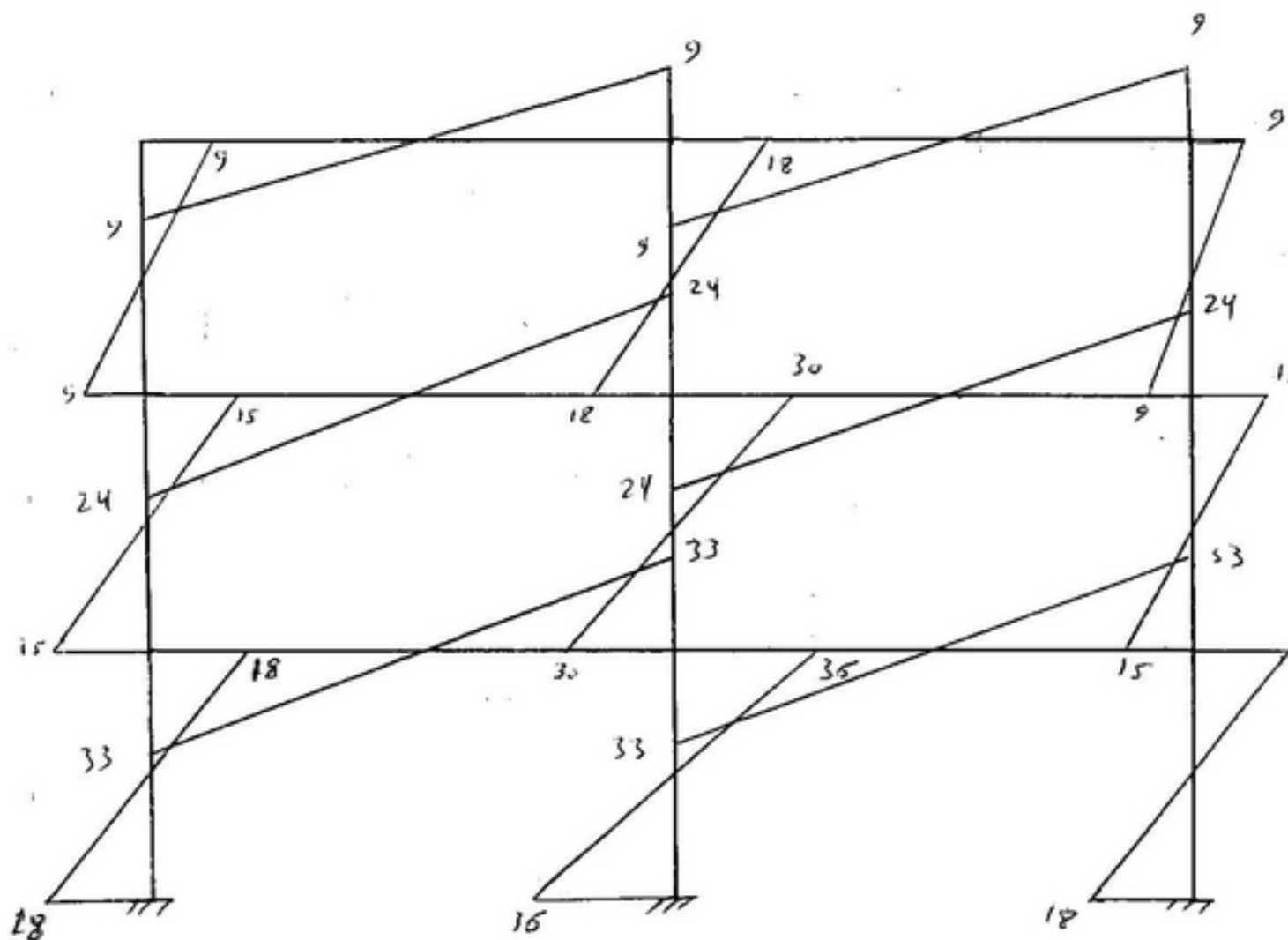




N-f.D



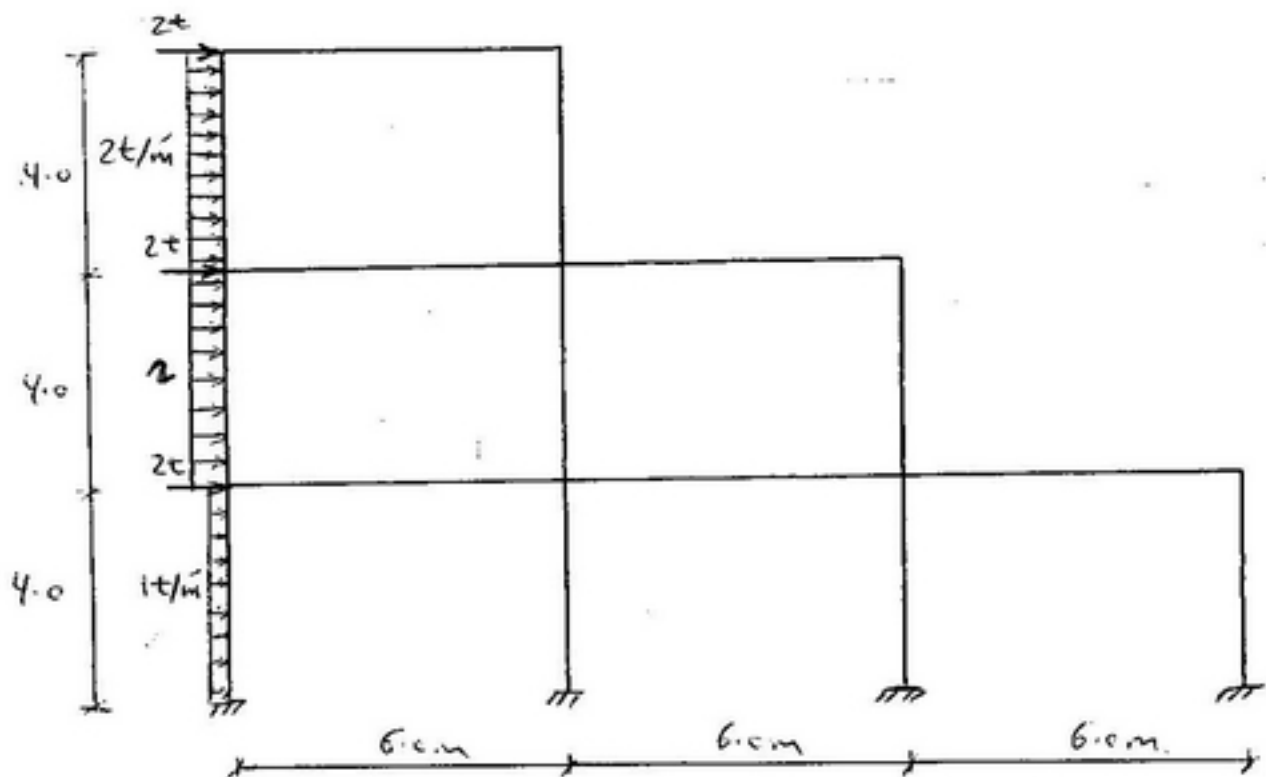
S-f.D



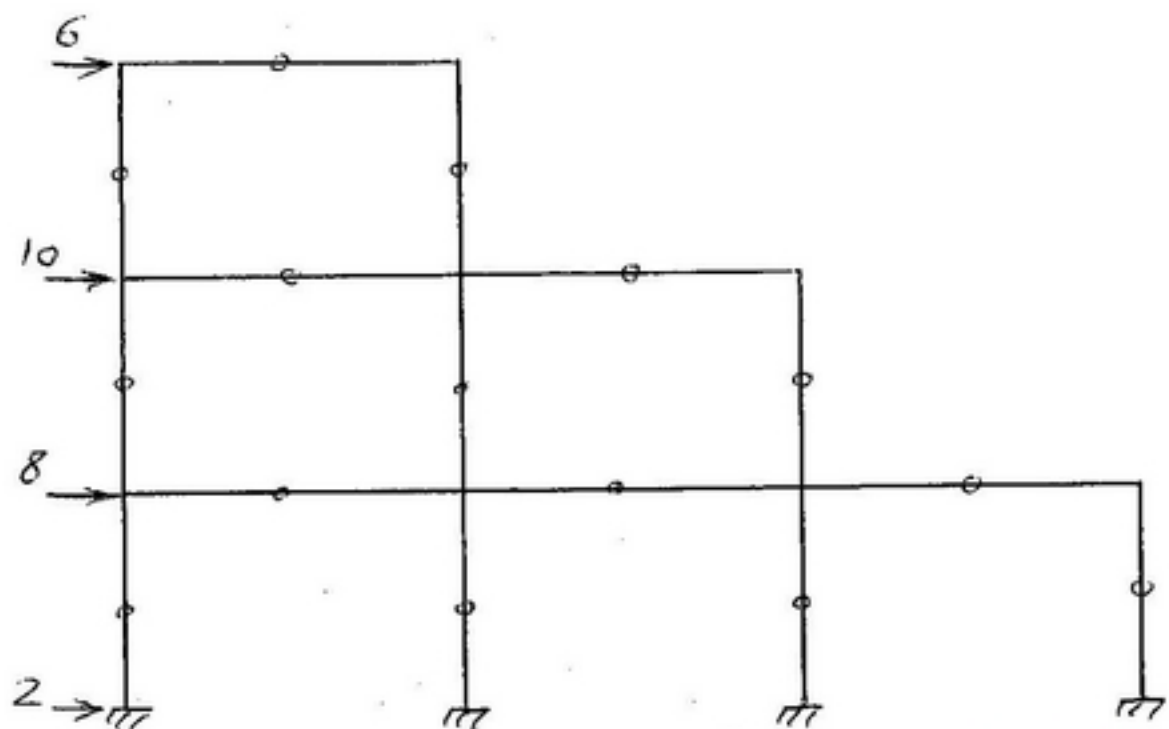
15 ↓  
 18 ← 33

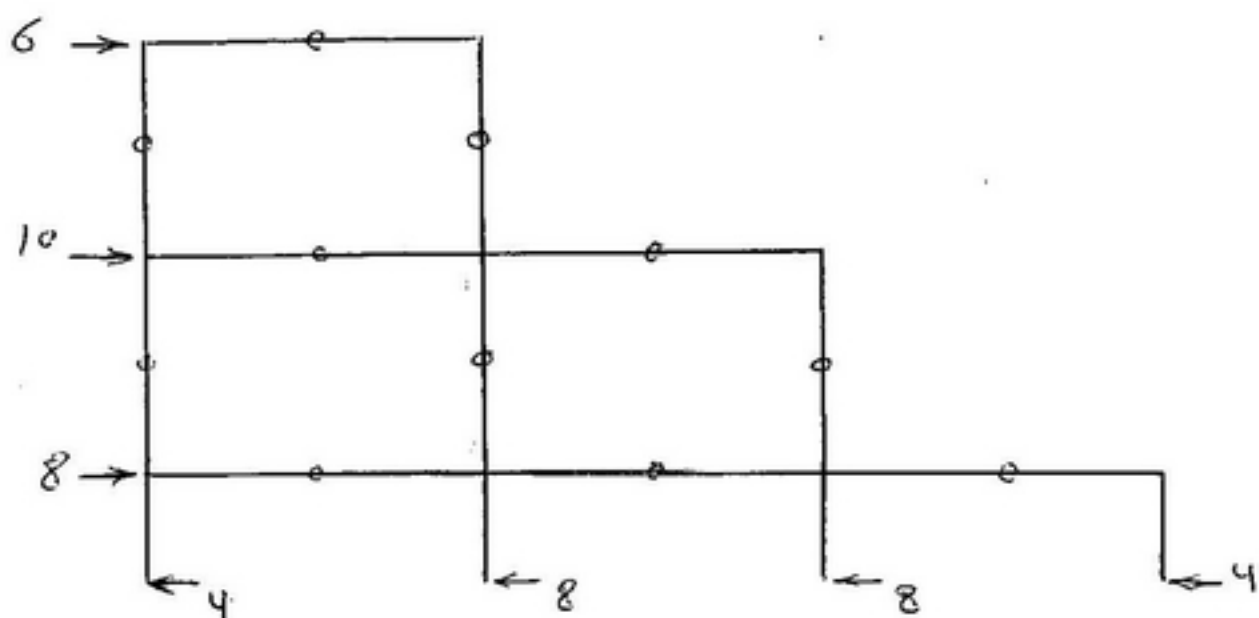
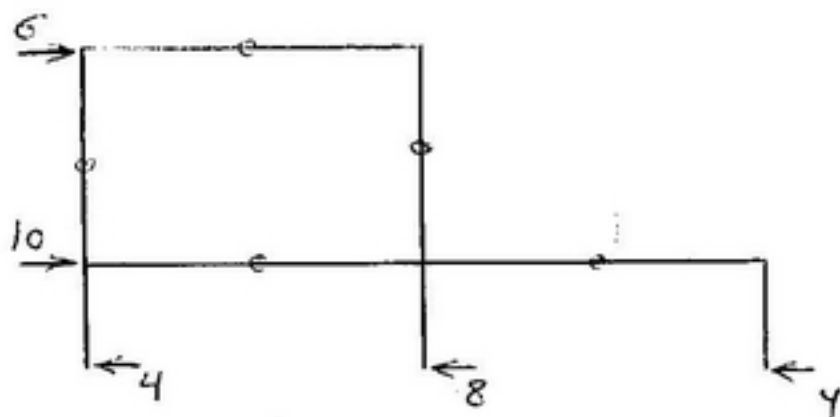
B.M.D

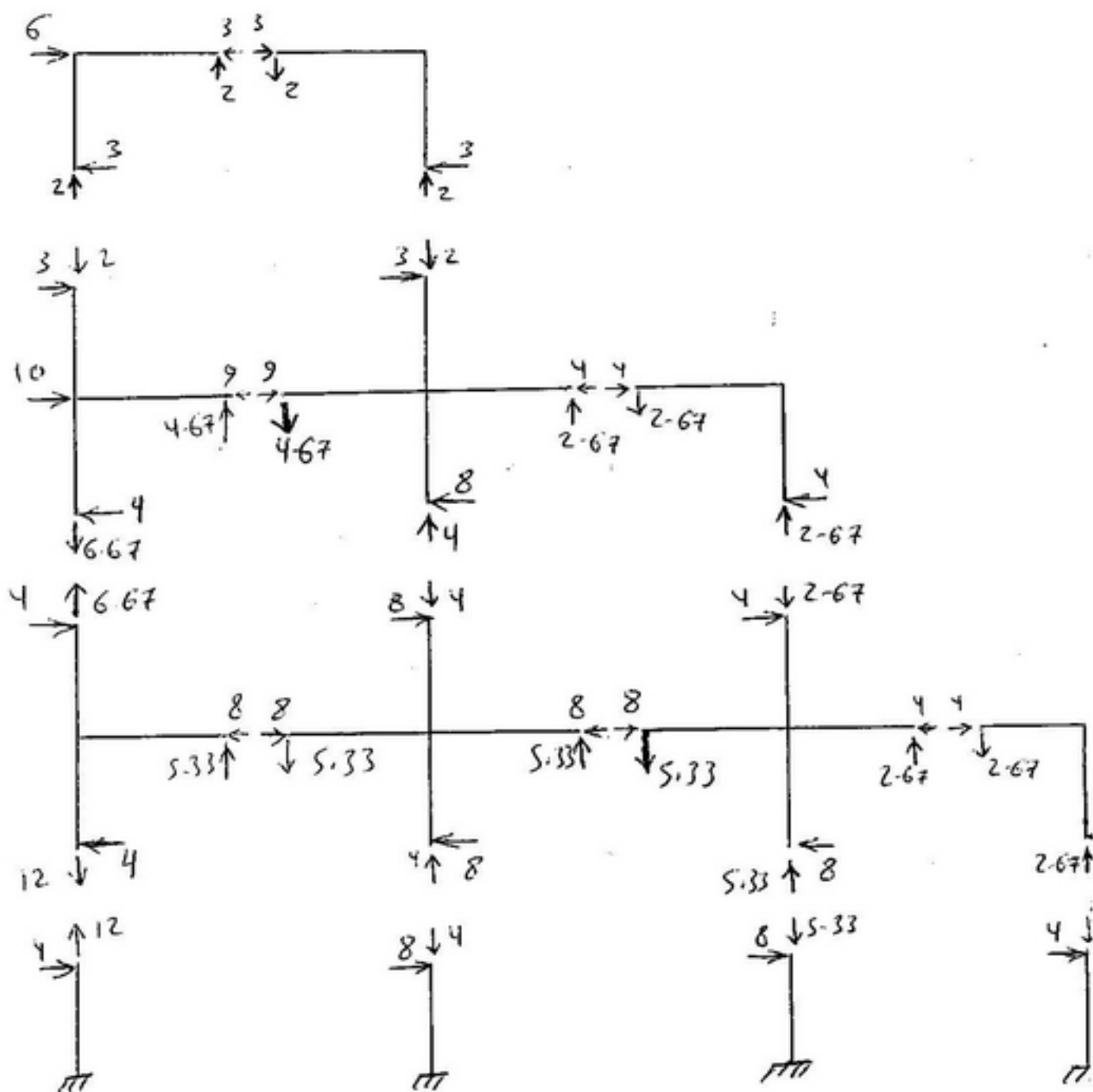




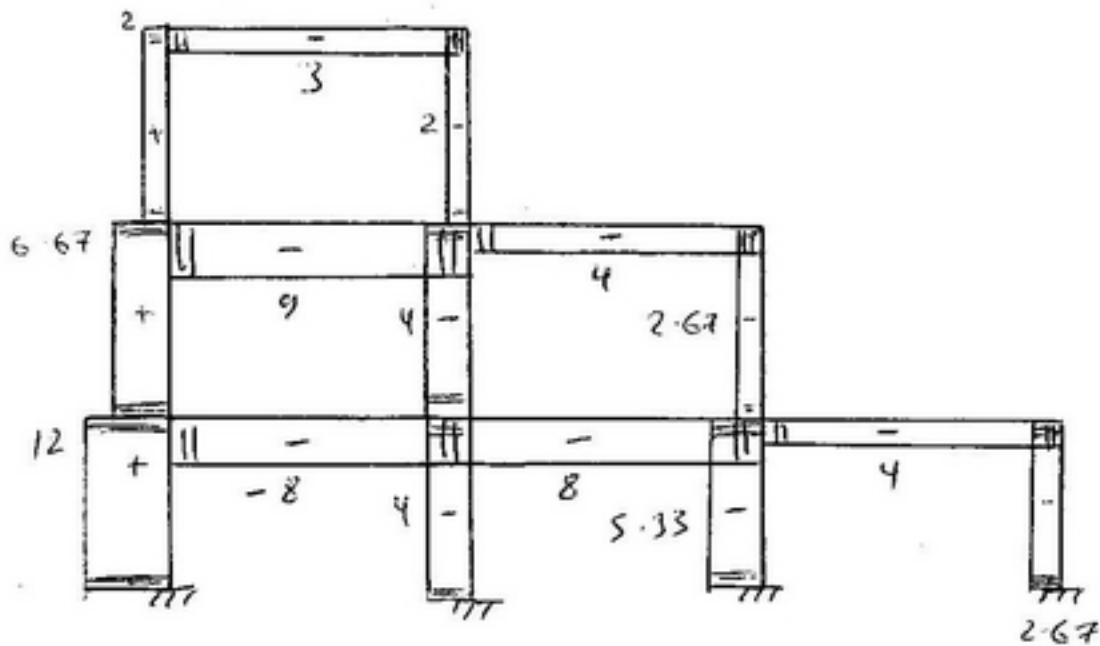
— Sol —



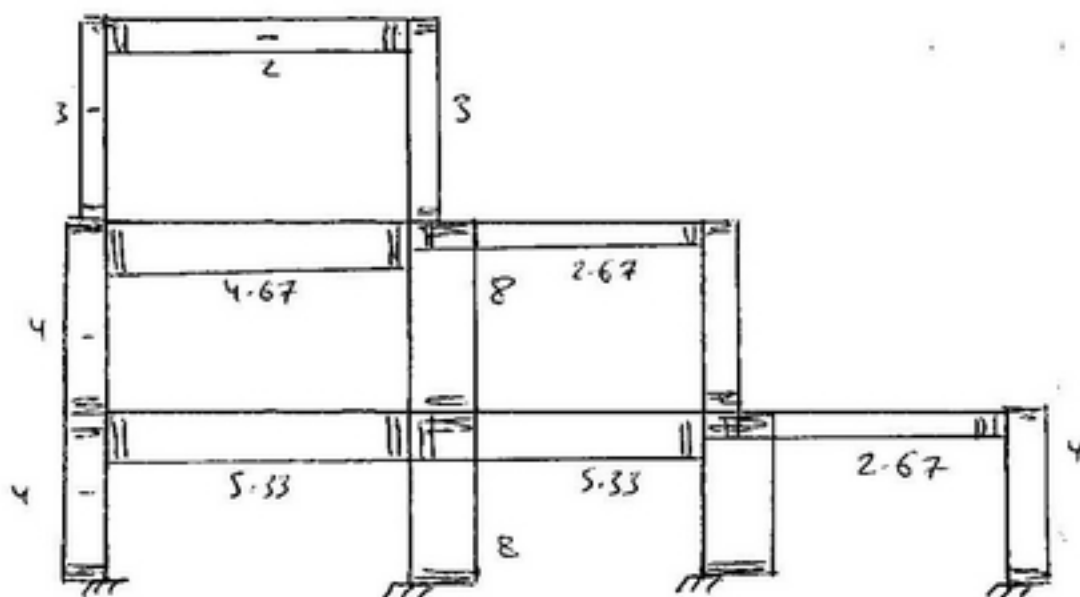




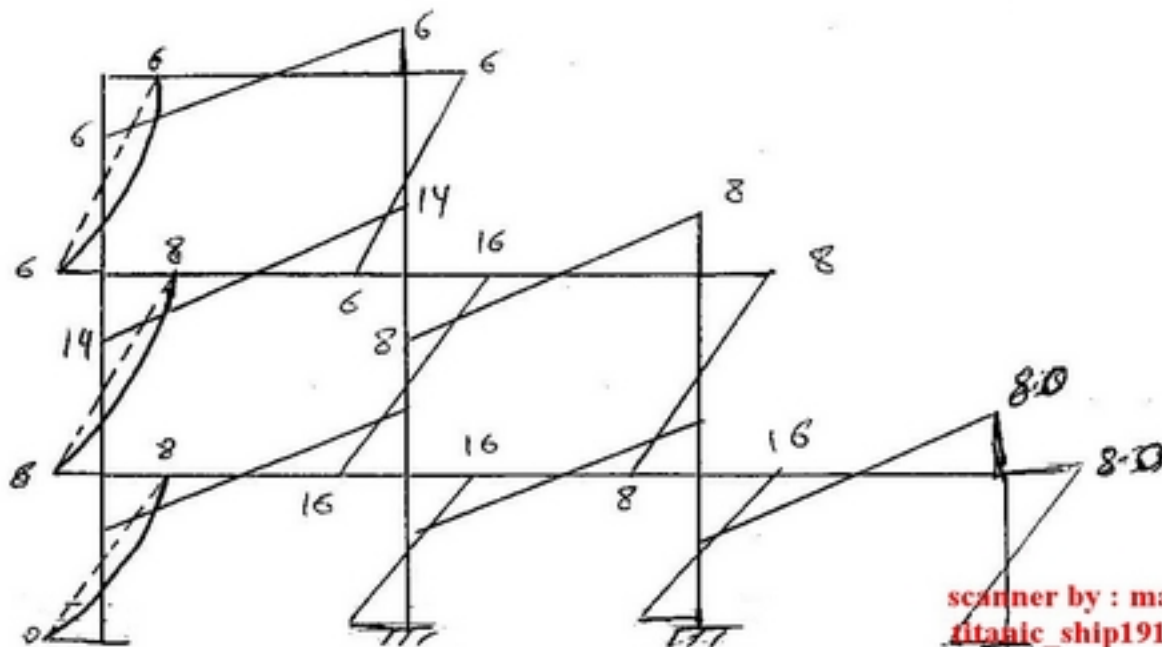




N.F.D

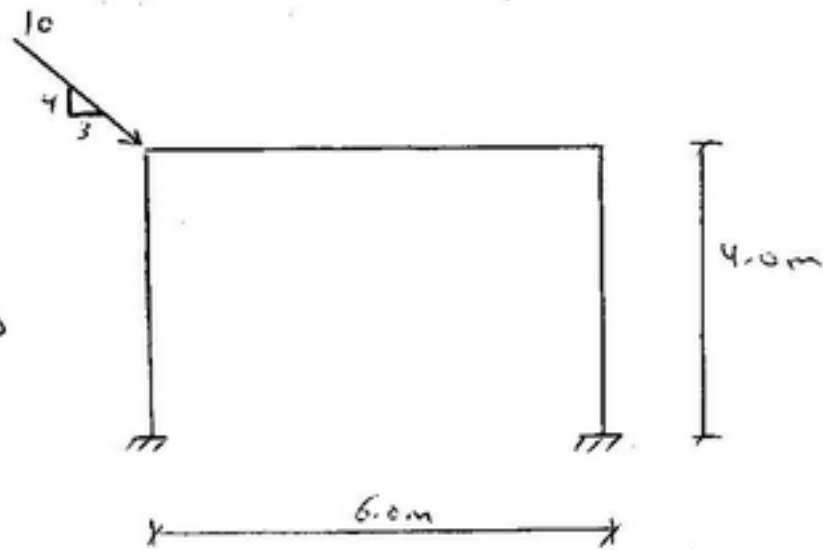


S.F.D

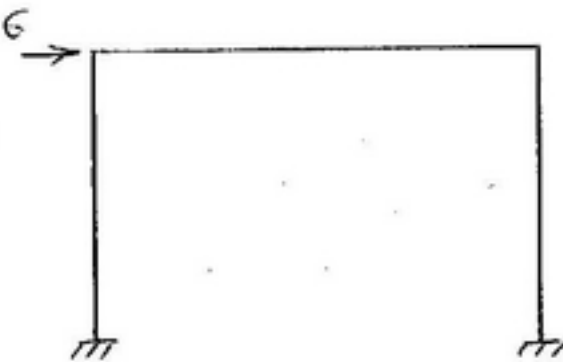
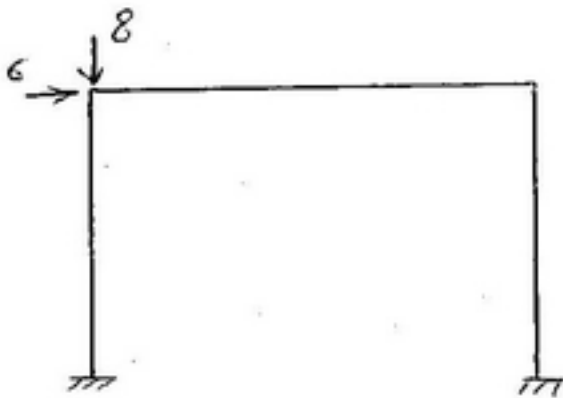


B.M.D

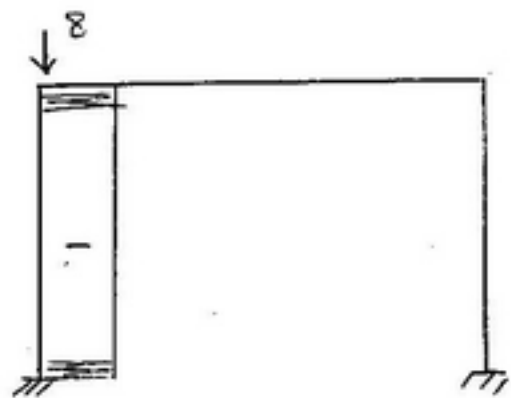
draw BMD  
NFD, SFD



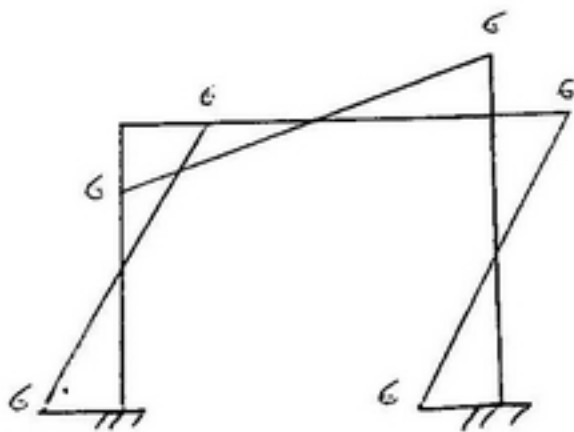
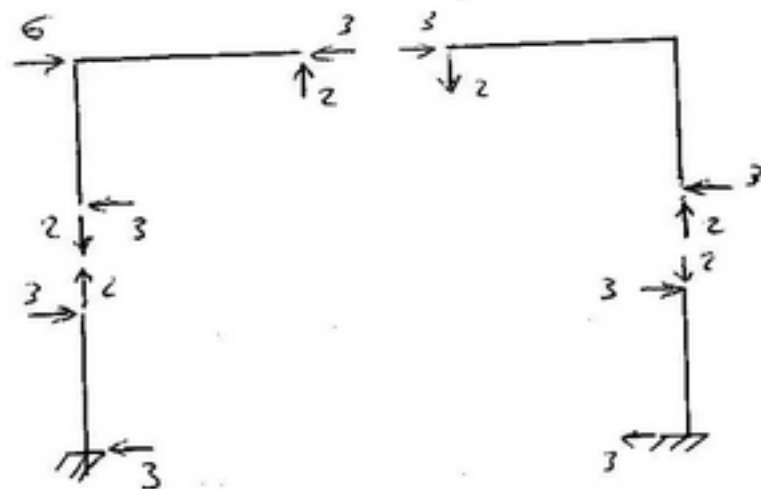
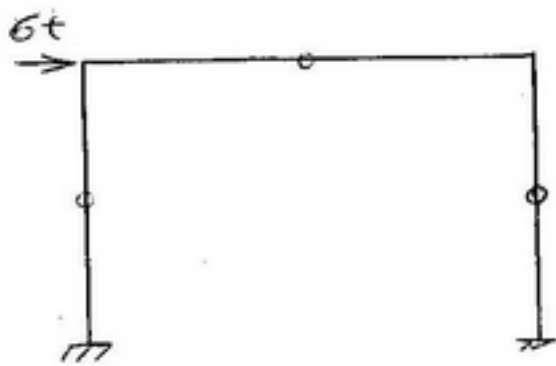
— Sol —



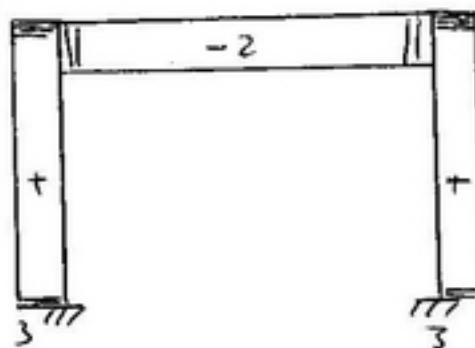
+



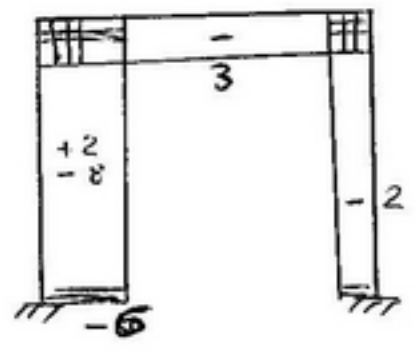
حمل 8 + ينزل  
الحمود normal



B.M.D

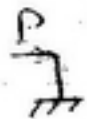


S.F.D

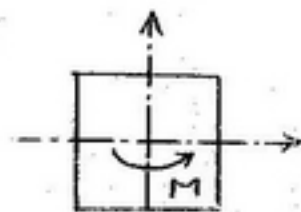
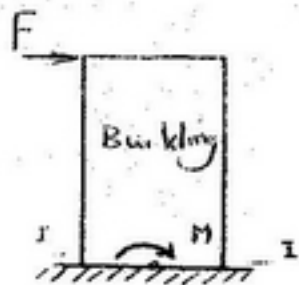


N.F.D

# Cantilever method



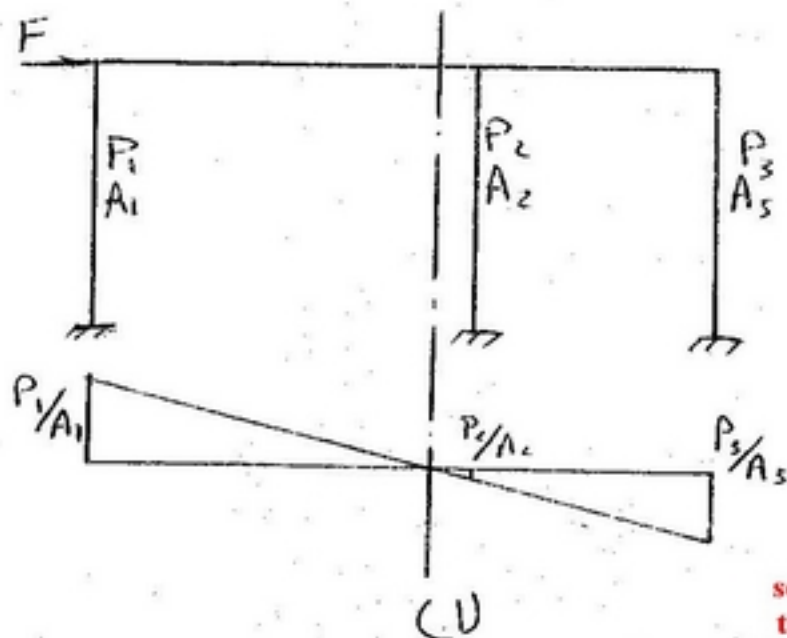
بنی اعتبار، نسبت کل (کارتی) و انحراف کلیه اجزای Cantilever



نسبت انحراف کلیه اجزای



تقریباً انحراف کلیه اجزای مساوی است  
 $\epsilon = \frac{P}{A} \Rightarrow$





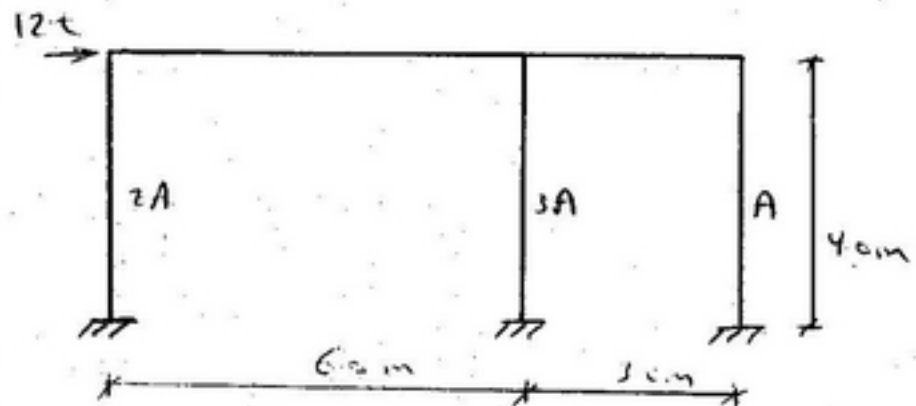
من نقطة ثالثة في جميع الحالات  $P_1, P_2, P_3$

$$P_1 + P_2 + P_3 = 0$$

في جميع الحالات  $P_1, P_2, P_3$

Example ①

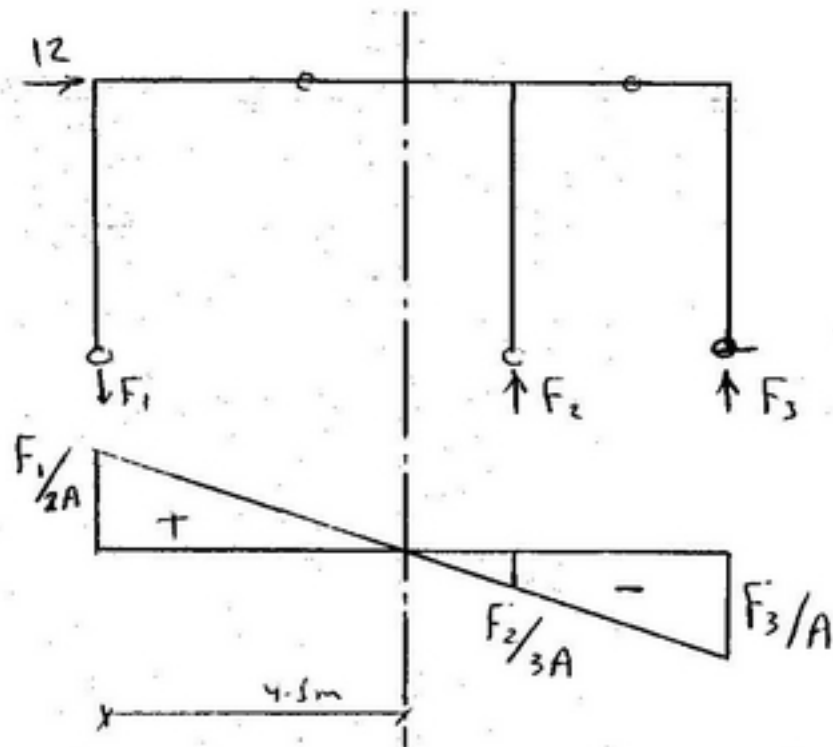
draw B.M.D  
N.F.D & S.F.D



— Sol —

نقطة ثالثة في جميع الحالات N.A

$$\bar{X} = \frac{2A \times 0 + 3A \times 6 + A \times 9}{2A + 3A + A} = 4.5m$$



$$\frac{F_1/2A}{4.5} = \frac{F_2/3A}{1.5}$$

$$F_1 = 2F_2 \longrightarrow \textcircled{1}$$

$$\frac{F_3/A}{4.5} = \frac{F_2/3A}{1.5}$$

$$F_2 = F_3 \longrightarrow \textcircled{2}$$

$$\sum M @ cg = 0$$

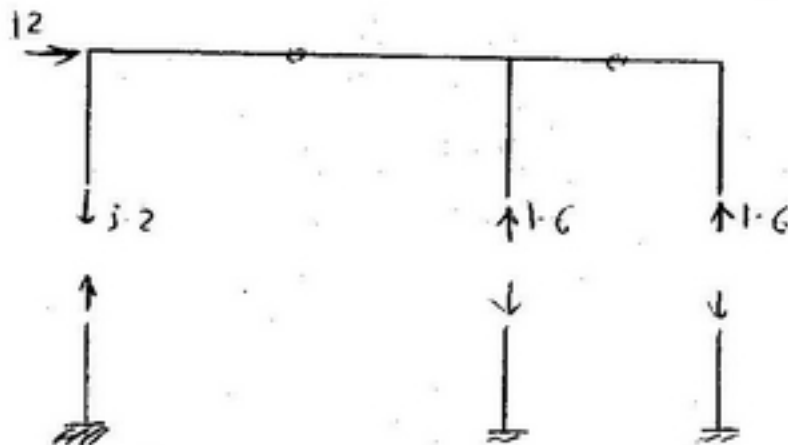
$$F_1 \times 4.5 + F_2 \times 1.5 + F_3 \times 4.5 = 12 \times 2$$

$$2F_2 \times 4.5 + F_2 \times 1.5 + F_2 \times 4.5 = 24$$

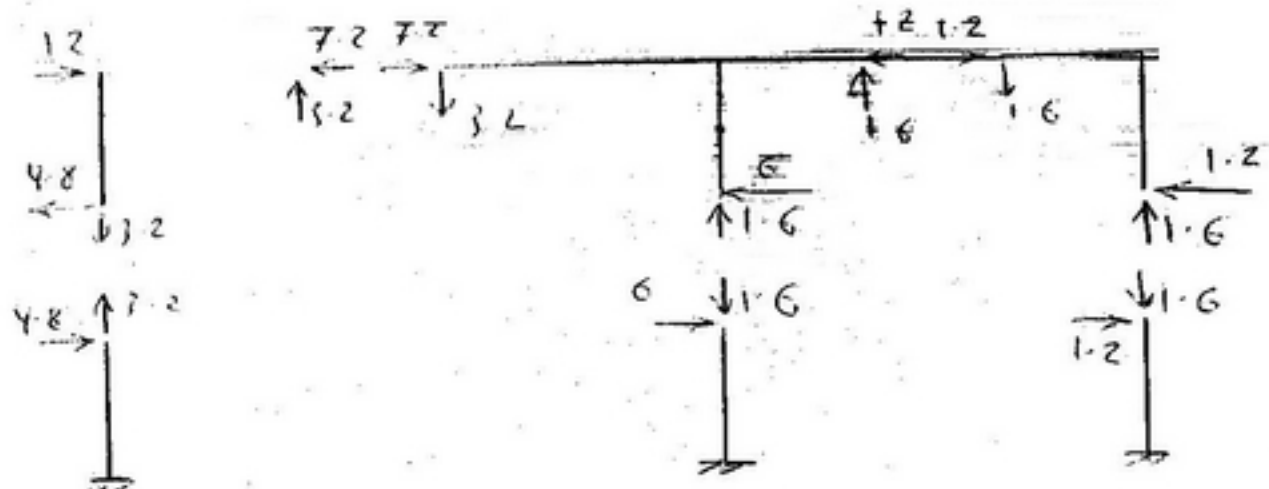
$$15F_2 = 24$$

$$F_2 = F_3 = 1.6 \text{ ton}$$

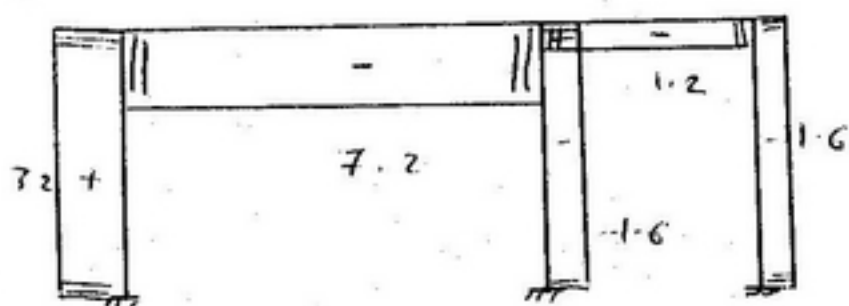
$$\therefore F_1 = 3.2 \text{ ton}$$



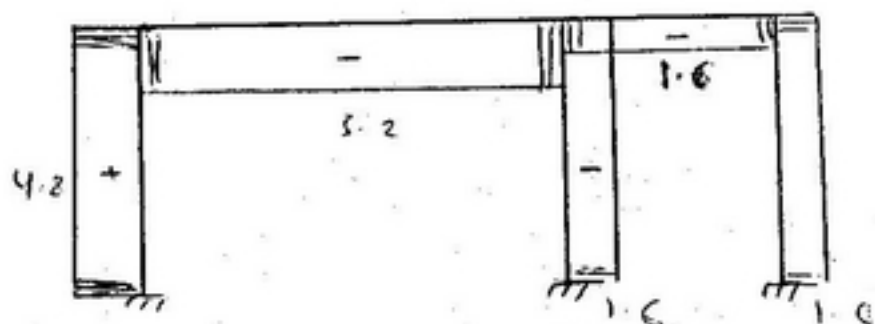
(4)



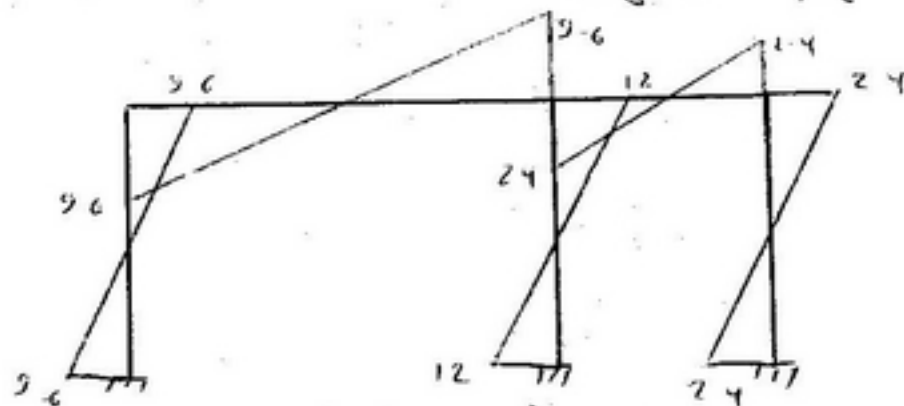
N.F.D



S.F.D



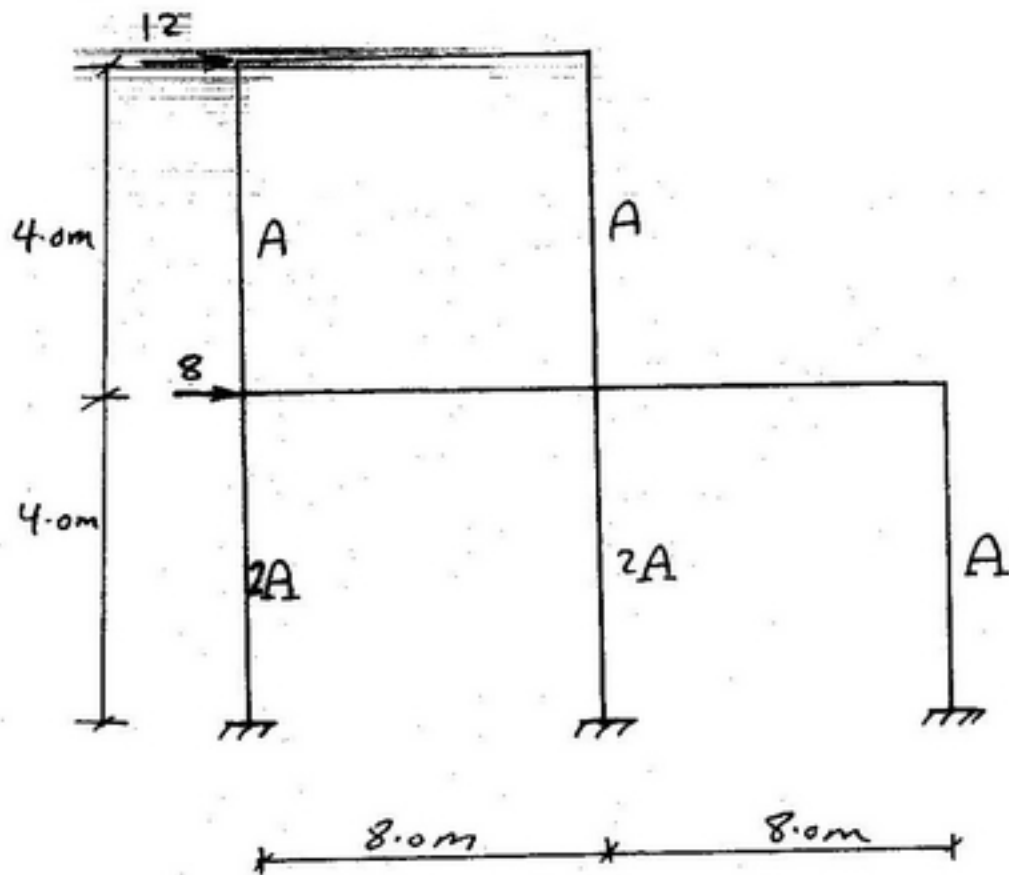
B.M.D



(5)



## approximate method

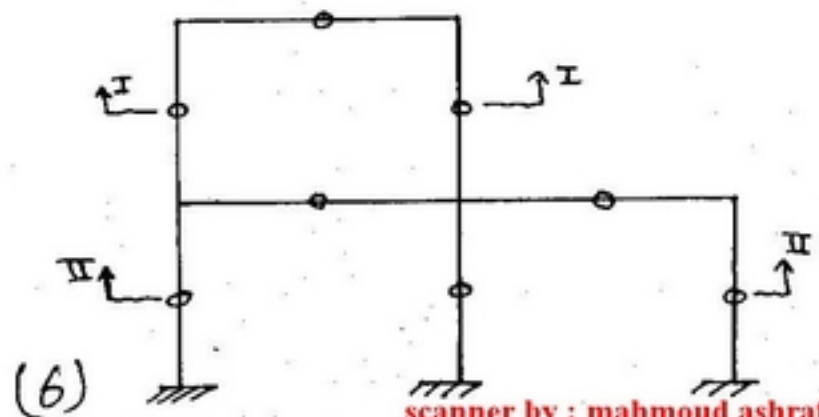


For the following frame draw N.F.D, S.F.D  
B.M.D due to :-

- (i) Portal Frame.
- (ii) Cantilivar method.

— Sol —

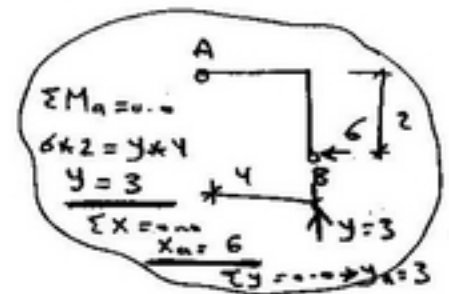
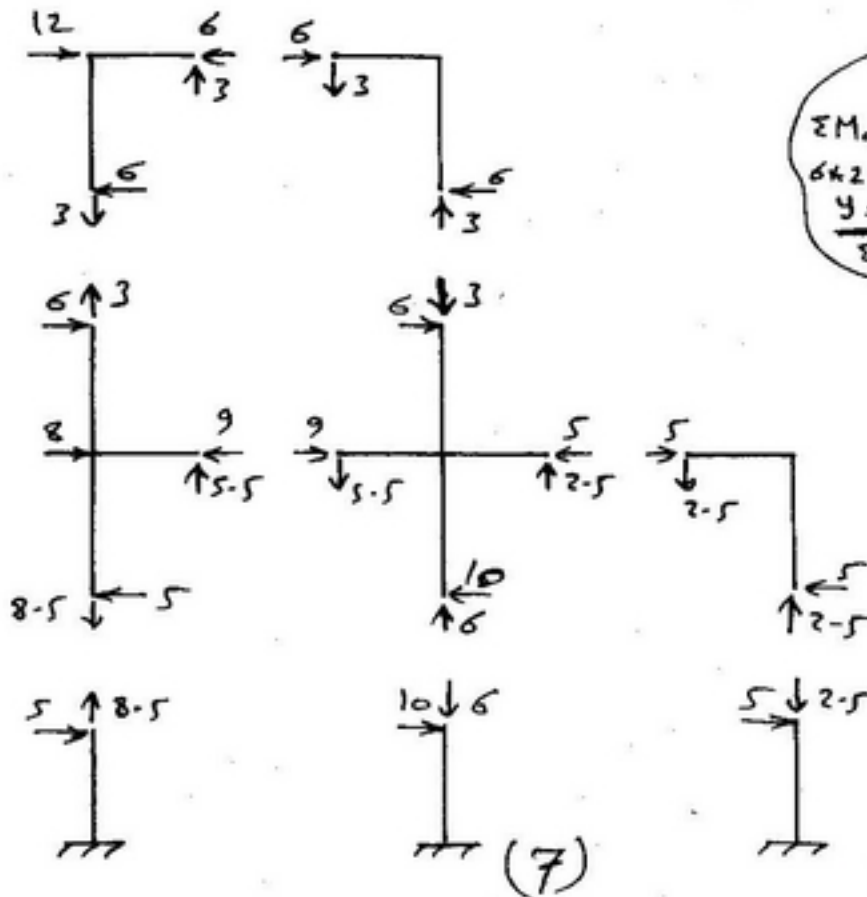
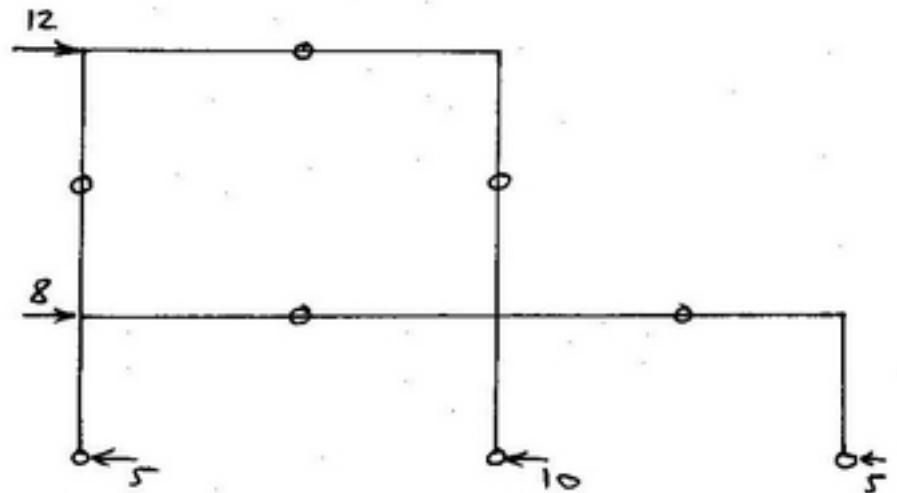
□ Portal Frame

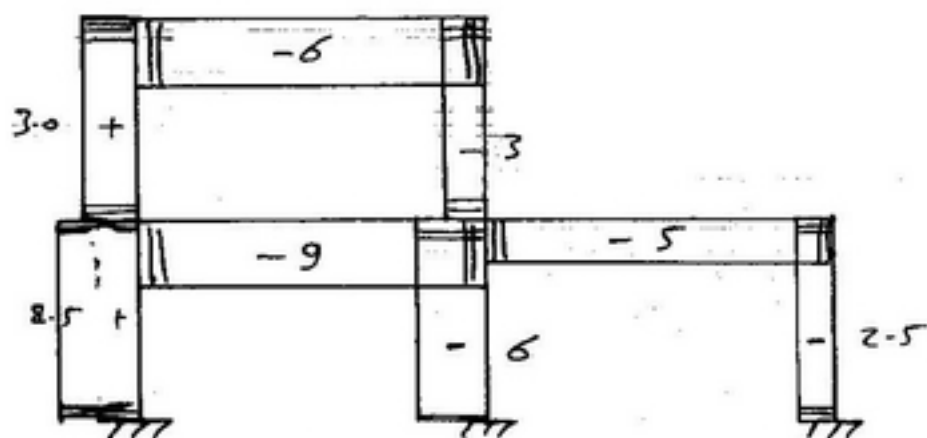


Sec I

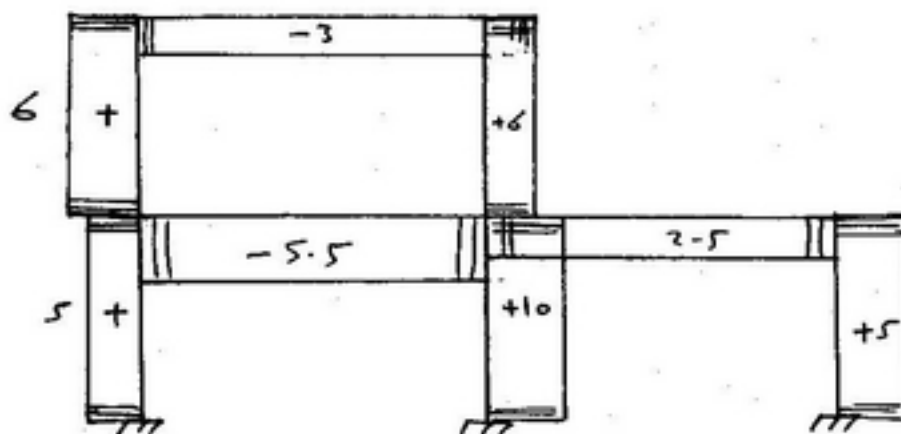


Sec II

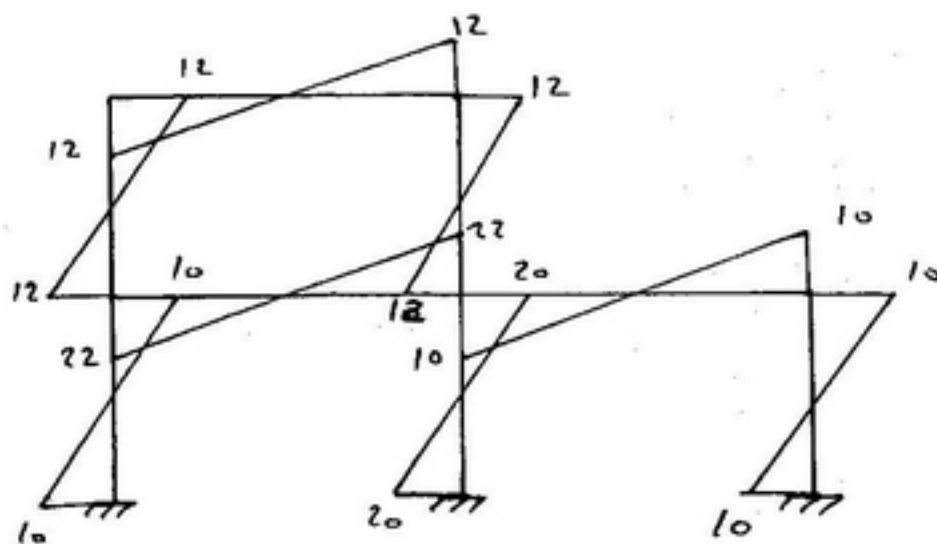




N.F.D



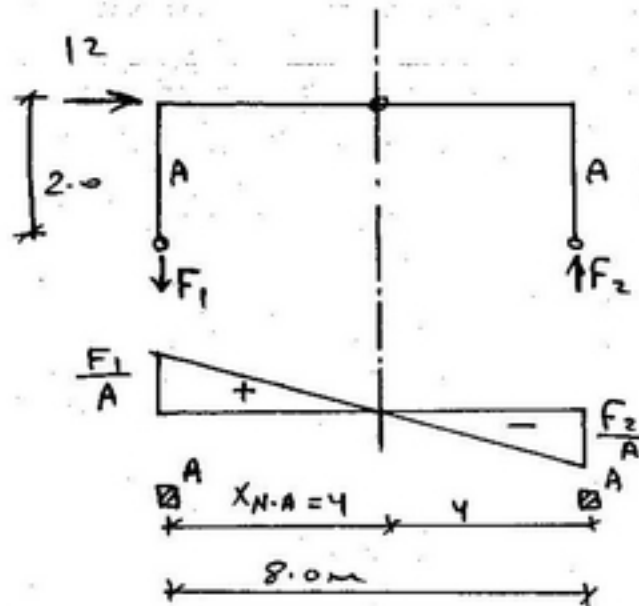
S.F.D



B.M.D

## [2] Cantilever method

Sec I-I



$$\times X_{N.A} = \frac{A \times 8}{A + A} = 4m$$

$$\times \frac{F_1/A}{4} = \frac{F_2/A}{4} \Rightarrow F_1 = F_2$$

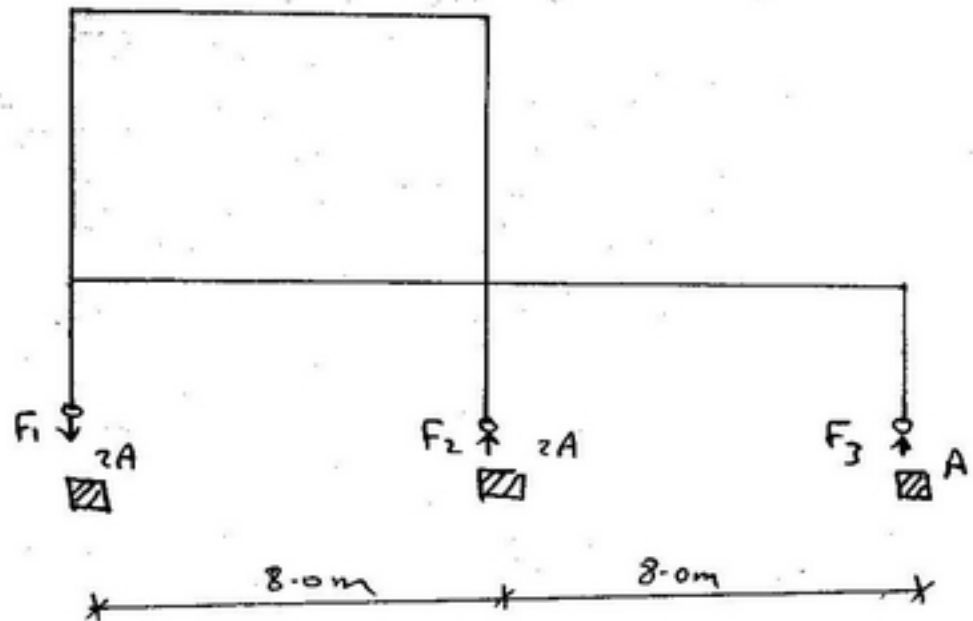
$$\times \sum M_{N.A} = 0$$

$$F_1 \times 4 + F_2 \times 4 = 12 \times 2$$

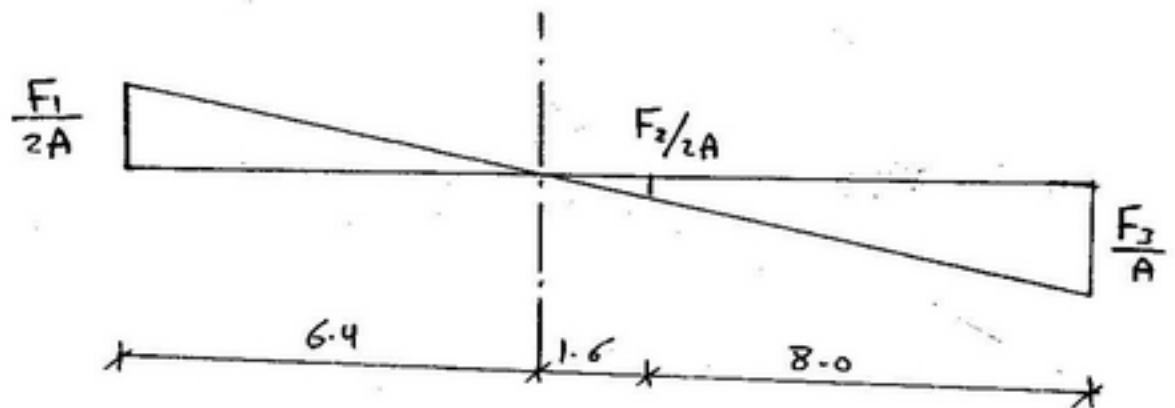
$$F_1 = F_2 = 3 \text{ ton}$$



Sec II-II



$$X_{N.A} = \frac{2A \times 8 + A \times 16}{(2A + 2A + A)} = 6.4 \text{ m}$$



$$\frac{F_1}{2A(6.4)} = \frac{F_2}{2A(1.6)} \Rightarrow F_1 = 4.0 F_2$$

$$\frac{F_2}{2A(1.6)} = \frac{F_3}{A(9.6)} \Rightarrow F_3 = 3 F_2$$

$$\underline{\Sigma M_{N-A} = 0}$$

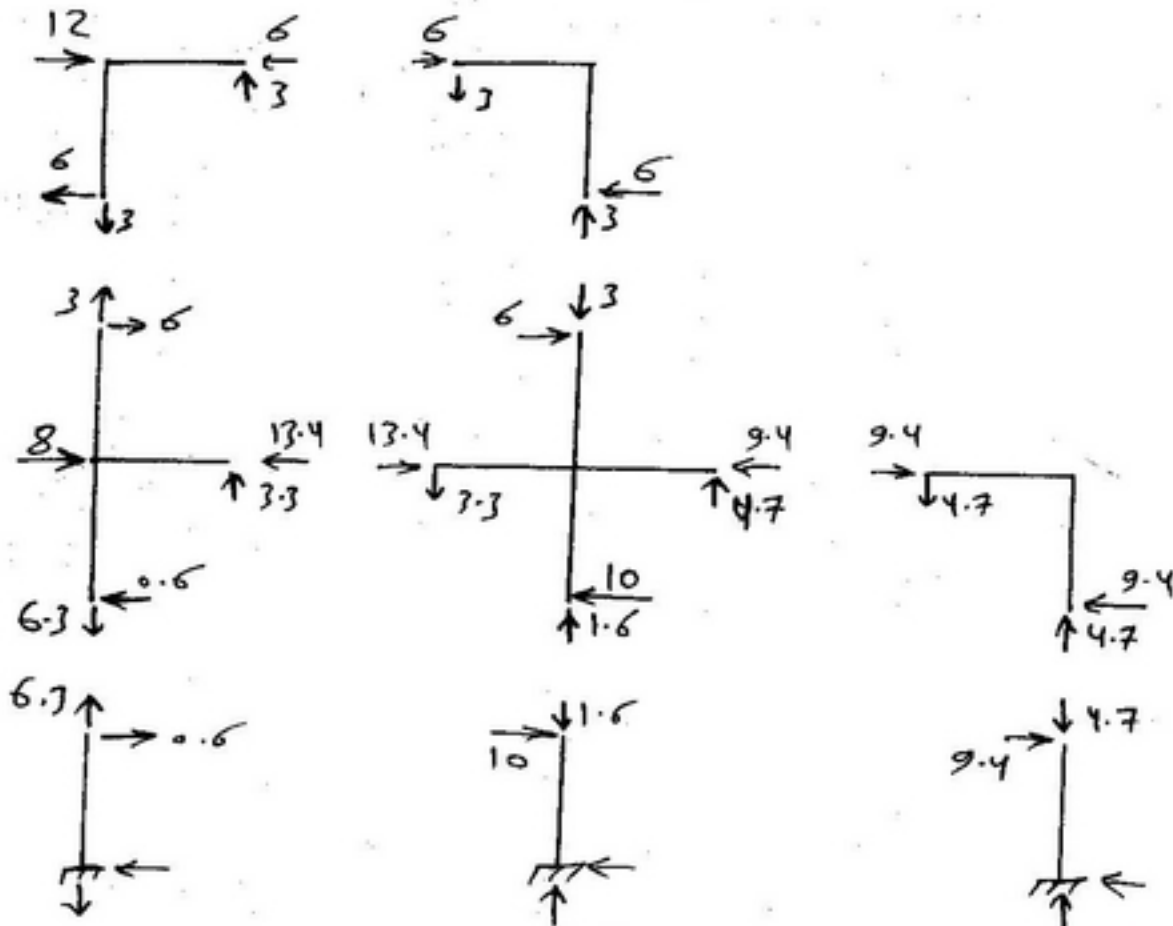
$$6.4 F_1 + 1.6 F_2 + 9.6 F_3 = 12 \times 6 + 8 \times 2$$

$$4 F_2 \times 6.4 + 1.6 F_2 + 9.6 (3 F_2) = 88$$

$$F_2 = 1.57 \text{ ton} \approx 1.6$$

$$F_3 = 4.714 \text{ ton} \approx 4.7$$

$$F_1 = 6.286 \text{ ton} \approx 6.3$$



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# structural analysis

No ( )

سید احمد اشرف

## Plates

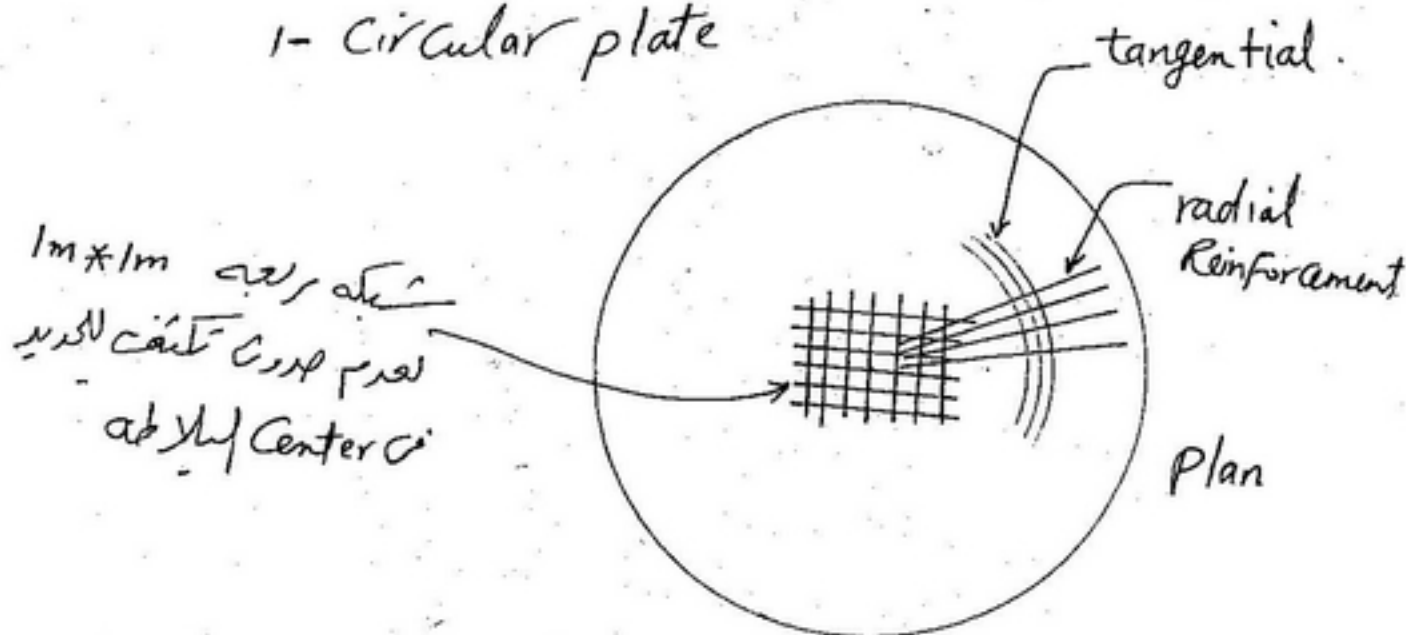
البلطات :- هي عبارة عن مساحة مستوية في اتجاهين لها

(شكل عدد 1) قد تكون دائرية (Coarse) - مستطيلة - مربعة

### \*\* classification of plates

#### (i) according to shape

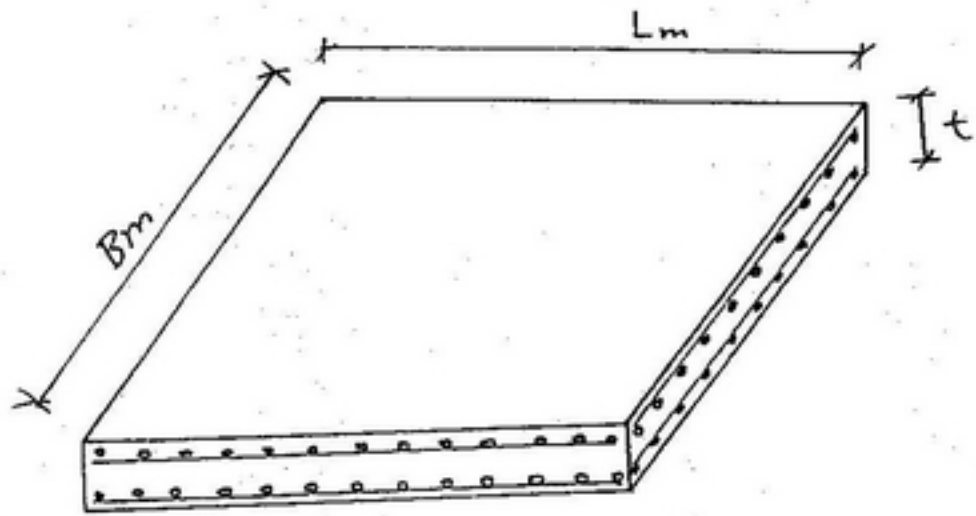
##### 1- Circular plate



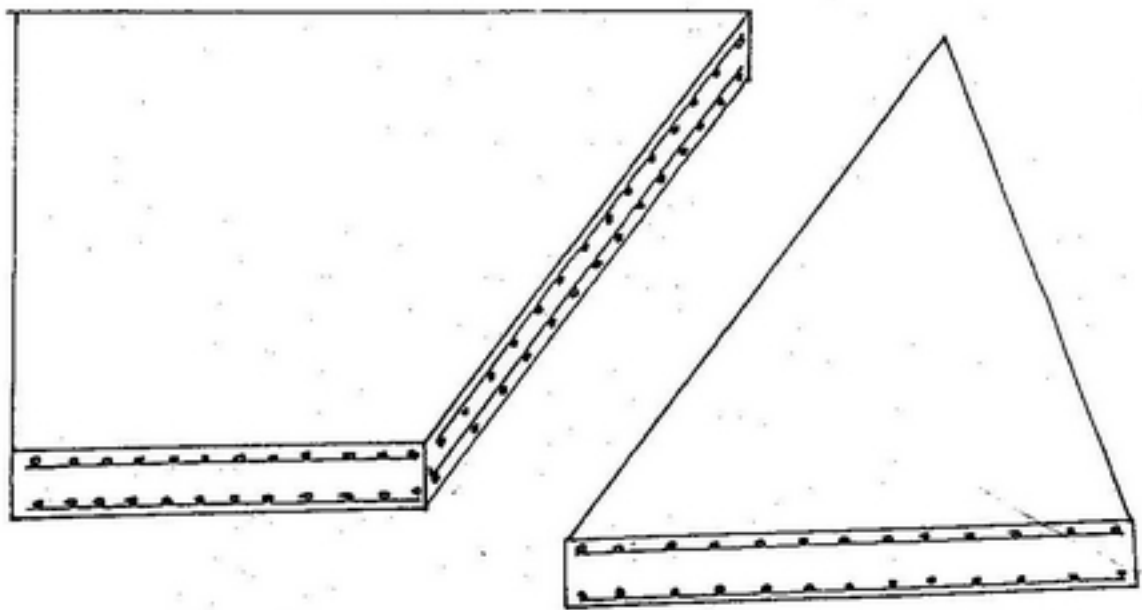
#### Section

شرح هذا النوع من  
البلطات Coarse



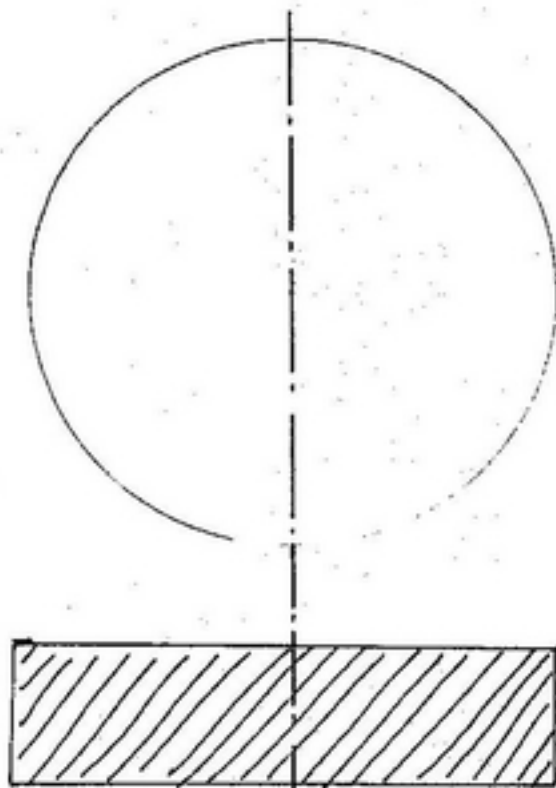


Rectangular

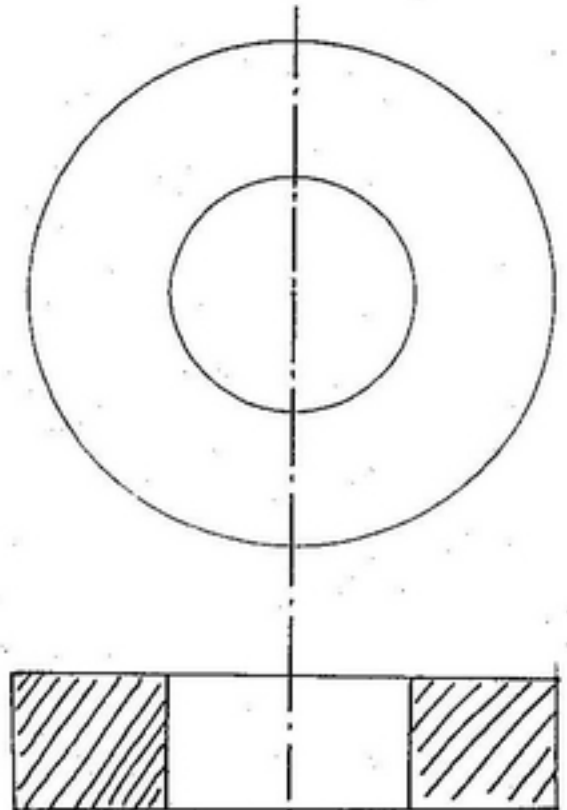


Trapezoidal

## type of circular plate

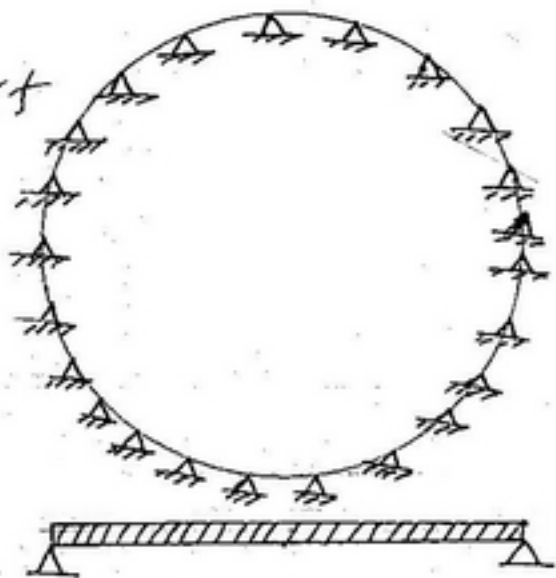


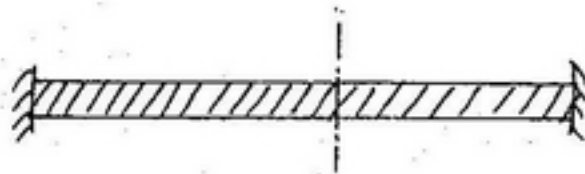
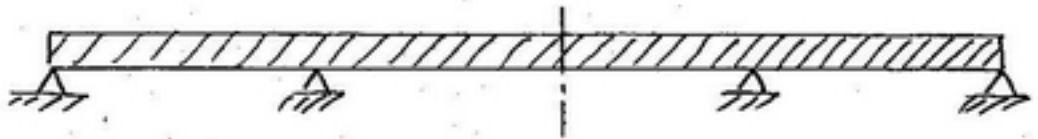
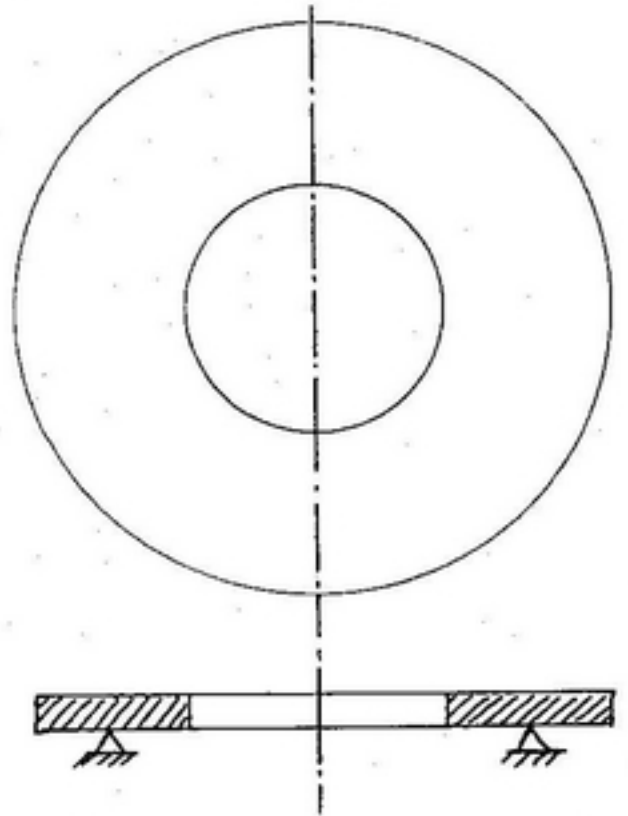
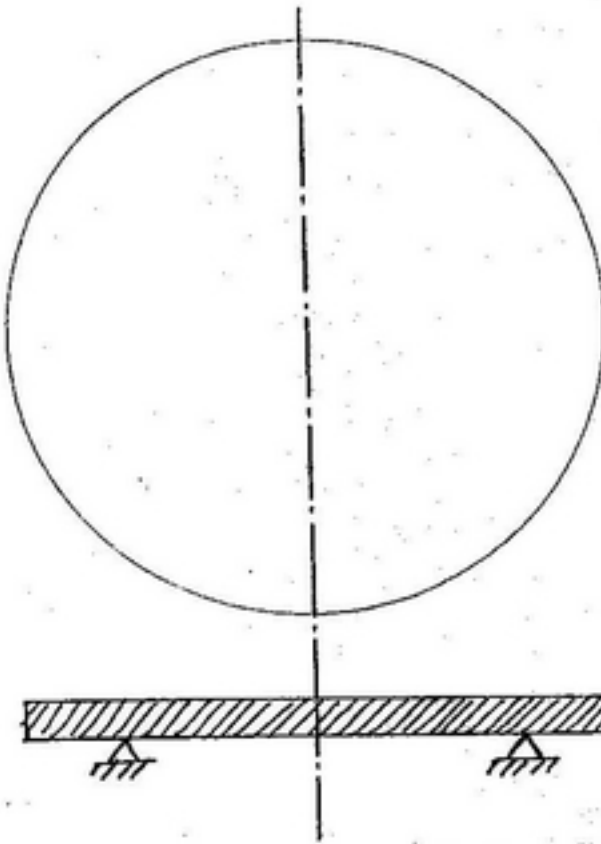
Solid

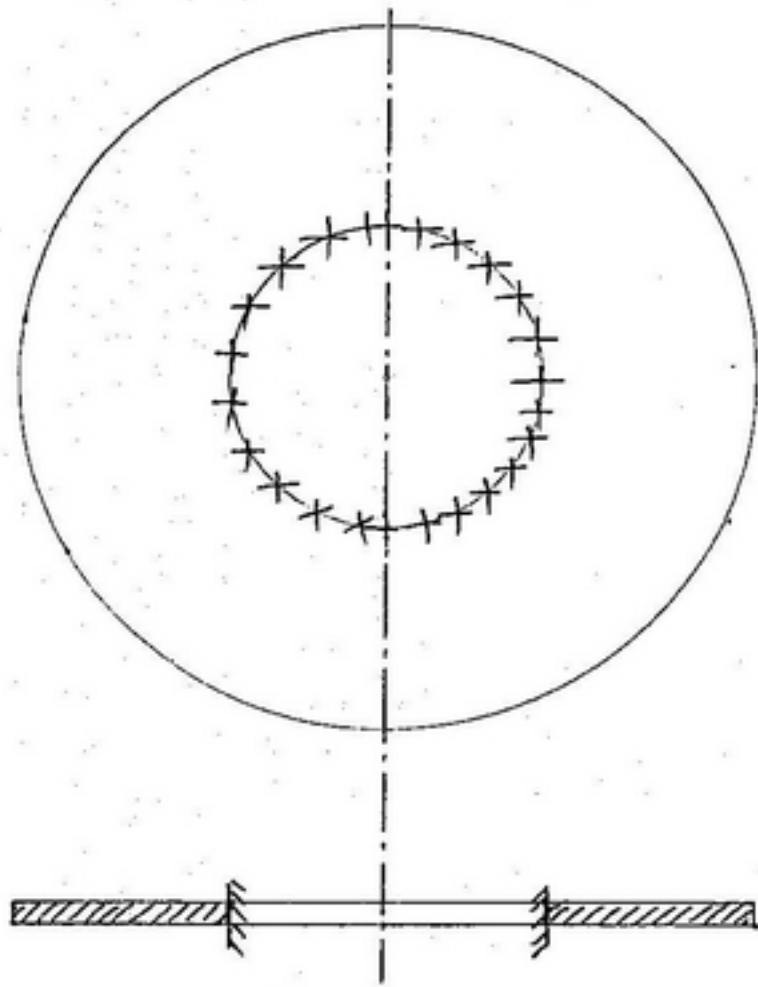


hollow slab

② according to support







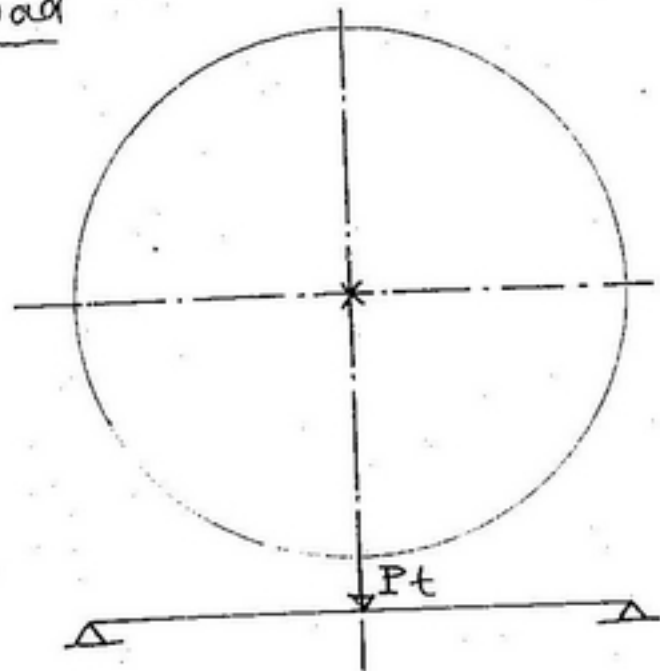
3- according to analysis point

- 1- thin plate with small deflection
- 2- thin plate with large deflection
- 3- thick plates.

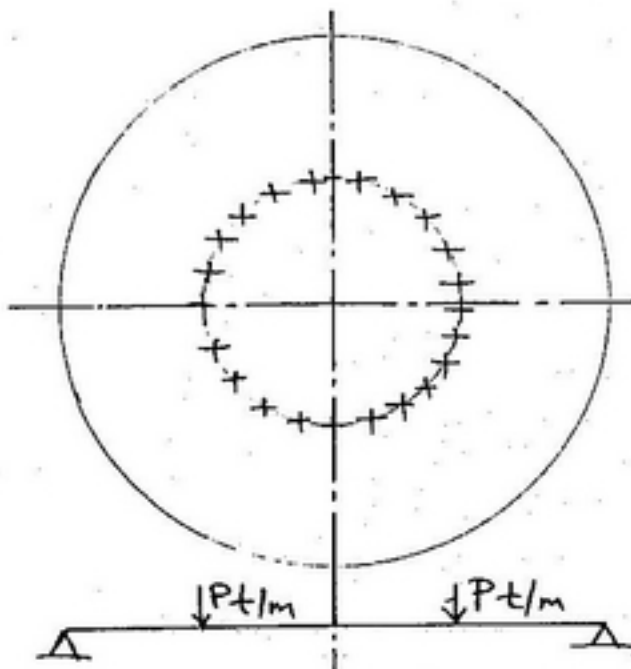


due to loads

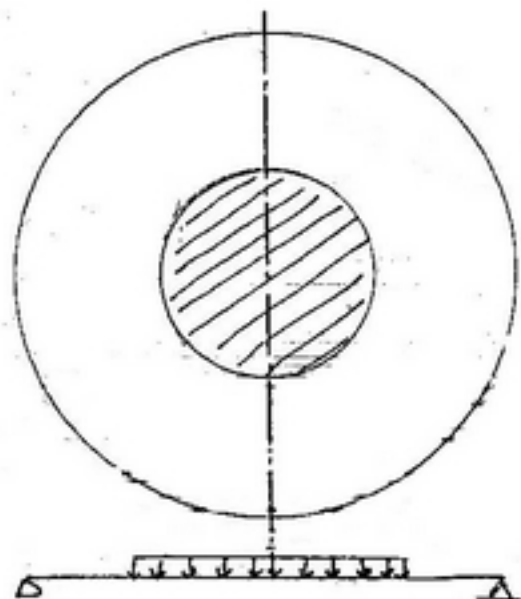
(i) Concentrated Load

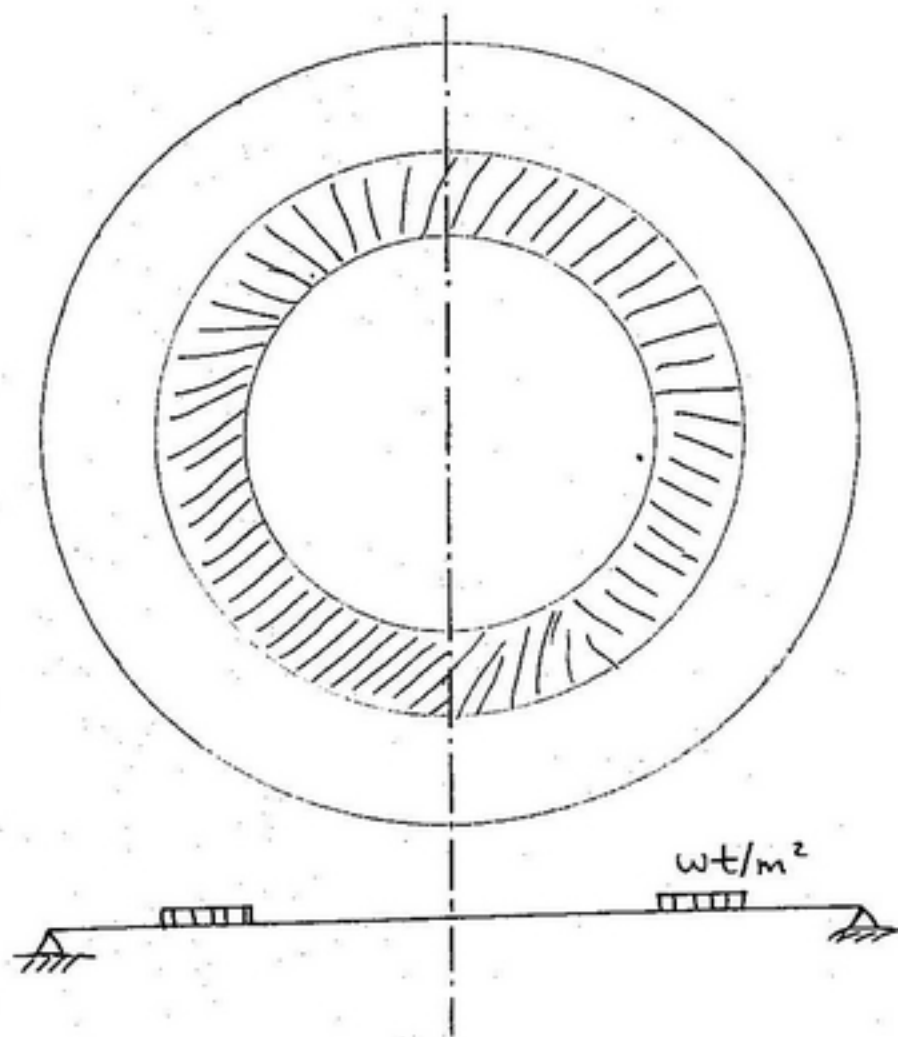


(ii) Line Load



(iii) area load





Some Integration Required

$$- \int r^n \cdot dr = \boxed{\frac{r^{n+1}}{n+1}}$$

$$- \int \frac{1}{r} \cdot dr = \boxed{\ln r}$$

$$= \int F \cdot \ln r - \frac{r^2}{2} \ln r - \int \frac{r^2}{2} \cdot \frac{1}{r} = \boxed{\frac{r^2}{2} \ln r - \frac{r^2}{4}}$$

Some differentiation Required

$$\frac{\partial}{\partial r} \cdot r^n = n r^{n-1}$$

$$\frac{\partial}{\partial r} \cdot \frac{1}{r} = - \frac{1}{r^2}$$

$$\frac{\partial}{\partial r} (r \cdot \ln r) = \ln r + 1$$

$$\ln 1 = 0.0$$

$$\ln 0 = \infty$$

$$\ln \infty = \infty$$

Here

1- Calculate the equation for  $(Q)$  for each Interval.

2- Sub - in  $\boxed{\frac{d}{dr} \left[ \frac{1}{r} \cdot \frac{d}{dr} (r\phi) \right] = - \frac{Q}{D}}$

3- double Integration

obtain equation  $(\phi)$  with Constants  $(C_1, C_2)$

4- take Boundary Condition obtain Constants

5-  $M_r, M_\theta$

$\boxed{\mu = 0.3}$

$$M_r = D \left[ \frac{\partial \phi}{\partial r} + \mu \frac{\phi}{r} \right]$$

$$M_\theta = D \left[ \frac{\phi}{r} + \mu \frac{\partial \phi}{\partial r} \right]$$

6-  $w \longrightarrow \phi = - \frac{\partial w}{\partial r}$



$$\mu \rightarrow \text{poission Ratio} = \frac{\text{الانفعال العرضي}}{\text{الانفعال الطولي}}$$

$$D = \text{plate rigidity} = \frac{Et^3}{12(1-\mu)}$$

## Boundary Condition

Closed plate v. sh.  $\leftarrow \phi = 0.0$  f. l. h. i. s. ①

$\phi = 0.0$   $\leftarrow$  Fixed f. h. i. s. ②

$M_r = 0.0$   $\leftarrow$  hinge f. h. i. s. ③

Tr. is  $\xrightarrow{M}$  f. h. i. s. v. b. l. d. l. v. l.   
  $M_r = M$

$M_r = 0.0$   $\leftarrow$  Cantilever f. h. i. s. ④

Tr. is  $\xrightarrow{M}$  f. h. i. s. v. b. l. d. l. v. l.   
  $M_r = M$

$\Phi_1 = \Phi_2$   $\leftarrow$  v. l. v. l. i. s. ⑤

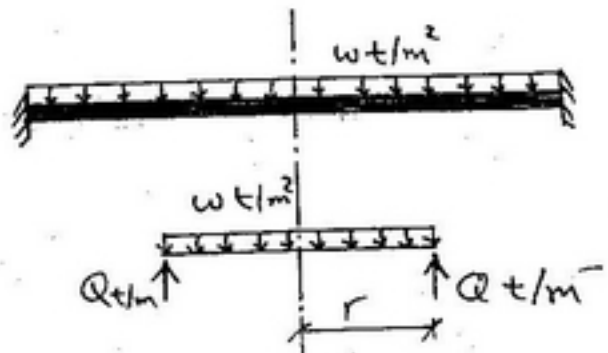
$M_1 = M_2$   $\leftarrow$

$Q \leftarrow$  Shear force

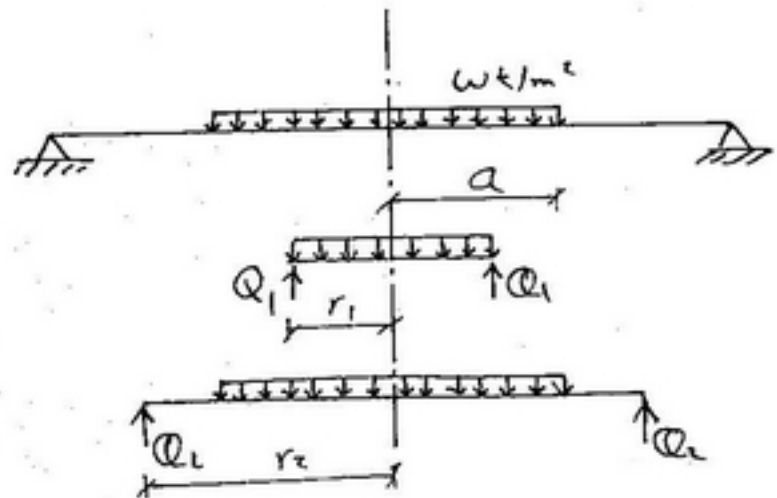
III one Interval

$$w \pi r^2 = Q (2\pi r)$$

$$Q = \frac{wr}{2}$$



II two Interval



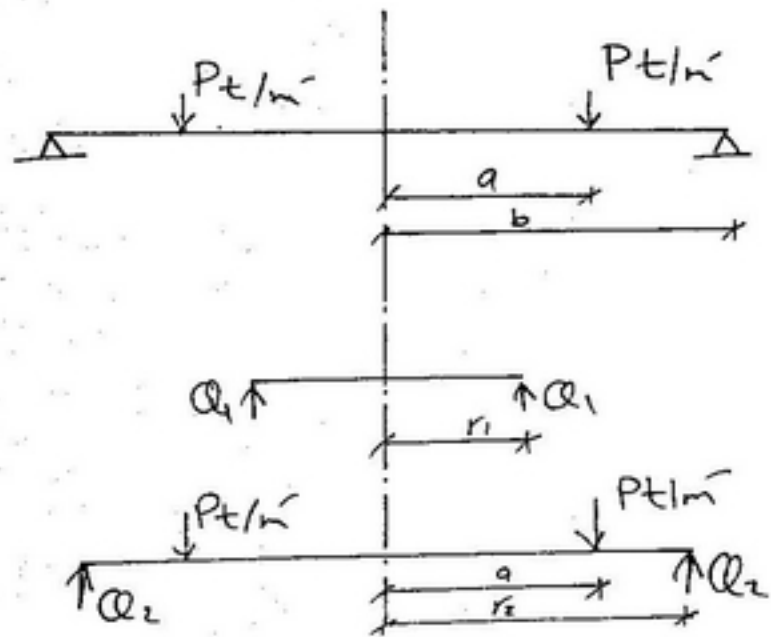
First

$$Q_1 (2\pi r_1) = w (\pi r_1^2)$$

Second  $Q_1 = \frac{w r_1}{2}$

$$w (\pi a^2) = Q_2 (2\pi r_2)$$

$$Q_2 = \frac{w a^2}{2\pi r_2}$$



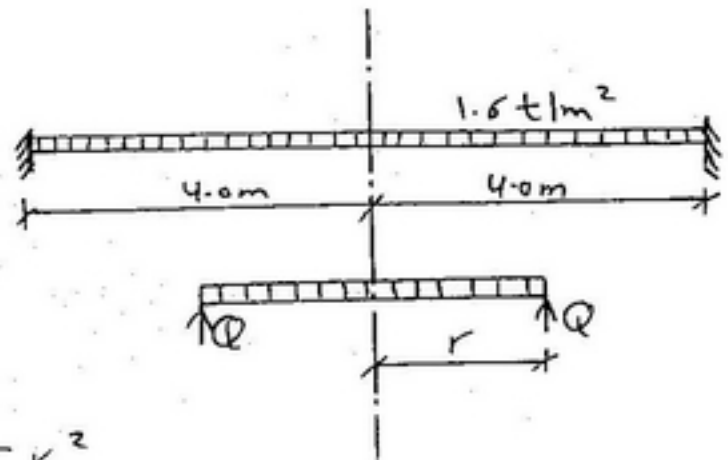
$$Q_1 = \text{Zero}.$$

$$Q_2 (2\pi r_2) = P_* (2\pi a)$$

$$\therefore Q_2 = P \cdot \frac{a}{r_2}$$



Final 2005



$$Q(2\pi r) = 1.6 \times \pi r^2$$

$$Q = \frac{1.6r}{2} = 0.8r$$

$$\frac{\partial}{\partial r} \left[ \frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) \right] = - \frac{Q}{D}$$

$$\frac{\partial}{\partial r} \left[ \frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) \right] = - \frac{1}{D} [0.8r]$$

$$\frac{\partial}{\partial r} \left[ \frac{1}{r} \cdot \frac{\partial}{\partial r} [r\phi] \right] = - \frac{1}{D} [0.8r]$$

$$\frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) = - \frac{1}{D} \left[ \frac{0.8r^2}{2} + C_1 \right]$$

$$\frac{\partial}{\partial r} (r\phi) = \frac{1}{D} [0.4r^3 + C_1 \cdot r]$$

$$r\phi = -\frac{1}{D} \left[ 0.1 r^4 + \frac{C_1}{2} r^2 + C_2 \right]$$

$$\phi = -\frac{1}{D} \left[ 0.1 r^3 + \frac{C_1}{2} r + \frac{C_2}{r} \right]$$

B.C ① at  $r=0.0 \Rightarrow \phi = 0.0$

$$\Rightarrow C_2 = 0.0$$

② at  $r=4.0 \Rightarrow \phi = 0.0$

$$4 \times 0 = -\frac{1}{D} \left[ 0.1 (4)^4 + \frac{C_1}{2} (4)^2 \right]$$

$$\Rightarrow C_1 = -3.2$$

$$\therefore \phi = -\frac{1}{D} \left[ 0.1 r^3 - 1.6 r \right]$$

$$\frac{\partial \phi}{\partial r} = -\frac{1}{D} \left[ 0.3 r^2 - 1.6 \right]$$

at  $\mu = 0.0$

$$M_r = D \left[ -\frac{1}{D} [0.3r^2 - 1.6] + \mu \times -\frac{1}{D} [0.1r^2 - 1.6] \right]$$

$$\therefore M_r = - (0.3r^2 - 1.6)$$

$$M_\theta = D \left[ -\frac{1}{D} [0.1r^2 - 1.6] + \mu \times -\frac{1}{D} [0.3r^2 - 1.6] \right]$$

$$\therefore M_\theta = - (0.1r^2 - 1.6)$$

|            |     |     |     |      |      |
|------------|-----|-----|-----|------|------|
| $r$        | 0.0 | 1.0 | 2.0 | 3.0  | 4.0  |
| $M_r$      | 1.6 | 1.3 | 0.4 | -1.1 | -3.2 |
| $M_\theta$ | 1.6 | 1.5 | 1.2 | 0.7  | 0.0  |

for drawing deformation

$$\phi = \frac{\partial w}{\partial r}$$

$$\partial w = \phi \cdot \partial r$$

$$w = \int \left( -\frac{1}{D} [0.1 r^3 - 1.6 r] \right) dr$$

$$= -\frac{1}{D} \left[ \frac{0.1 r^4}{4} - \frac{1.6 r^2}{2} + C_3 \right]$$

$$\text{at } r = 4.0 \quad \Rightarrow \quad w = 0.0$$

$$0.0 = \frac{0.1 (4)^4}{4} - \frac{1.6 (4)^2}{2} + C_3$$

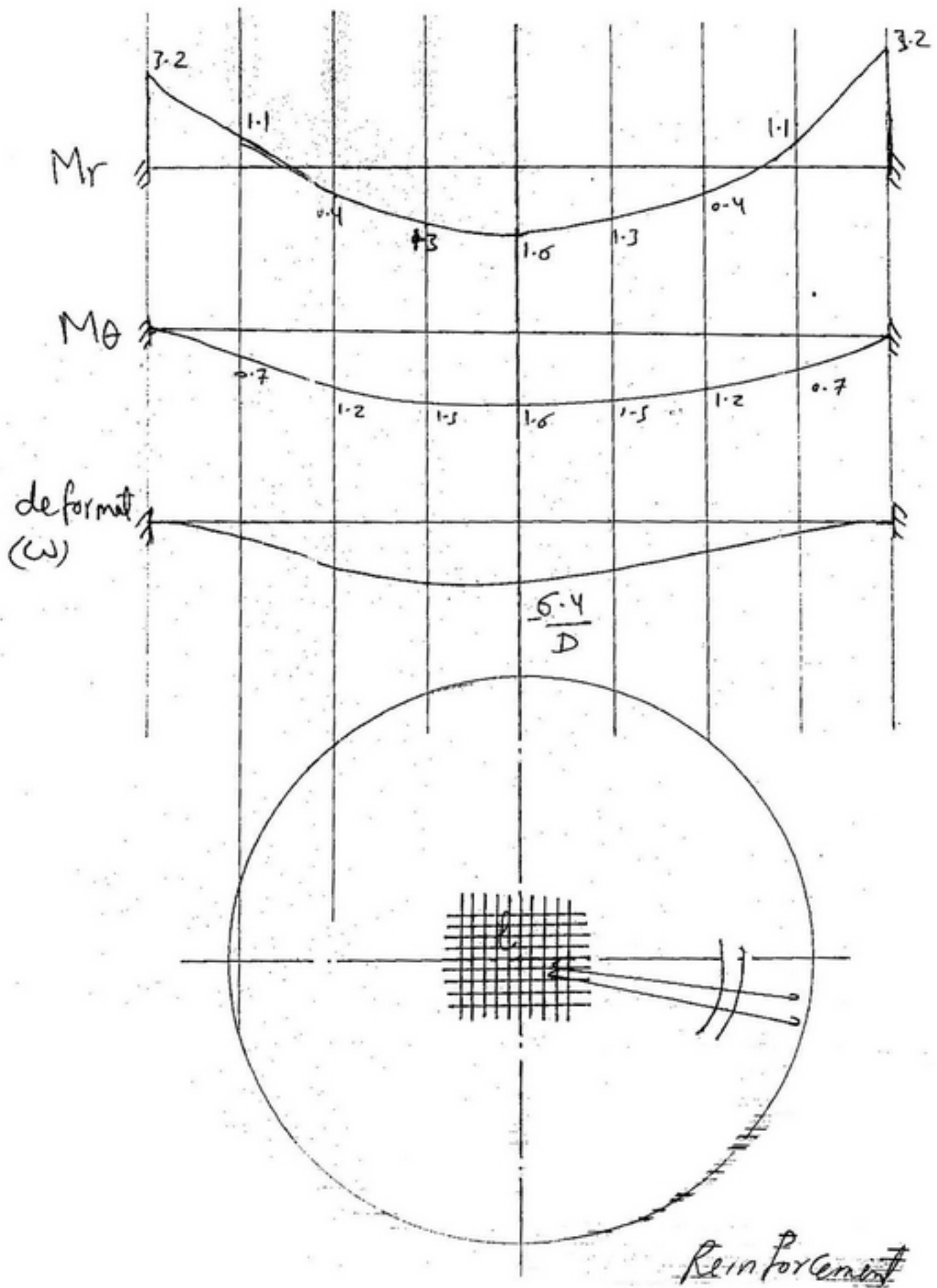
$$C_3 = 6.4$$

$$\therefore w = -\frac{1}{D} \left[ \frac{0.1}{4} r^4 - 0.8 r^2 + 6.4 \right]$$

$$\therefore w_{\max} \text{ at } r = 0.0$$

$$\therefore w = -\frac{6.4}{D}$$





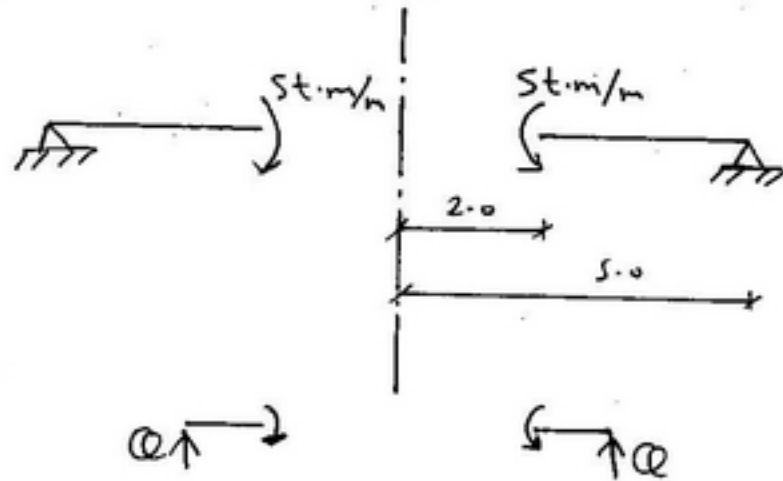
Faculty of engineering  
3rd year

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# structural analysis

No (9)

b-



$$Q = 0.0$$

$$\frac{\partial}{\partial r} \left[ \frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) \right] = - \frac{1}{D} [0]$$

$$\frac{1}{r} \cdot \frac{\partial}{\partial r} [r\phi] = - \frac{1}{D} [C_1] \quad *r$$

$$\frac{\partial}{\partial r} (r\phi) = - \frac{1}{D} [C_1 r]$$

$$r\phi = - \frac{1}{D} \left[ C_1 \frac{r^2}{2} + C_2 \right]$$

$$\phi = - \frac{1}{D} \left[ \frac{C_1 r}{2} + \frac{C_2}{r} \right]$$

B.C

$$\text{at } r = 2 \longrightarrow M_r = -5$$

$$\text{at } r = 5 \longrightarrow M_r = 0.0$$

(11)

$$\frac{\partial \phi}{\partial r} = -\frac{1}{D} \left[ \frac{C_1}{2} - \frac{C_2}{r^2} \right]$$

$$\therefore M_r = D \left[ \frac{\partial \phi}{\partial r} + \frac{1}{r} \left( \frac{\partial \phi}{\partial r} \right) \right]$$

$$M_r = -\frac{C_1}{2} + \frac{C_2}{r^2}$$

$$\text{at } r=2 \longrightarrow M_r = -5$$

$$\therefore -5 = -\frac{C_1}{2} + \frac{C_2}{4}$$

$$2C_1 - C_2 = 20 \longrightarrow \textcircled{1}$$

$$\text{at } r=5 \longrightarrow M_r = 0.0$$

$$0.0 = -\frac{C_1}{2} + \frac{C_2}{25}$$

$$12.5C_1 - C_2 = 0.0 \longrightarrow \textcircled{2}$$

$$C_1 = -1.9$$

$$C_2 = -23.8$$

(2)



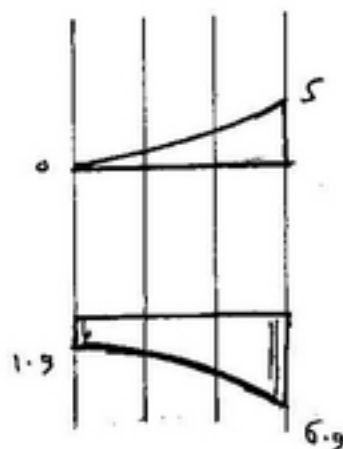
$$\therefore \phi = -\frac{1}{D} \left[ -0.95 r - \frac{23.8}{r} \right]$$

$$\frac{\phi}{r} = -\frac{1}{D} \left[ -0.95 - \frac{23.8}{r^2} \right]$$

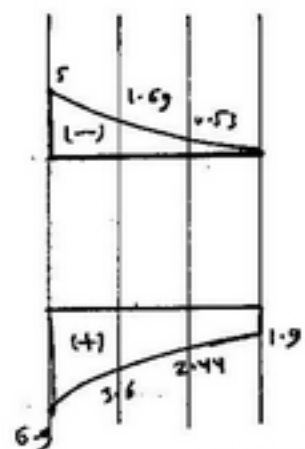
$$M_r = 0.95 - \frac{23.8}{r^2}$$

$$M_\theta = +0.95 + \frac{23.8}{r^2}$$

| r          | 2   | 3     | 4     | 5   |
|------------|-----|-------|-------|-----|
| $M_r$      | -5  | -1.69 | -0.53 | 0.0 |
| $M_\theta$ | 6.9 | 3.6   | 2.44  | 1.9 |



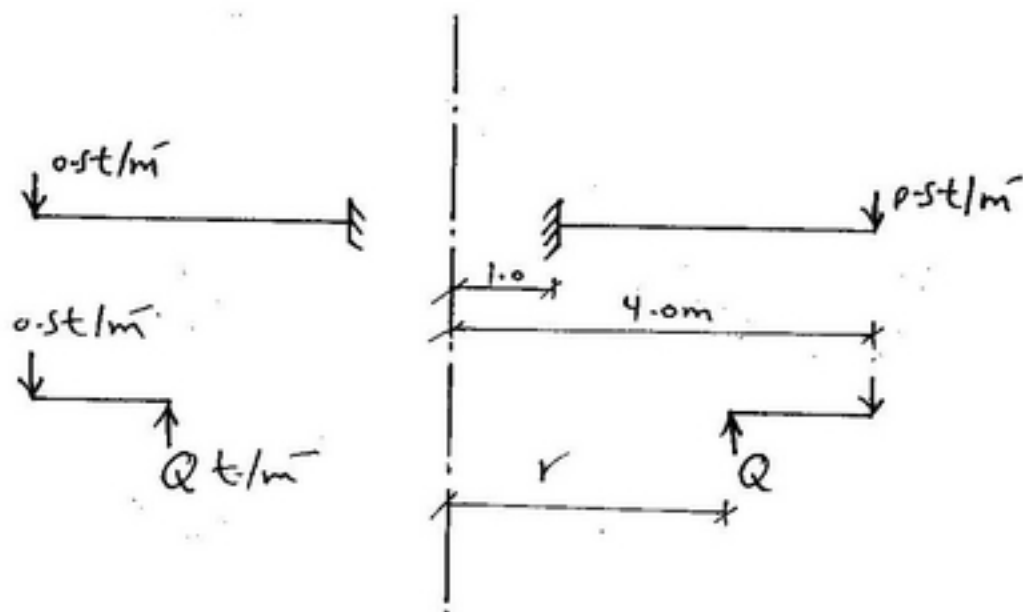
(3)



$M_r$

$M_\theta$

Final 2004



$$Q \times 2\pi r = -0.5 \times (2\pi \times 4)$$

$$Q = -\frac{2}{r}$$

$$\frac{\partial}{\partial r} \left( \frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) \right) = -\frac{1}{D} \left[ -\frac{2}{r} \right]$$

$$\frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) = \frac{1}{D} [2 \ln r + C_1]$$

$$\frac{\partial}{\partial r} [r\phi] = \frac{1}{D} [2r \ln r + C_1 r]$$

$$r\phi = \frac{1}{D} \left[ 2 \left( \frac{r^2}{2} \ln r - \frac{r^2}{4} \right) + \frac{C_1 r^2}{2} + C_2 \right]$$

$$\phi = \frac{1}{D} \left[ r \ln r - \frac{r}{2} + \frac{C_1}{2} r + \frac{C_2}{r} \right]$$

at  $r=1.0$

$$\Rightarrow \Phi = 0.0$$

$$0.0 = 0.0 = \Phi$$

$$= r \ln r - \frac{r}{2} + r \cdot \frac{C_1}{2} + \frac{C_2}{r}$$

$$= \ln 1 - \frac{1}{2} + \frac{C_1}{2} + C_2$$

$$\boxed{C_1 + 2C_2 = 1.0 \longrightarrow (1)}$$

at  $r=4.0$

$$\Rightarrow M_r = 0.0$$

$$M_r = \left[ \frac{\partial \Phi}{\partial r} + 0 \right]$$

$$0.0 = 1 + \ln r - \frac{1}{2} + \frac{C_1}{2} - \frac{C_2}{r^2}$$

$$= 1 + \ln 4 - \frac{1}{2} + \frac{C_1}{2} - \frac{C_2}{16}$$

$$\boxed{8C_1 - C_2 = -30.18 \longrightarrow (2)}$$

by solve ① & ②

$$C_1 = -3.5$$

$$C_2 = 2.25$$

$$\therefore \phi = \frac{1}{D} \left[ r \ln r - \frac{r}{2} - 1.75 r + \frac{2.25}{r} \right]$$

$$\frac{\partial \phi}{\partial r} = \frac{1}{D} \left[ \ln r + 1 - \frac{1}{2} - 1.75 - \frac{2.25}{r^2} \right]$$

$$M_r = D \left[ \frac{\partial \phi}{\partial r} + \nu \frac{\phi}{r} \right]$$

$$\therefore M_r = \ln r - 1.25 - \frac{2.25}{r^2}$$

$$M_\theta = D \left[ \frac{\phi}{r} \right]$$

$$= \ln r - \frac{1}{2} - 1.75 + \frac{2.25}{r^2}$$

$$\therefore M_\theta = \ln r - 2.25 + \frac{2.25}{r^2}$$

⑦

$$w = \int \phi \cdot dr$$

$$= \frac{1}{D} \left[ \frac{r^2}{2} \ln r - \frac{r^2}{4} - \frac{r^2}{4} - \frac{1.75r^2}{2} + 2.25 \ln r + C_3 \right]$$

$$= \frac{1}{D} \left[ \frac{r^2}{2} \ln r - 1.375 r^2 + 2.25 \ln r + C_3 \right]$$

$$\text{at } r = 1.0 \Rightarrow w = 0.0$$

$$0.0 = \frac{1}{D} \left[ \frac{r^2}{2} \ln 1 - 1.375 + 2.25 \ln 1 + C_3 \right]$$

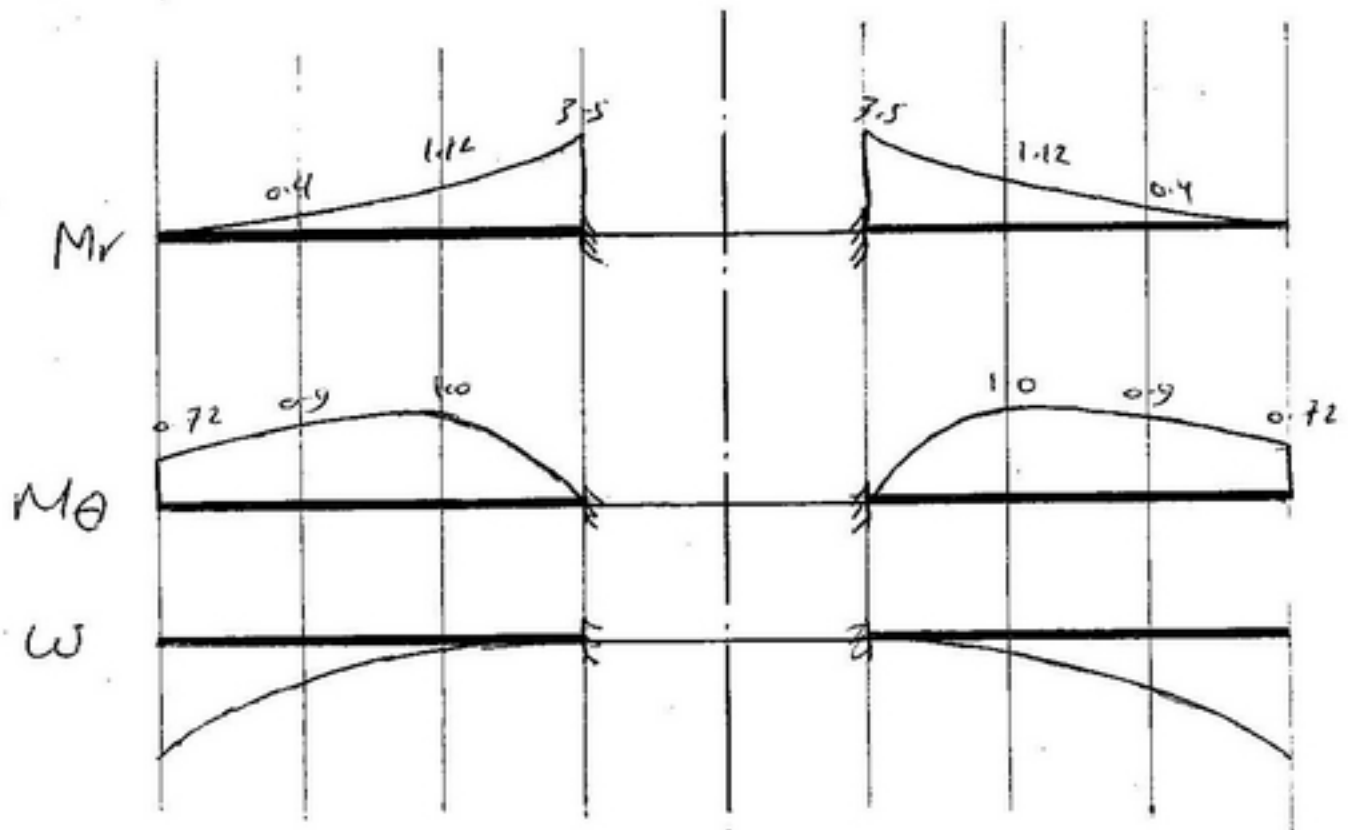
$$\Rightarrow \boxed{C_3 = 1.375}$$

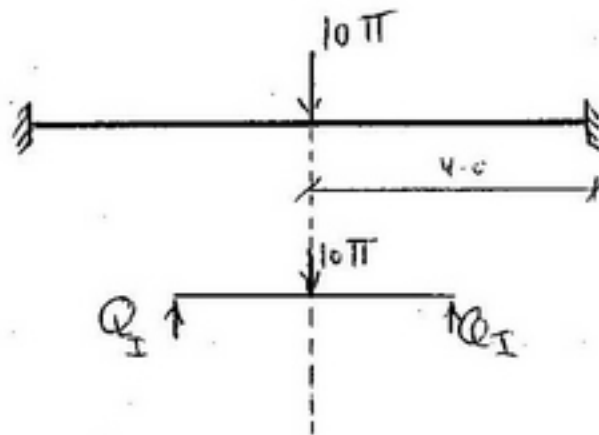
$$\therefore w = \frac{1}{D} \left[ \frac{r^2}{2} \ln r - 1.375 r^2 + 2.25 \ln r + 1.375 \right]$$

$$D = \frac{E t^3}{12(1-\mu^2)} = \frac{200 \times 10^4 \times 0.3^3}{12} = 4500$$

| r  | 1    | 2     | 3     | 4     |
|----|------|-------|-------|-------|
| Mr | -3.5 | -1.12 | -0.4  | 0.0   |
| Mθ | 0.0  | -1    | -0.9  | -0.72 |
| w  | 0.0  | -0.26 | -1.16 | 1.4   |







$$Q = \frac{10\pi}{2\pi r} = \frac{5}{r}$$

$$\frac{\partial}{\partial r} \left[ \frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) \right] = - \frac{1}{D} \left[ \frac{5}{r} \right]$$

$$\frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) = - \frac{5}{D} [\ln r + c_1]$$

$$\frac{\partial}{\partial r} (r\phi) = - \frac{5}{D} [r \ln r + c_1 \cdot r]$$

$$r\phi = - \frac{5}{D} \left[ \frac{r^2}{2} \ln r - \frac{r^2}{4} + \frac{c_1}{2} r^2 + c_2 \right]$$

$$\phi = - \frac{5}{D} \left[ \frac{r}{2} \ln r - \frac{r}{4} + \frac{c_1}{2} r + \frac{c_2}{r} \right]$$

B.C

at  $r = 0$

$$\Rightarrow \phi = 0$$

$$c.r.o = 0 \cdot \overset{\infty}{\ln 0} - 0 + 0 + C_2$$

(0  $\times$   $\infty$ )

By L'Hopital's law.

$$\lim_{r \rightarrow 0} \left( \frac{r^2}{2} \ln r \right) = \frac{1}{2} \lim_{r \rightarrow 0} \left( \frac{\ln r}{(1/r^2)} \right)$$

$$= \frac{1}{2} \lim_{r \rightarrow 0} \left( \frac{1/r}{-2/r^3} \right)$$

$$= \lim_{r \rightarrow 0} \left( -\frac{r^2}{4} \right) = 0.0$$

$$\therefore C_2 = 0.0$$

At  $r = 4.0 \Rightarrow \phi = 0.0$

$$c.r.o = [2 \ln 4 - 1 + 2C_1 + 0]$$

$$C_1 = -0.886$$

$$\therefore \phi = -\frac{S}{D} \left[ \frac{r}{2} \ln r - \frac{r}{4} - 0.443r \right]$$

$$\frac{\partial \phi}{\partial r} = -\frac{S}{D} \left[ \frac{1}{2} [\ln r + 1] - \frac{1}{4} - 0.443 \right]$$

$$= -\frac{S}{D} \left[ \frac{1}{2} \ln r - 0.193 \right]$$

$$\therefore M_r = D \left[ -\frac{S}{D} \left[ \frac{1}{2} \ln r - 0.193 \right] \right]$$

$$= -2.5 \ln r + 0.965$$

$$M_\theta = D \left[ -\frac{S}{D} \left[ \frac{1}{2} \ln r - 0.693 \right] \right]$$

$$= -2.5 \ln r + 3.465$$

$$\omega = \int \phi \cdot \partial r$$

$$= -\frac{S}{D} \left[ \frac{1}{2} \left[ \ln r \cdot \frac{r^2}{2} - \frac{r^2}{4} \right] - \frac{0.69 r^2}{2} + C_3 \right]$$

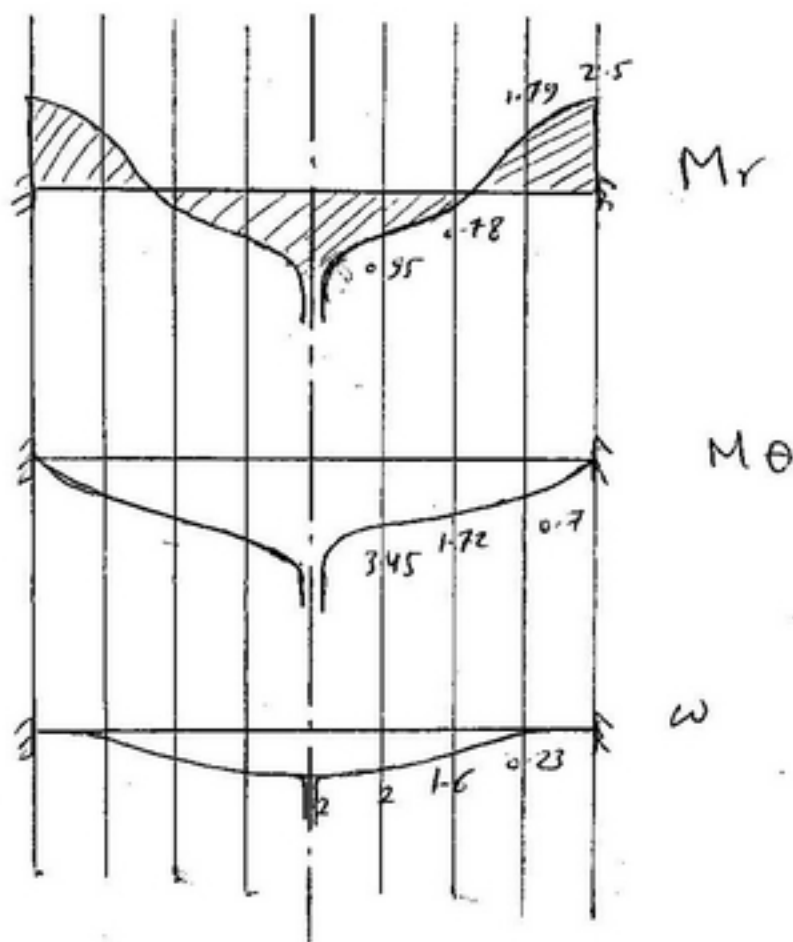
$$\text{at } r = 4.0 \Rightarrow \omega = 0.0$$

$$\therefore D = \frac{200 \times 10^4 \times 0.3^3}{12} = 4500$$

$$C_3 = 1.975$$

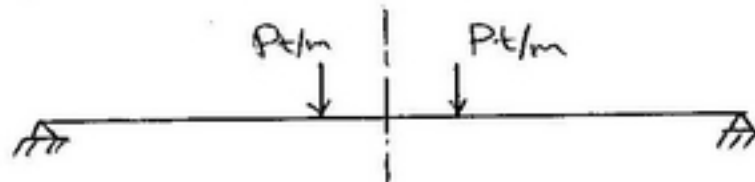
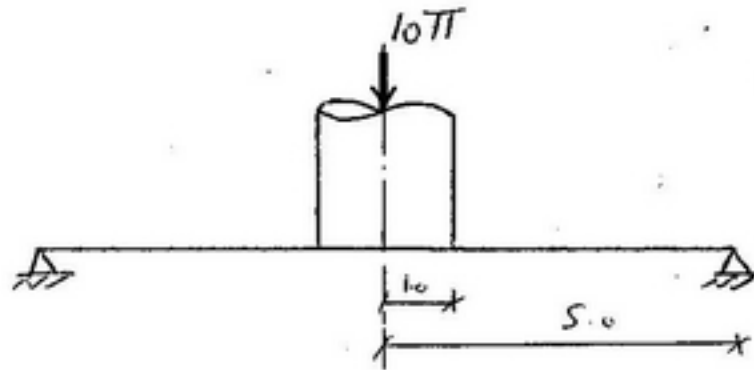
$$0.0 \quad \omega = -\frac{S}{D} \left[ \frac{r^2}{4} \ln r - \frac{r^2}{8} - \frac{0.69}{2} r^2 + 1.975 \right]$$

| r              | 0.0 | 0.1 | 1    | 2    | 3     | 4    |
|----------------|-----|-----|------|------|-------|------|
| M <sub>r</sub> | ∞   | 6.7 | 0.95 | 0.78 | -1.79 | -2.5 |
| M <sub>θ</sub> | ∞   | 9.2 | 3.45 | 1.72 | 0.7   | 0    |
| ω              | ∞   | 2   | 2    | 1.6  | 0.23  | 0    |





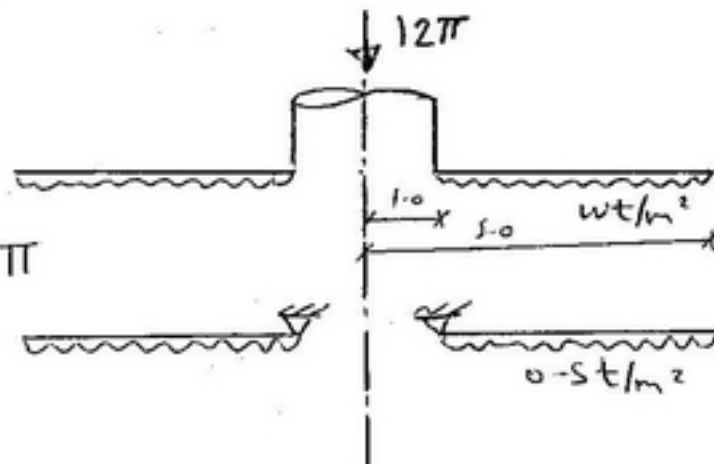
معادل گشتی، نقطه میانی



$$10\pi = P \times 2\pi(1)$$

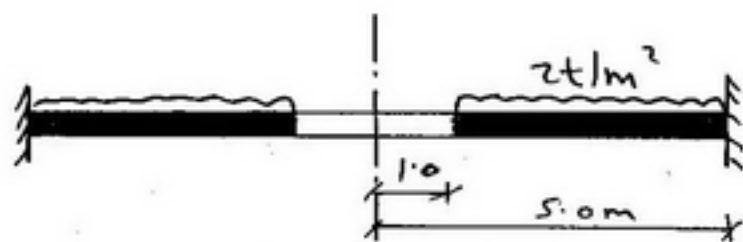
$$P = 5 \text{ t/m}$$

-----



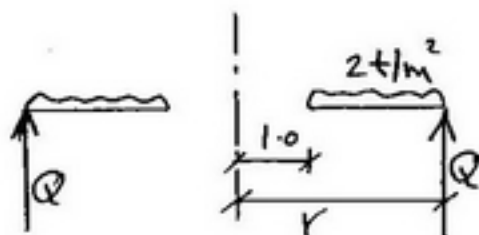
$$W \times \pi [5^2 - 1^2] = 12\pi$$

$$W = 0.5 \text{ t/m}^2$$



draw  $M_r, M_\theta$

— sol —



$$2\pi r Q = 2\pi [r^2 - 1^2]$$

$$Q = r - \frac{1}{r}$$

$$\rightarrow \frac{\partial}{\partial r} \left[ \frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) \right] = -\frac{1}{D} \left[ r - \frac{1}{r} \right]$$

$$\Rightarrow \frac{1}{r} \cdot \frac{\partial}{\partial r} (r\phi) = -\frac{1}{D} \left[ \frac{r^2}{2} - \ln r + C_1 \right]$$

$$\Rightarrow \frac{\partial}{\partial r} (r\phi) = -\frac{1}{D} \left[ \frac{r^3}{2} - r \ln r + C_1 \cdot r \right]$$

$$\Rightarrow (r\phi) = -\frac{1}{D} \left[ \frac{r^4}{8} - \frac{r^2}{2} \ln r + \frac{r^2}{4} + \frac{C_1 r^2}{2} + C_2 \right]$$

$$\Rightarrow \phi = -\frac{1}{D} \left[ \frac{r^3}{8} - \frac{r}{2} \ln r + \frac{r}{4} + \frac{C_1 r}{2} + \frac{C_2}{r} \right]$$

$$\frac{\phi}{r} = -\frac{1}{D} \left[ \frac{r^2}{8} - \frac{1}{2} \ln r + \frac{1}{4} + \frac{C_1}{2} + \frac{C_2}{r^2} \right]$$

$$\frac{\partial \phi}{\partial r} = -\frac{1}{D} \left[ \frac{3r^2}{8} - \frac{1}{2} [\ln r + 1] + \frac{1}{4} + \frac{C_1}{2} - \frac{C_2}{r^2} \right]$$

$$M_r = -\frac{1}{D} \left( \frac{3r^2}{8} - \frac{\ln r}{2} - \frac{1}{2} + \frac{1}{4} + \frac{C_1}{2} - \frac{C_2}{r^2} \right)$$

$$M_r = -\frac{3r^2}{8} + \frac{\ln r}{2} + \frac{1}{4} - \frac{C_1}{2} + \frac{C_2}{r^2}$$

$$M_\phi = -\frac{r^2}{8} + \frac{1}{2} \ln r - \frac{1}{4} - \frac{C_1}{2} - \frac{C_2}{r^2}$$

B.C at  $r=5 \rightarrow \phi=0.0$

$$\frac{5^3}{8} - \frac{5}{2} \ln 5 + \frac{5}{4} + 2.5 C_1 + 0.2 C_2 = 0.0$$

$$2.5 + 0.2 C_2 = -12.85 \rightarrow \textcircled{1}$$

at  $r=1.0 \rightarrow M_r=0.0$

$$-\frac{3}{8} + \frac{\ln 1}{2} + \frac{1}{4} - \frac{C_1}{2} + C_2 = 0.0$$

$$-0.5 C_1 - C_2 = -0.125 \rightarrow \textcircled{2}$$

$$C_1 = -4.95$$

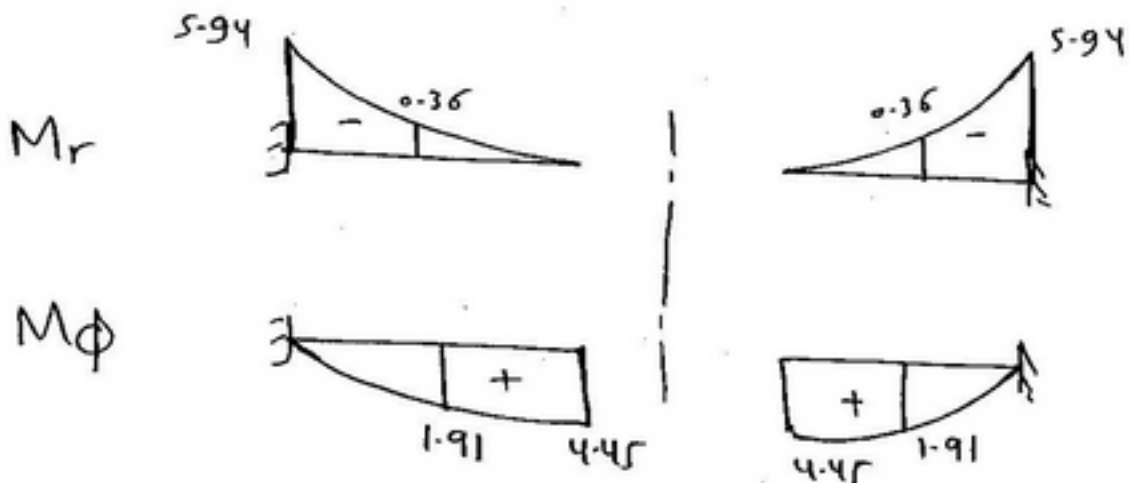
$$C_2 = -2.35$$

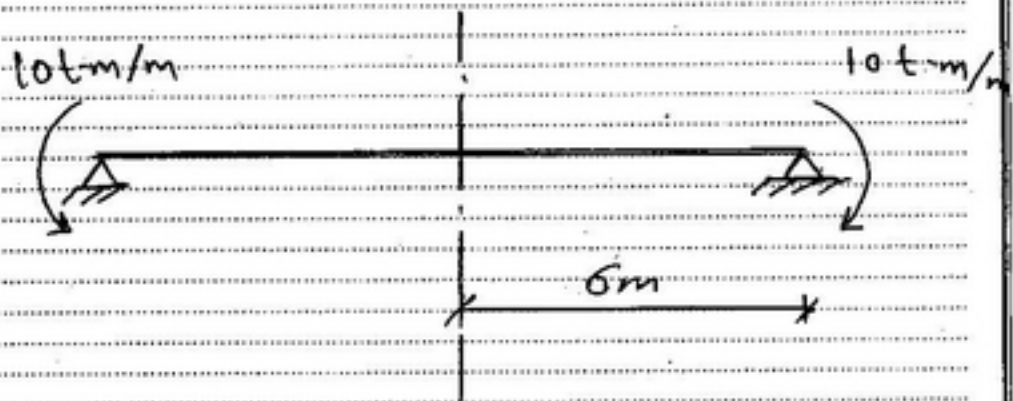
$$\therefore M_r = -\frac{3r^2}{8} + \frac{\ln r}{2} + \frac{1}{4} + 2.475 - \frac{2.35}{r^2}$$

$$M_r = \frac{-3r^2}{8} + \frac{\ln r}{2} + 2.725 - \frac{2.35}{r^2}$$

$$\therefore M_\phi = -\frac{r^2}{8} + \frac{\ln r}{2} + 2.225 + \frac{2.35}{r^2}$$

| r        | 1    | 3     | 5     |
|----------|------|-------|-------|
| $M_r$    | 0    | -0.36 | -5.94 |
| $M_\phi$ | 4.45 | 1.91  | 0     |

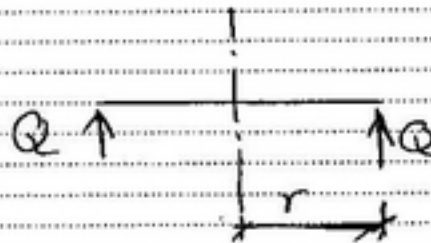




draw  $(M_r, M_\theta)$

given  $(\mu = 0.0)$

solution



$$2\pi r Q = 0.0$$

$$Q = 0.0$$

$$\frac{\partial}{\partial r} \left( \frac{1}{r} - \frac{\partial}{\partial r} (r\phi) \right) = -\frac{1}{D} (0)$$

$$\frac{1}{r} - \frac{\partial}{\partial r} (r\phi) = -\frac{1}{D} (C_1)$$

$$\frac{\partial}{\partial r} (r\phi) = -\frac{1}{D} [C_1 r]$$

$$r\phi = -\frac{1}{D} \left[ \frac{C_1 r^2}{2} + C_2 \right]$$

$$\phi = -\frac{1}{D} \left[ \frac{C_1}{2} r + \frac{C_2}{r} \right]$$



\* at  $r=0.0 \rightarrow \phi=0.0$

$$0.0 = -\frac{1}{D} [0 + C_2] \rightarrow C_2 = 0.0$$

\* at  $r=6.0 \quad M_r = -10$

$$M_r = D \left[ \frac{\partial \phi}{\partial r} \right]$$

$$= D \left[ -\frac{1}{D} \times \left( \frac{C_1}{2} - \frac{C_2}{r^2} \right) \right]$$

$$= -1 \left[ \frac{C_1}{2} - \frac{C_2}{r^2} \right]$$

$$-10 = -1 \left[ \frac{C_1}{2} - \frac{C_2}{36} \right]$$

0.0

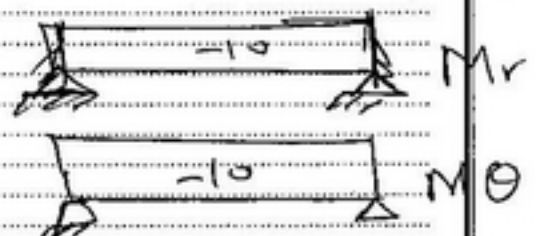
$$\frac{C_1}{2} = 10$$

$$C_1 = 20$$

$$\phi = -\frac{1}{D} [10r]$$

$$M_r = [-10]$$

$$M_\theta = [-10]$$



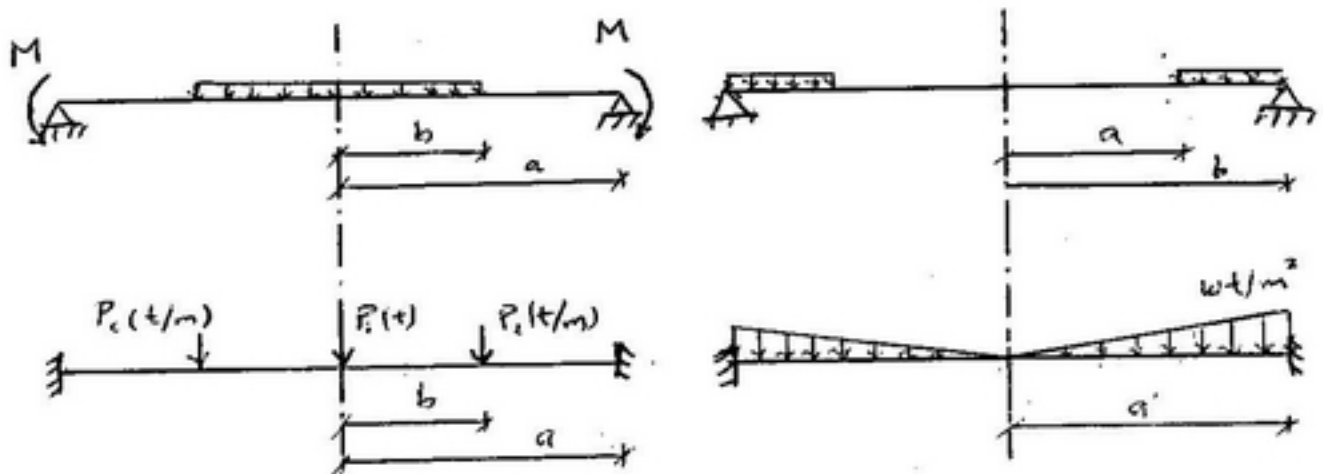
11201  
Faculty of engineering  
3rd year

# structural analysis

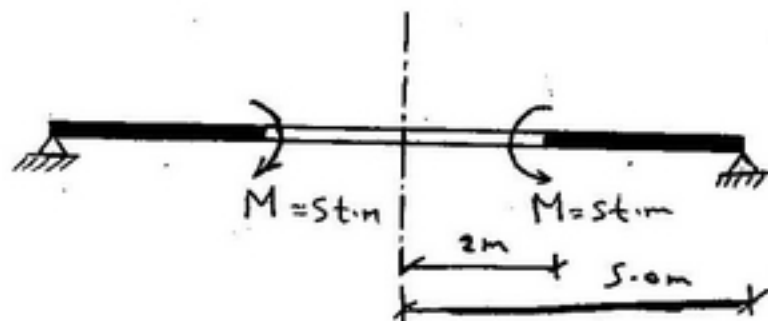
No (10)

final 2007

- a) state the no of Interval, state the boundary conditions and derive the expression of the shear in each Interval.

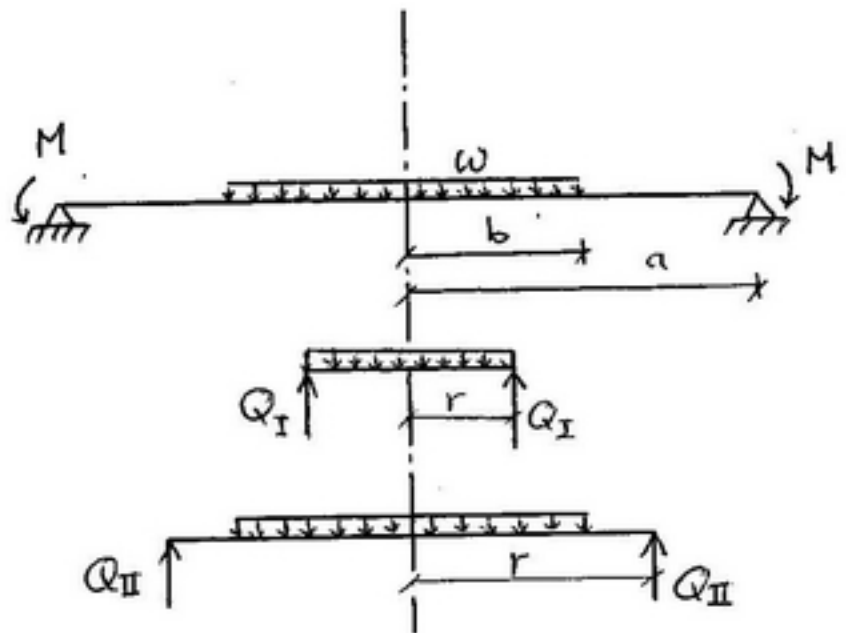


- b) for the plate shown draw bending moment diagrams ( $M_r, M_\theta$ ) ( $\mu = 0.0$ )



a

(i)



\* two interval .

$$\begin{cases} * Q_I = \frac{\pi w r^2}{2\pi r} = \frac{w r}{2} \\ * Q_{II} = \frac{w \pi b^2}{2\pi r} = \frac{w b^2}{2r} \end{cases}$$

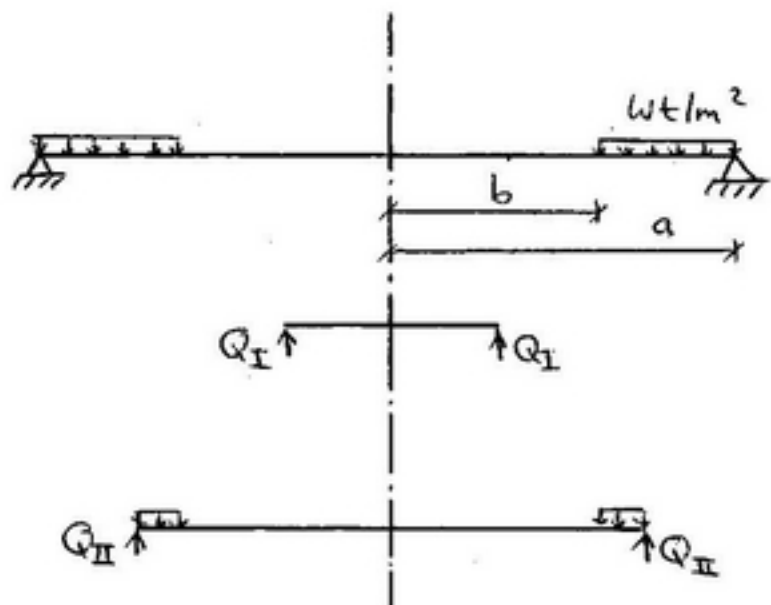
\* B.C

$$\text{at } r = 0.0 \Rightarrow \phi_I = 0.0$$

$$\text{at } r = a \Rightarrow M_{r_{II}} = -M$$

$$\text{at } r = b \Rightarrow \begin{cases} \phi_I = \phi_{II} \\ M_{r_{II}} = M_{r_I} \end{cases}$$

(ii)



\* two interval.

$$\left\{ \begin{array}{l} * Q_I = 0.0 \end{array} \right.$$

$$* Q_{II} * 2\pi r = \pi(r^2 - b^2) * w$$

$$\Rightarrow Q_{II} = \frac{w(r^2 - b^2)}{2r} = \frac{wr}{2} - \frac{wb^2}{2r}$$

\* B-C

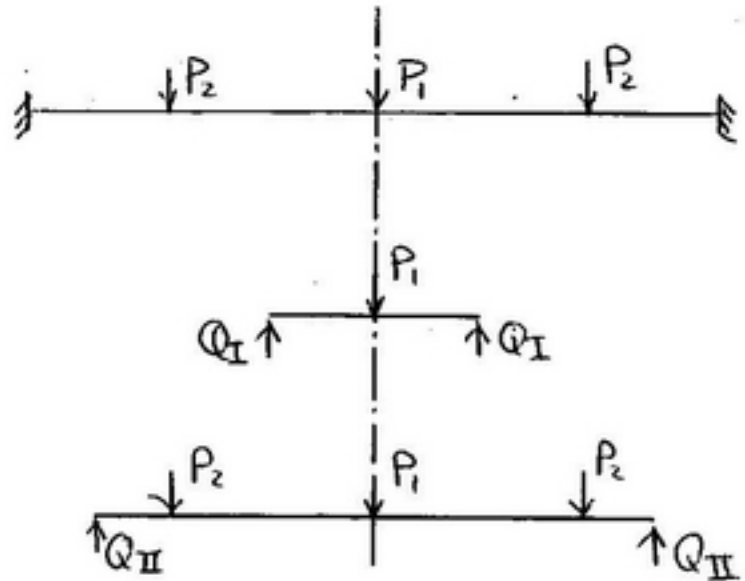
$$\text{at } r=0.0 \Rightarrow \phi_I = 0.0$$

$$\text{at } r=a \Rightarrow M_{r_{II}} = 0.0$$

$$\text{at } r=b \left\{ \begin{array}{l} \Rightarrow \phi_I = \phi_{II} \\ \Rightarrow M_{r_I} = M_{r_{II}} \end{array} \right.$$



(iv)



\* two interval.

$$\begin{cases} * Q_I = \frac{P_1}{2\pi r} \\ * Q_{II} = \frac{P_1 + P_2 \times 2\pi b}{2\pi r} \end{cases}$$

\* B.C

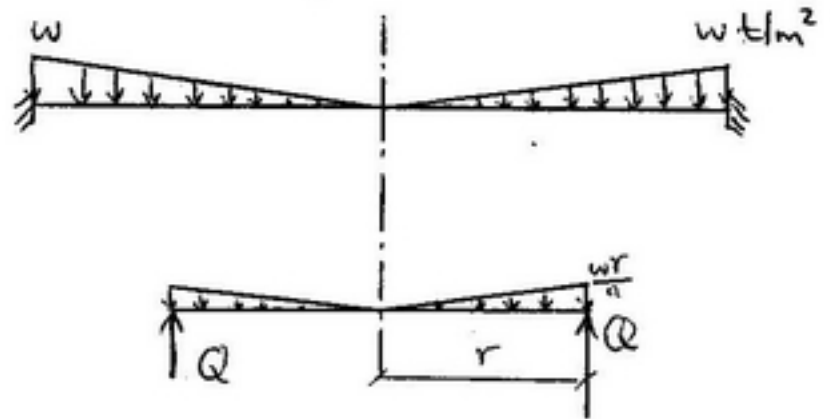
$$r = 0 \Rightarrow \Phi_I = 0$$

$$r = a \Rightarrow \Phi_{II} = 0$$

$$r = b \Rightarrow M_{r_I} = M_{r_{II}}$$

$$\Phi_I = \Phi_{II}$$

(iv)



same Interval

جمع جزاء =  $\frac{1}{3}$  جزء لا متكامل

$$Q \times 2\pi r = \left( \pi r^2 \times \frac{wr}{a} \right)^{2/3}$$

$$Q = \frac{wr^2}{3a}$$

\* B.C

at  $r = 0$

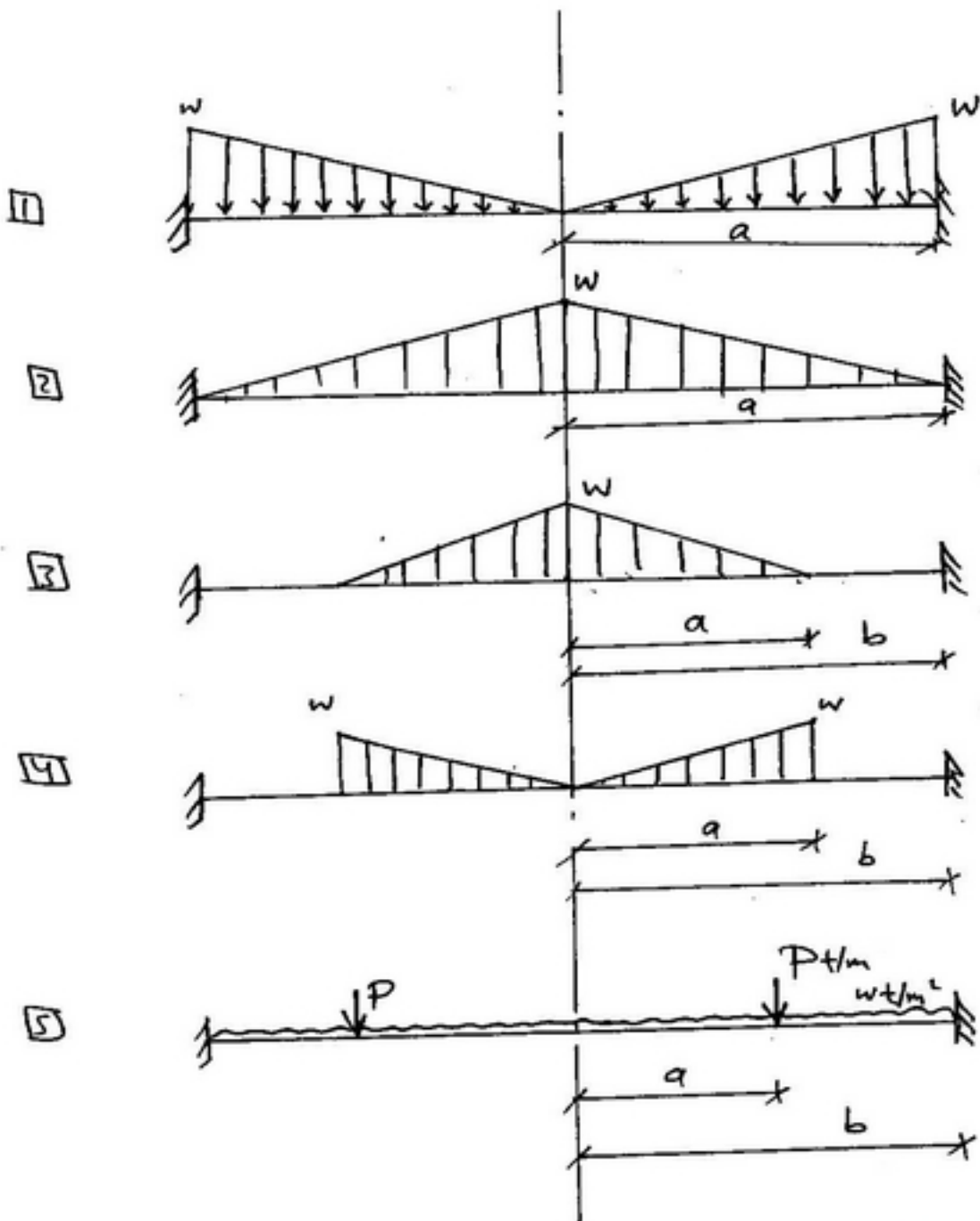
$$\phi = 0$$

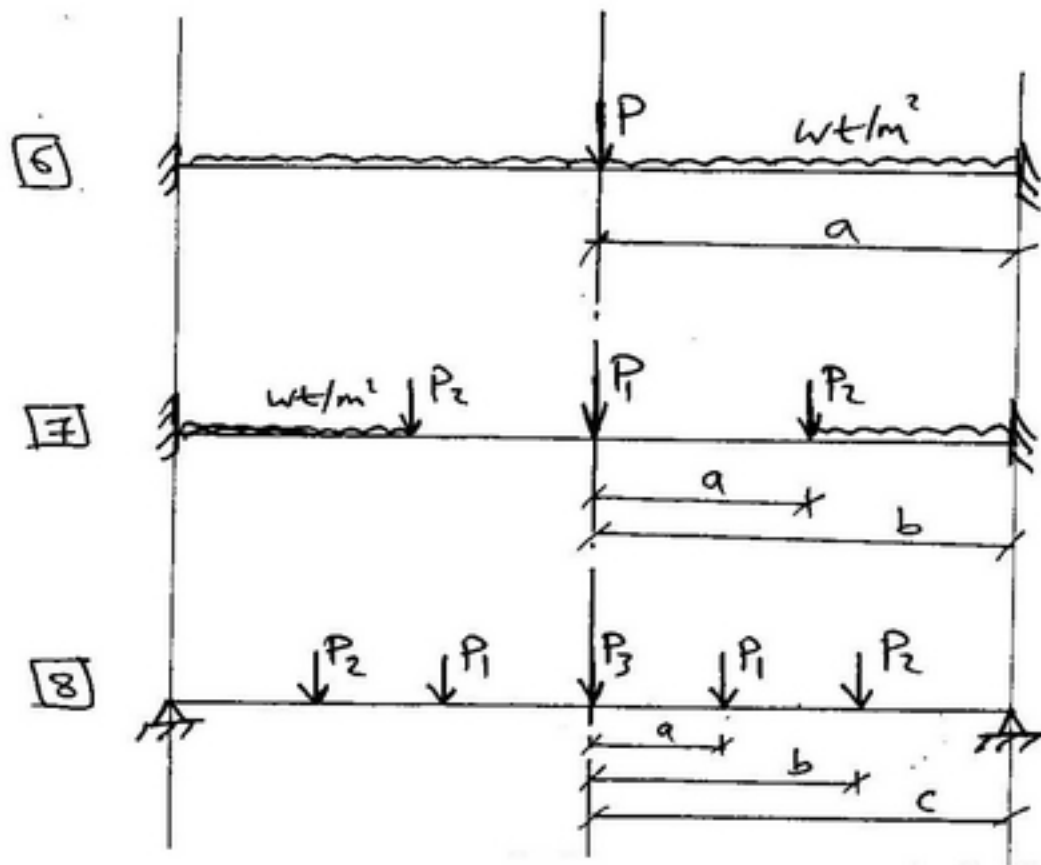
at  $r = a$

$$\phi = 0$$

## Plates

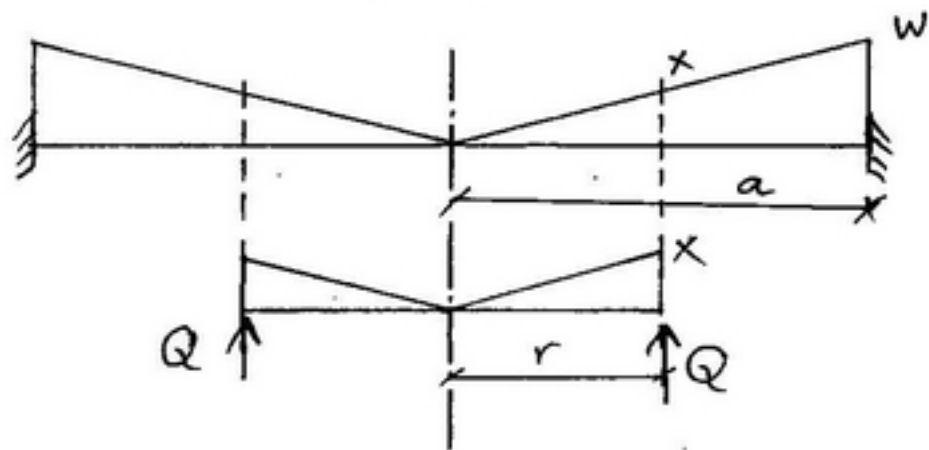
\* For the following plate find the no of Interval  
and the equation (expression of shear eqn)  
and the Boundary Condition .





III

— Sol —



\* one Interval

$$* 2\pi r Q = \frac{2}{3} (x \cdot \pi r^2)$$

② ← مجموع طوائف  
- جميع الخواص

$$Q = \frac{x \cdot r}{13}$$

$$\frac{x}{w} = \frac{r}{a} \Rightarrow x = w \cdot \frac{r}{a}$$

$$Q = \frac{w r^2}{3a}$$

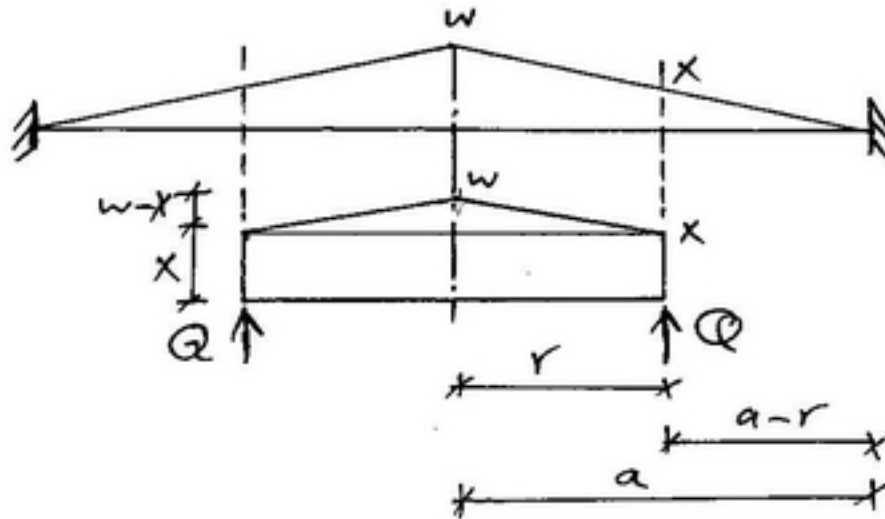
\* Boundary Conditions

$$\text{at } r=0 \rightarrow \phi = 0.0$$

$$\text{at } r=a \rightarrow \phi = 0.0$$



[2]



\* one Interval

\* eqn

$$2\pi r Q = x \cdot \pi r^2 + \frac{1}{3} (w-x) \cdot \pi r^2$$

$$\frac{x}{w} = \frac{a-r}{a}$$

$$x = w - \frac{r}{a} w$$

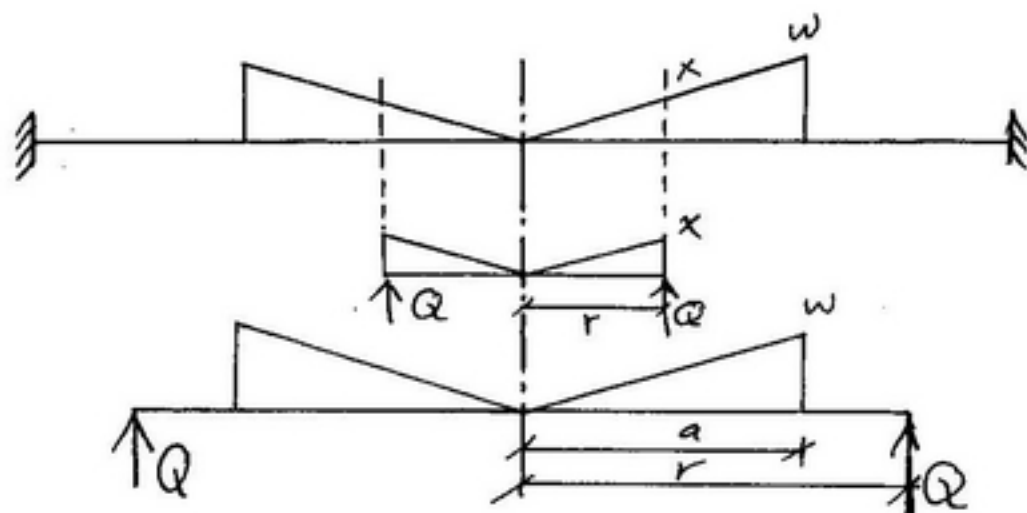
$$\therefore 2\pi r Q = (w - \frac{r}{a} w) \cdot \pi r^2 + \frac{1}{3} (\frac{r}{a} w) \cdot \pi r^2$$

\* B-C

$$\text{at } r=0 \rightarrow \phi=0$$

$$\text{at } r=a \rightarrow \phi=0$$

3



\* Two Interval

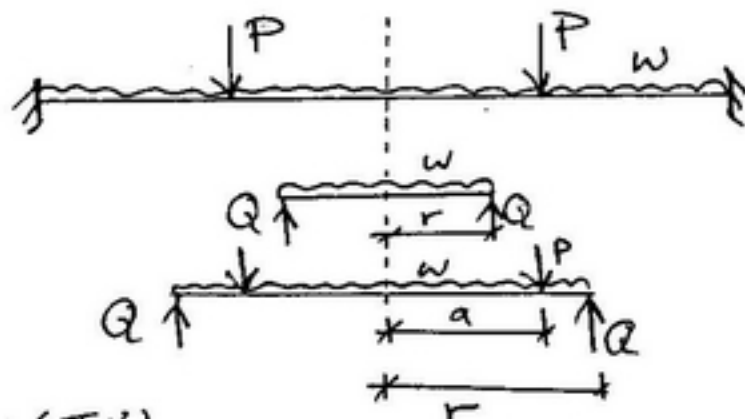
$$\begin{aligned} * \text{I} &\Rightarrow 2\pi rQ = \frac{2}{3} \cdot \pi r^2(x) \quad \& \quad x = w \cdot \frac{r}{a} \\ * \text{II} &\Rightarrow 2\pi rQ = \frac{2}{3} \cdot \pi (a)^2(w) \end{aligned}$$

\* B.C at

$$\begin{aligned} r=0 &\rightarrow \phi_I = 0 \\ r=b &\rightarrow \phi_{II} = 0 \\ r=a &\rightarrow \phi_I = \phi_{II} \\ &\rightarrow M_{rI} = M_{rII} \end{aligned}$$

4 try

5



\* Two Interval.

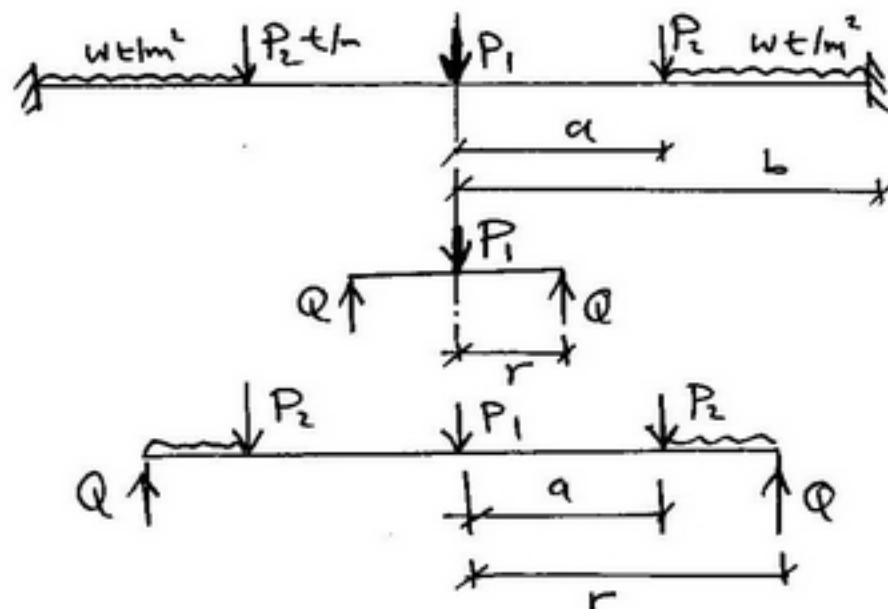
$$\begin{aligned} * \text{I} &\left\{ \begin{aligned} 2\pi rQ &= w(\pi r^2) \\ * \text{II} &\left\{ \begin{aligned} 2\pi rQ &= w\pi r^2 + P(2\pi a) \end{aligned} \right. \end{aligned} \right.$$

\* B.C

$$\begin{aligned} \text{at } r=0 &\rightarrow \phi_I = 0 \\ \text{at } r=b &\rightarrow \phi_{II} = 0 \\ \text{at } r=a &\rightarrow \phi_I = \phi_{II} \end{aligned}$$

6 + ry.

7



\* two Interval.

\* I  $\left\{ \begin{aligned} 2\pi r Q &= P_1 \end{aligned} \right.$

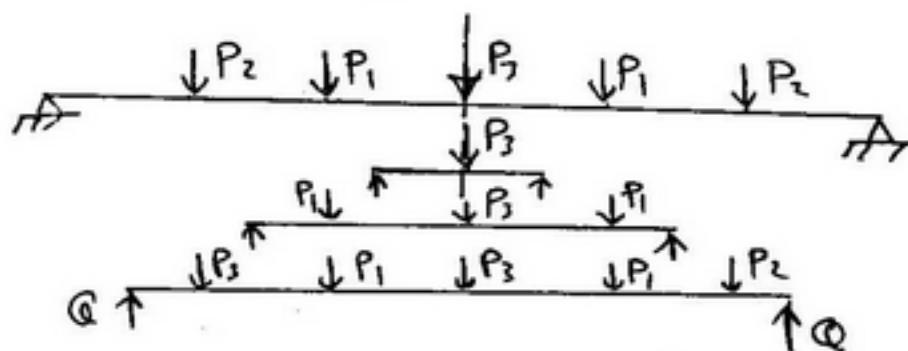
II  $\left\{ \begin{aligned} 2\pi r Q &= P_1 + 2\pi a P_2 + w \times \pi [r^2 - a^2] \end{aligned} \right.$

\* B.C  $\rightarrow$  at  $r = 0 \dots \rightarrow \phi_I = 0$

$\rightarrow$  at  $r = L \rightarrow \phi_{II} = 0$

$\rightarrow$  at  $r = a \rightarrow \phi_I = \phi_{II} \text{ \& } M_{rI} = M_{rII}$

8



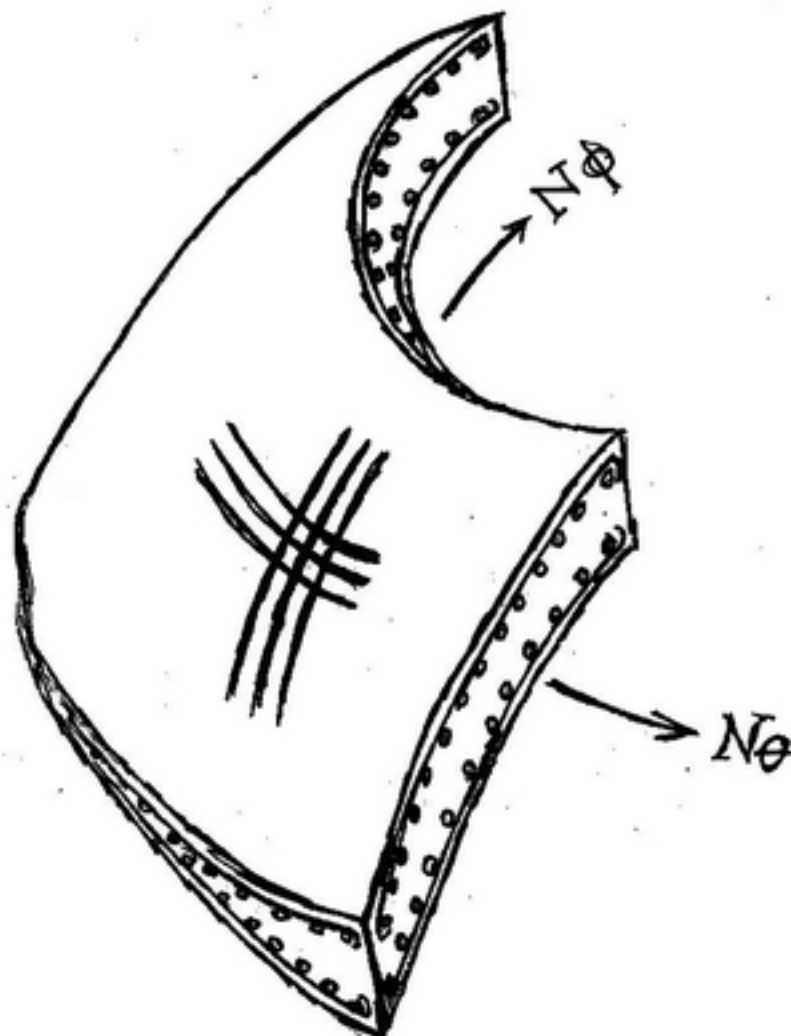
\* 3 Interval

\* I  $\rightarrow 2\pi r Q = P_3$

II  $\rightarrow 2\pi r Q = P_3 + P_1 (2\pi a)$

III  $\rightarrow 2\pi r Q = P_3 + P_1 (2\pi a) + P_2 (2\pi b)$

B.C  $\left\{ \begin{aligned} \text{at } r = 0 \dots &\rightarrow \phi_I = 0 \\ \text{at } r = c &\rightarrow M_{rIII} = 0 \\ \text{at } r = a &\rightarrow \phi_I = \phi_{II} \\ &M_{rI} = M_{rII} \\ \text{at } r = b &\rightarrow \phi_{II} = \phi_{III} \end{aligned} \right.$



SELLS  
التسريات

بسم الله الرحمن الرحيم

shell

لقشريات

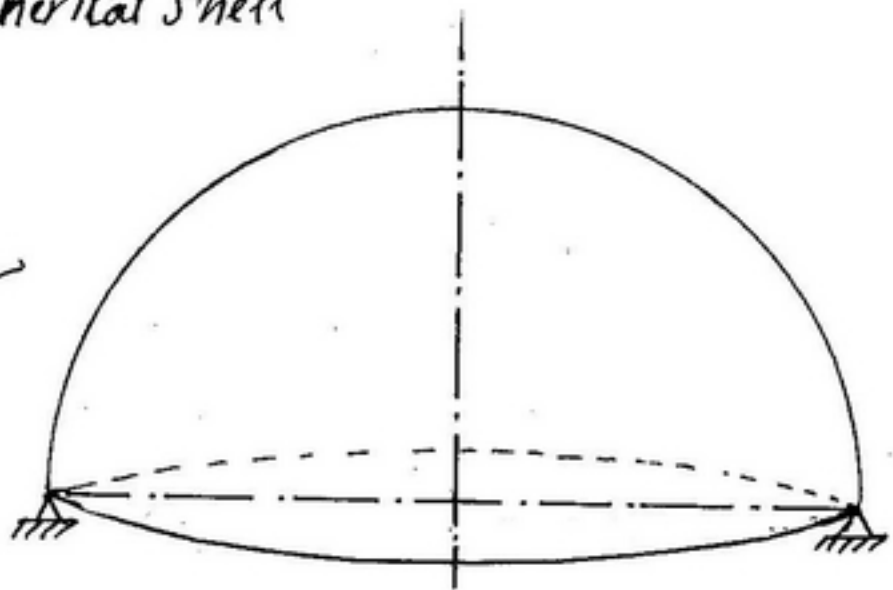
\* تظهر هذه الانواع من خشب المساجد - الصاري (tanks)

\* انساس من لقشريات انظر تحمل normal fire فقط

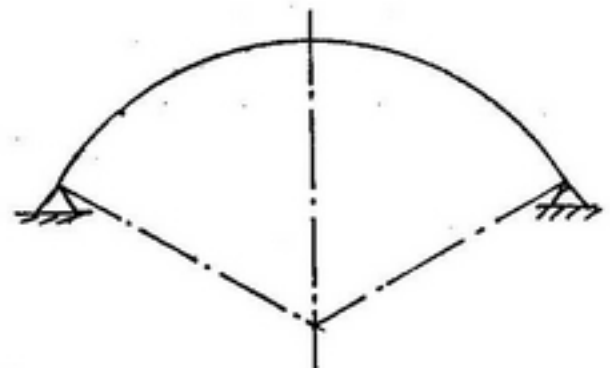
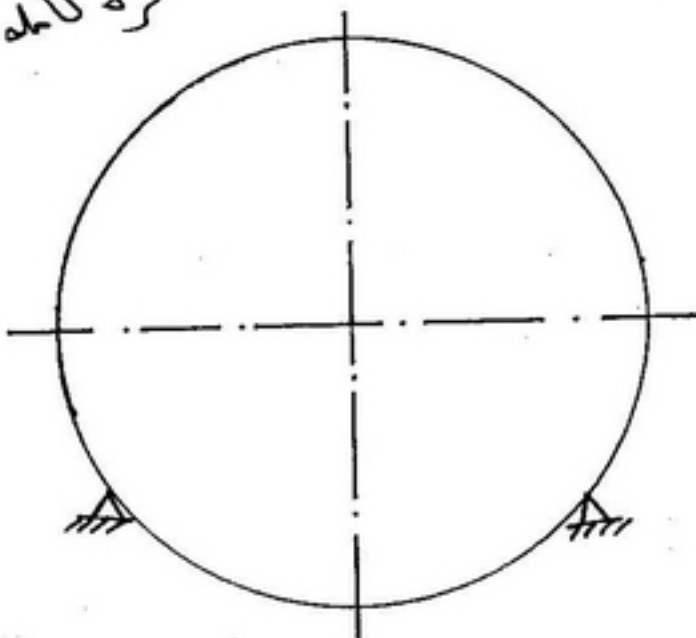
types of shells

"spherical shell"

semi circular  
نصف كورة



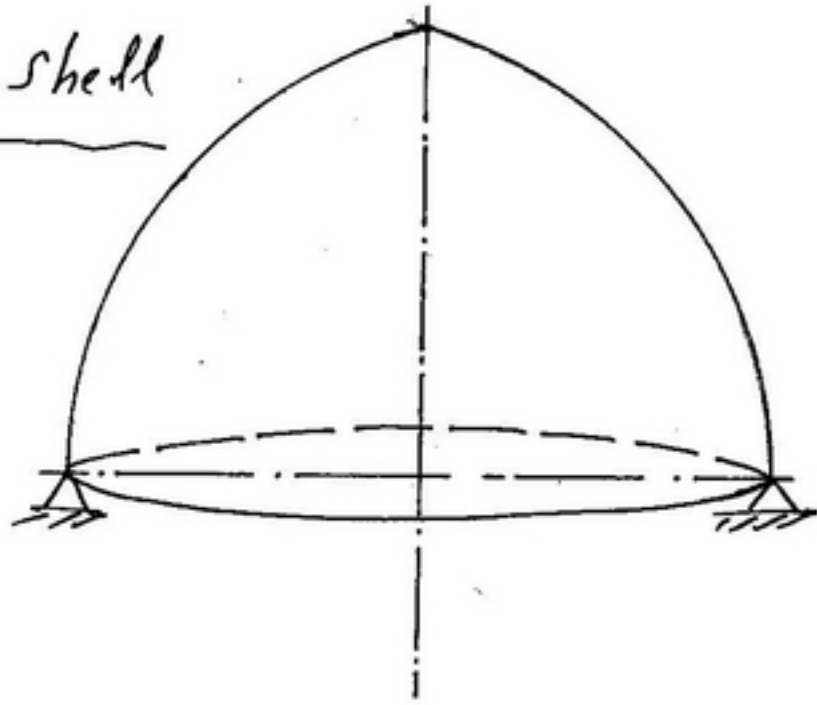
كرة كاملة



جزء من كورة

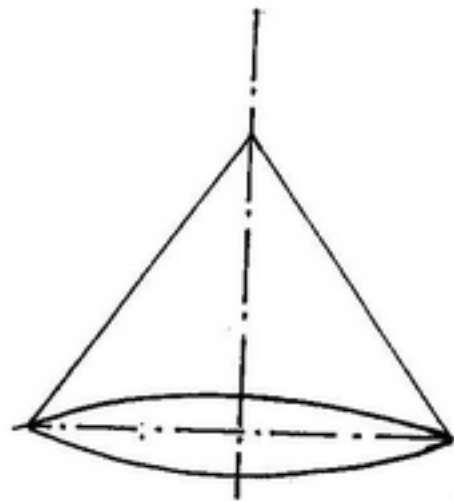


2- parabolic shell

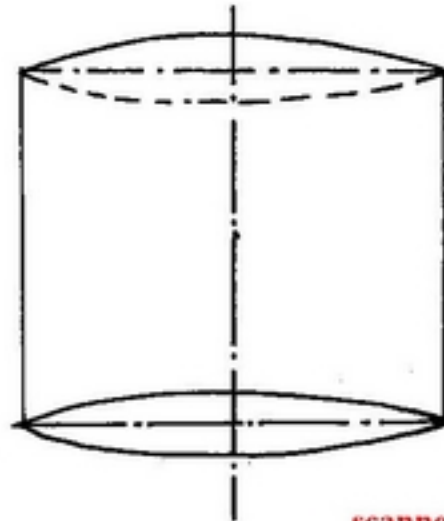


3- Conical shell

مخروط



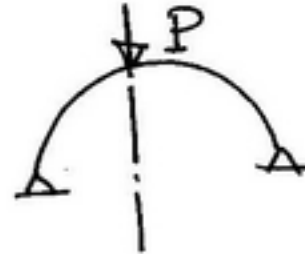
4- cylindrical shell



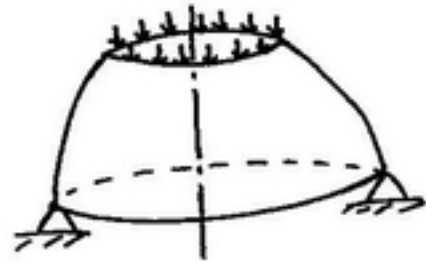
القذائف السطحية هي التي  
تستخدم في الجدران

## type of load

### ① point load



### ② Ring load



### ③ uniform load

(i) D.L "dead load"

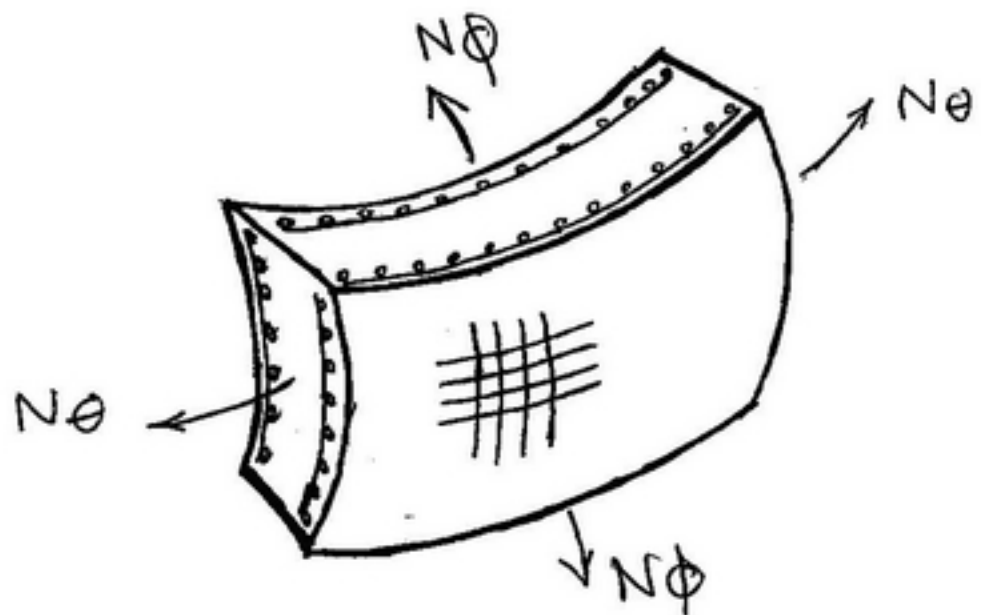
\* یتم توزیع دل ۴ ل

(ii) L.L "live load"

\* "H.P" یتم توزیع دل ۴ ل

\* سيتم دراسة ثلاثة أشكال (Spherical, conical, cylindr)

والهدف من دراسة هذه القوى هي  $(N_\phi, N_\theta)$  وهما normal force  
 يتولد داخل هذه البلاطات حيث تكون العزوم ضالها ضئيلة بالنسبة  
 لل normal force .



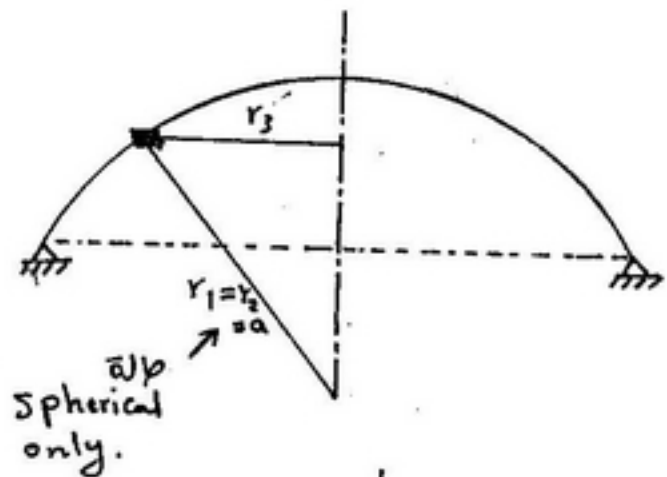
\* يتم الحصول على  $N_\phi$  من معادله  $N_\theta$  تم استويين في معادله

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = -P_r$$

## 1 - spherical shells

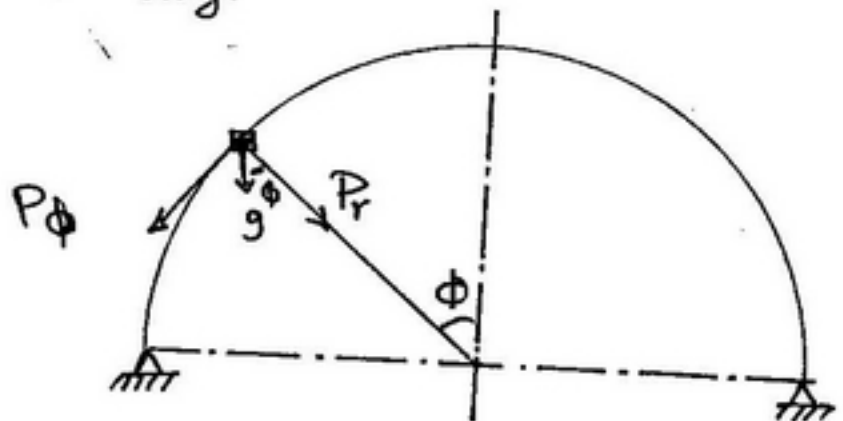
- \* dead load ( $g$ )
- \* live load ( $w$ )
- \* water load ( $\gamma h$ )

(i) due to dead load ( $g$ )  $= g t / m^2$



$$P_r = g \cos \phi$$

$$P_\phi = g \sin \phi$$



$P_\phi \rightarrow +ve$  in the direction of increase of  $\phi$

$P_r \rightarrow +ve$  . . . . . of  $R(\text{center})$

بالتعويض في المعادلة الأساسية

$$N\phi = \frac{-1}{r_2 \cdot \sin^2 \phi} \int r_1 r_2 [P_r \cos \phi + P_\phi \sin \phi] \sin \phi \cdot d\phi$$

$$\begin{cases} r_1 = r_2 = a \\ P_r = g \cos \phi \\ P_\phi = g \sin \phi \end{cases}$$

$$\therefore N\phi = \frac{-1}{a \sin^2 \phi} \int a^2 [g \cos^2 \phi + g \sin^2 \phi] \sin \phi \cdot d\phi$$

$$= \frac{-a^2 \cdot g}{a \cdot \sin^2 \phi} \int \underbrace{(\cos^2 \phi + \sin^2 \phi)}_{=1.0} \sin \phi \cdot d\phi$$

$$= - \frac{a \cdot g}{\sin^2 \phi} \int \sin \phi \cdot d\phi$$

$$= - \frac{a \cdot g}{\sin^2 \phi} [-\cos \phi + C_1]$$

Boundary Condition

$$\text{at } \phi = 0 \Rightarrow N\phi = 0 \Rightarrow \int \frac{1}{\sin^2 \phi} = 0$$

$$-\cos(0) + C_1 = 0 \Rightarrow \boxed{C_1 = 1.0}$$



$$\therefore N\phi = \frac{-a \cdot g}{\sin^2 \phi} [-\cos \phi + 1]$$

$$= \frac{-a \cdot g}{\sin^2 \phi} [1 + \cos \phi]$$

$$\sin^2 \phi = 1 - \cos^2 \phi = (1 - \cos \phi)(1 + \cos \phi)$$

$$\therefore N\phi = \frac{-a \cdot g}{(1 - \cos \phi)(1 + \cos \phi)} (1 - \cos \phi)$$

$$N\phi = \frac{-a \cdot g}{(1 + \cos \phi)}$$

$$\frac{N\phi}{r_1} + \frac{N\theta}{r_2} = -P_r$$

is correct

$$\frac{N\phi}{a} + \frac{N\theta}{a} = -g \cos \phi$$

$$\therefore N_{\theta} = -ag \cos \phi - \left( \frac{-ag}{1 + \cos \phi} \right)$$

$$N_{\theta} = \frac{ag}{(1 + \cos \phi)} - ag \cos \phi$$

at  $\phi = 0^\circ$

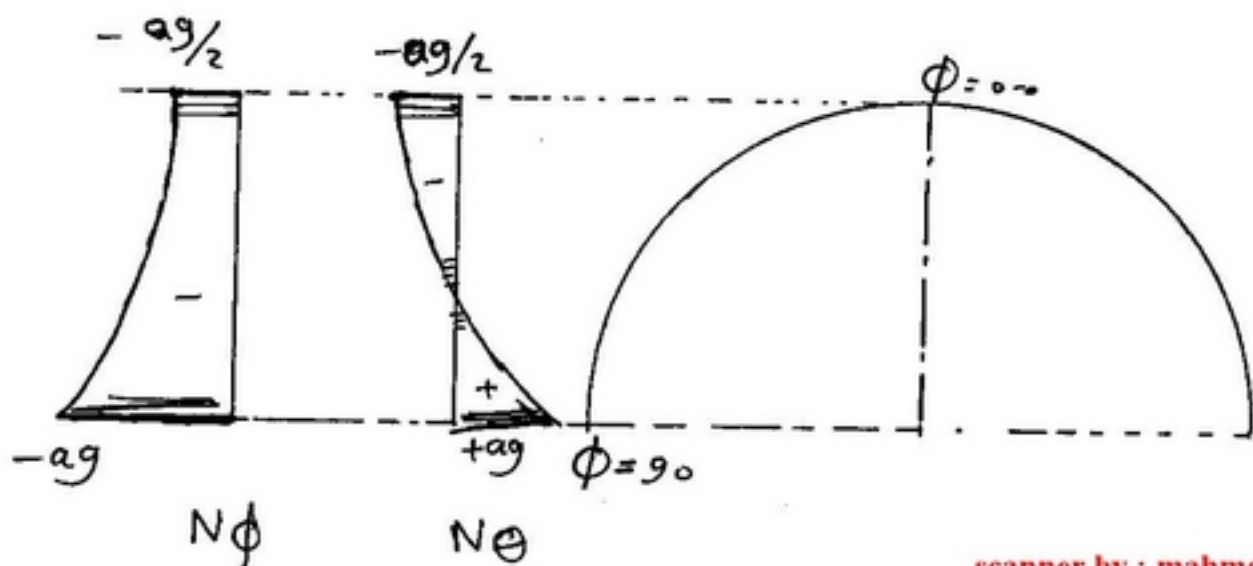
$$\Rightarrow N_{\phi} = - \frac{ag}{1 + \cos 0} = \left( - \frac{ag}{2} \right)$$

$$N_{\theta} = + \frac{ag}{2} - ag = \left( - \frac{ag}{2} \right)$$

at  $\phi = 90^\circ$

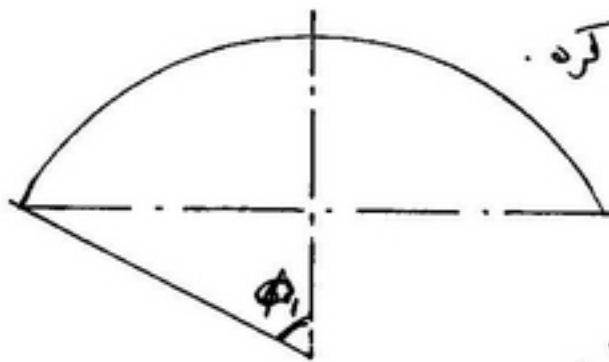
$$\Rightarrow N_{\phi} = - ag / (1 + 0) = (- ag)$$

$$N_{\theta} = ag - 0 = (ag)$$



لا حظ للحصول على الشكل السابق بالضبط

| $\phi$    | 0-0             | 30 | 60 | 90    |
|-----------|-----------------|----|----|-------|
| $N\phi$   | $-\frac{ag}{2}$ | ✓  | ✓  | $-ag$ |
| $N\theta$ | $-\frac{ag}{2}$ | ✓  | ✓  | $ag$  |



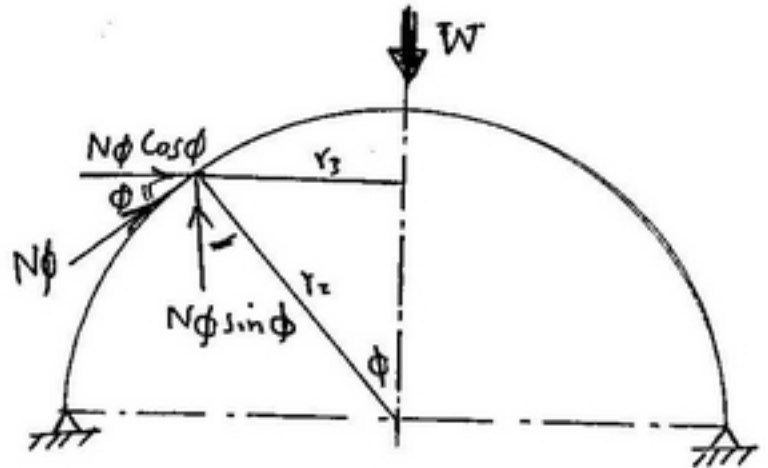
من غير حالة انه يكون أقل أو أكبر من نصف كره.

نصفه  $\phi$  له يغير الجردل بأعلى ولكن

$\phi = 0.0$   $\phi = \phi_1$  نقط

(ii) Case of live load

(H.P) live load  $\phi$  يوزع على الأضلاع



$$-W = \underbrace{N\phi \cdot \sin\phi}_{\text{تكون الأضلاع } N\phi} * \underbrace{[2\pi r_3]}_{\text{محيط الدائرة المنتهية}}$$

$$r_3 = r_2 \sin\phi$$

$$-W = N\phi \cdot \sin\phi \cdot 2\pi r_2 \sin\phi$$

$$N\phi = \frac{+W}{2\pi r_2 \cdot \sin^2\phi}$$

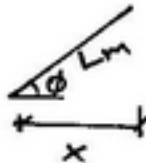
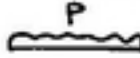
هنا

live load  $\phi$

• (live load) حمل

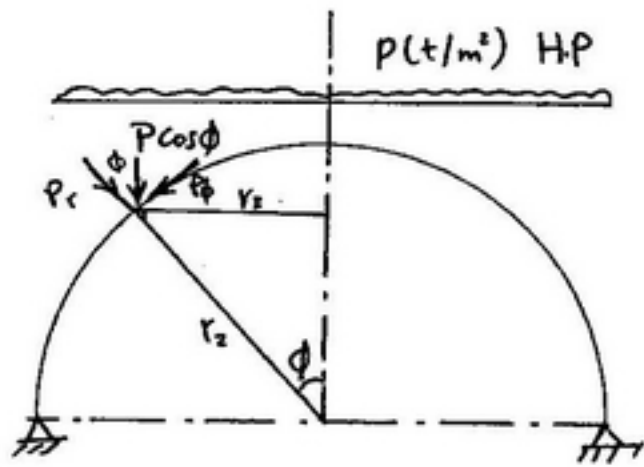
$P \cos \phi \rightarrow$  منسوب الى مركز

حمل H.P. على نصف كروي لا يسقط  
حمل  $P \cos \phi$  على كل shell  
مثال للعرض عليه



المساحة  $P_{\pi}(x) = W_{\pi}(L)$

$$W = P \cdot \frac{x}{L} = \boxed{P \cos \phi}$$



$$P_r = P \cos^2 \phi$$

$$P_{\phi} = P \cos \phi \cdot \sin \phi$$

$$W = P_{\pi} (\pi r_3^2)$$

$$r_3 = r_2 \sin \phi$$

$$W = P \cdot \pi \cdot r_2^2 \cdot \sin^2 \phi$$



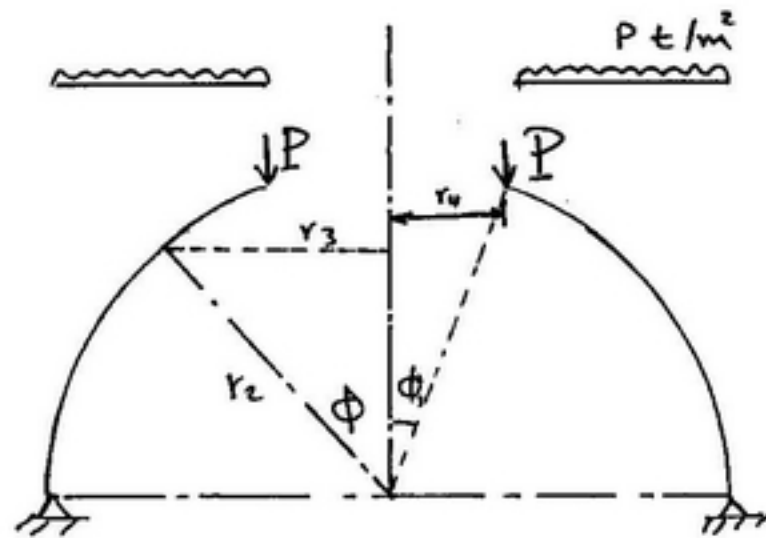
$$\begin{aligned} \therefore N_{\phi} &= \frac{-W}{2\pi r_2 \sin^2 \phi} \\ &= \frac{+P \cdot \pi \cdot r_2^2 \cdot \sin^2 \phi}{2\pi r_2 \cdot \sin^2 \phi} = \boxed{\frac{+Pa}{2}} \end{aligned}$$

$$\frac{N_{\phi}}{r_1} + \frac{N_{\theta}}{r_2} = -P_r$$

$$N_{\phi} + N_{\theta} = -P_r \cdot a = -P \cos^2 \phi \cdot a$$

$$\boxed{N_{\theta} = -P \cdot a \cdot \cos^2 \phi - \frac{Pa}{2}}$$

جذباتی لائن - سیب  
 صبح پیرا سے سہا  
 ورنہ انکو باقی = صبح



$$W = P * (2\pi r_4) + P * \pi [r_3^2 - r_4^2]$$

$$r_3 = r_2 \sin \phi$$

$$r_4 = r_2 \sin \phi$$

نعم! التعريف من الجاذبية

$$N\phi = \frac{W}{2\pi r_2 \cdot \sin^2 \phi} \Rightarrow (1)$$

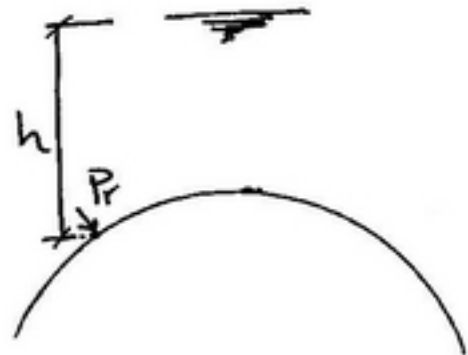
$$\frac{N\phi}{r_1} + \frac{N\theta}{r_2} = -P_r$$

$$\text{So } N\theta = \dots \Rightarrow (2)$$

(iii) Case of water load

$$P_r = \gamma h$$

$$P_\phi = 0.0$$



$$\therefore N_\phi = \frac{-1}{r_2 \cdot \sin^2 \phi} \int r_1 \cdot r_2 (P_r \cos \phi + P_\phi \sin \phi) \sin \phi \, d\phi$$

$$= \frac{-a^2}{a \sin^2 \phi} \int \gamma h \cos \phi \sin \phi \, d\phi$$

$$\Rightarrow \int \cos \phi \cdot \sin \phi \, d\phi = -\frac{\cos^2 \phi}{2}$$

$$\therefore N_\phi = -\frac{\gamma h \cdot a}{\sin^2 \phi} \left[ -\frac{\cos^2 \phi}{2} + C_1 \right]$$

$$\text{at } \phi = 0 \Rightarrow N_\phi = 0.0$$

$$\therefore C_1 = + \frac{\cos^2 0}{2} = 0.5$$

$$\therefore N_\phi = -\frac{\gamma h a}{\sin^2 \phi} \left[ -\frac{\cos^2 \phi}{2} + 0.5 \right]$$

$$\therefore \frac{N\phi}{r_1} + \frac{N\theta}{r_2} = -P_r$$

$$N\phi + N\theta = -P_r \cdot a$$

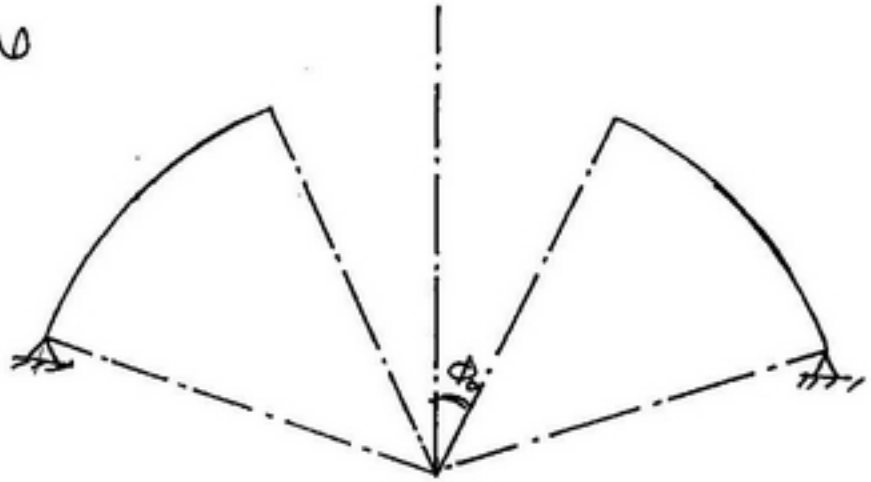
$$N\theta = -P_r \cdot a - N\phi$$

$$N\theta = -\tau h \cdot a - N\phi$$

OK

ملا فف

spherical  
مفوق



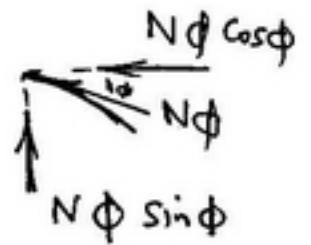
B-C

الف

at  $\phi_0$

$$N\phi \cdot \sin\phi = \dots$$

$$\Rightarrow N\phi = \dots$$



at  $\phi_0$

$$N\phi \sin\phi = g$$

$$\Rightarrow N\phi = \frac{g}{\sin\phi}$$

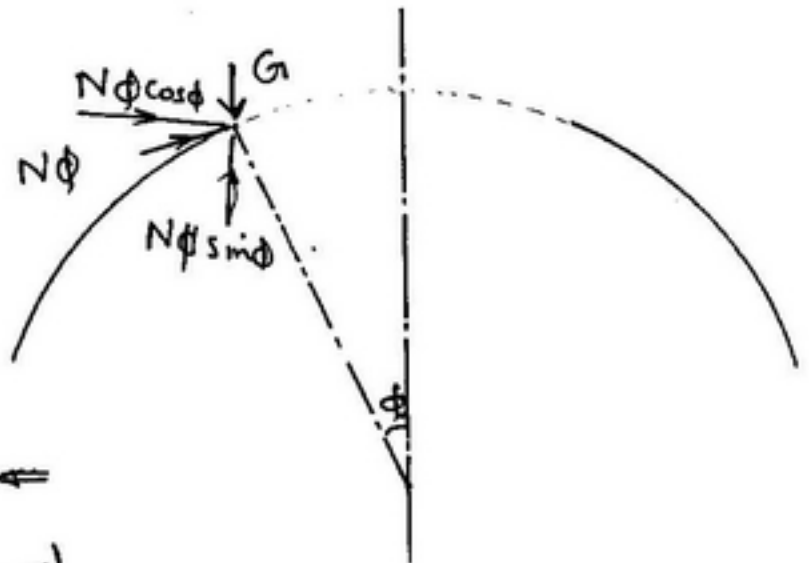
حالة وجود (g) مركز





# Ring beam

\* اگر یک خطای بر روی shell باشد  
حالات پتانسیل عدم حرکت تیرشی در shell می باشد



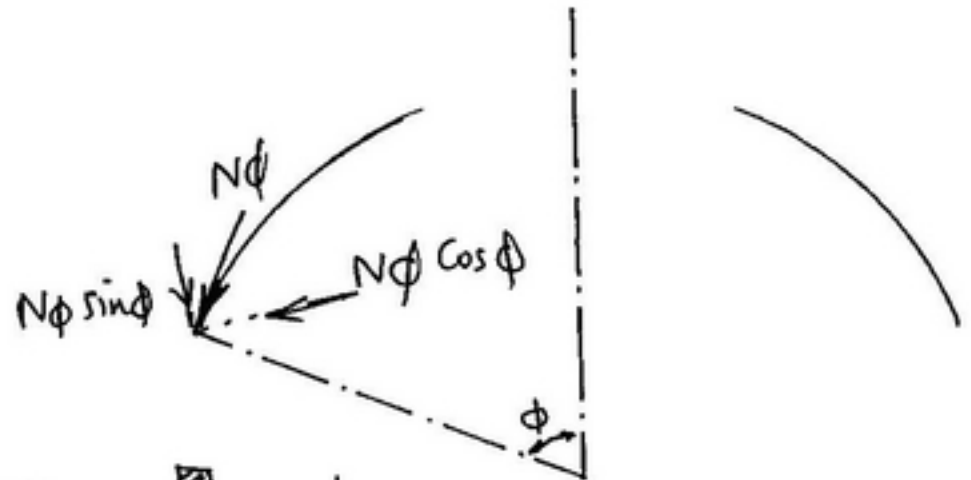
$$* N\phi \cdot \sin \phi = G \Leftarrow$$


نیست محتاج به این رابطه


\*  $N\phi \cos \phi \Rightarrow$  لا یزید مایل است  
هنگامی که shell فیت و جمع کرده افقیه  
تلاش هتاهت

horizontal beam

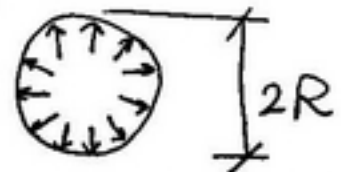
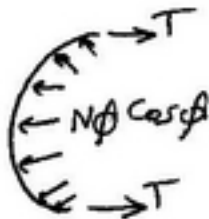
طریق عدم وجود (G) لا نیاز فکرات افقیه ولا، آسیه  
لا  $N\phi = 0 \Leftarrow$  اگر کسیه = صفر



$N\phi \sin \phi \rightarrow$   کمرہ ایسیہ

$N\phi \cos \phi \rightarrow$   کمرہ افقیہ

$$T = N\phi \cdot \cos \phi * R$$



$$\text{Force} = \text{Tension} = WR$$

$$W = N\phi \cos \phi$$

$$A_s = \frac{T}{F_s} = \dots (A_s) \leftarrow$$

$F_s \leftarrow \text{kg/cm}^2$

میں

OK

12  
113

Faculty of engineering  
3rd year

# structural analysis

No ( )

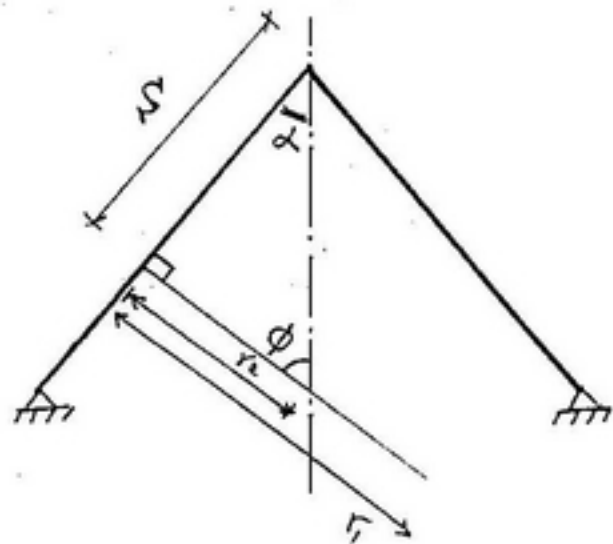
## 2- Conical shell

$$r_1 = \infty$$

$$\phi = \text{Const}$$

$$\phi = 90^\circ - \alpha$$

$$r_2 = \frac{s}{\tan \phi}$$



$$N_s = \frac{-1}{s \cdot \sin \phi} \int (P_r \cos \phi + P_s \sin \phi) s \cdot ds \rightarrow \textcircled{1}$$

$$\frac{N_s}{r_1} + \frac{N_\theta}{r_2} = -P_r$$

$$\therefore \frac{N_s}{\infty} + \frac{N_\theta}{r_2} = -P_r$$

$$\therefore 0 + \frac{N_\theta}{s/\tan \phi} = -P_r$$

$$\Rightarrow N_\theta = -P_r \cdot \frac{s}{\tan \phi} \rightarrow \textcircled{2}$$

(i) Case of dead load

$$g \sin \phi = P_s$$
$$P_r = g \cos \phi$$

$$P_r = g \cos \phi$$

$$P_s = g \sin \phi$$

$$\therefore N_s = \frac{-1}{s \cdot \sin \phi} \int g (\cos^2 \phi + \sin^2 \phi) s \, ds$$

$$= \frac{-g}{s \cdot \sin \phi} \int s \cdot ds$$

$$= \frac{-g}{s \cdot \sin \phi} \left[ \frac{s^2}{2} + C \right]$$

$$\therefore N_\theta = -P_r \cdot \frac{s}{\tan \phi} = -g \cos \phi \cdot \frac{s}{\tan \phi}$$

$$= -g \cdot \cos \phi \cdot \frac{s}{\sin \phi} \cdot \cos \phi$$

$$= \frac{-s \cdot g}{\sin \phi} \cdot \cos^2 \phi$$



B.C. at  $s = 0 \Rightarrow N_s = 0$

$$\Rightarrow \frac{s^2}{2} + C = 0$$

$$\Rightarrow C = 0$$

s.

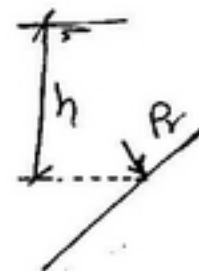
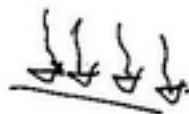
$$N_s = \frac{-g}{s \sin \phi} \cdot \frac{s^2}{2} = \frac{-g \cdot s}{2 \sin \phi}$$

$$N_\theta = \frac{g \cdot s}{\sin \phi} \cdot \cos^2 \phi$$

(ii) Case of water load

$$\begin{cases} P_r = \gamma h \\ P_s = 0 \end{cases}$$

→ كوز مادي



### 3- Cylindrical shell

$$r_1 = \infty$$

$$r_2 = a$$

$$P_s = 0$$

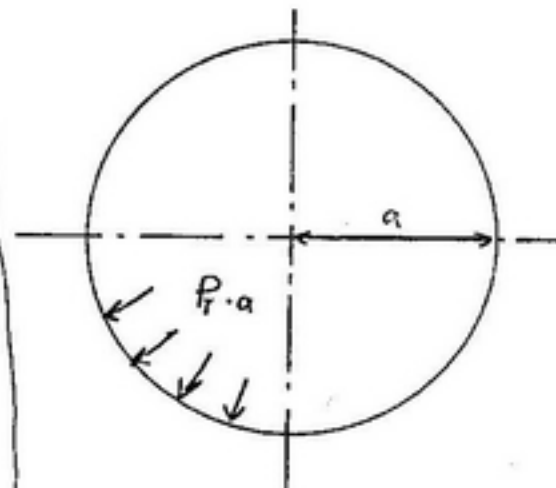
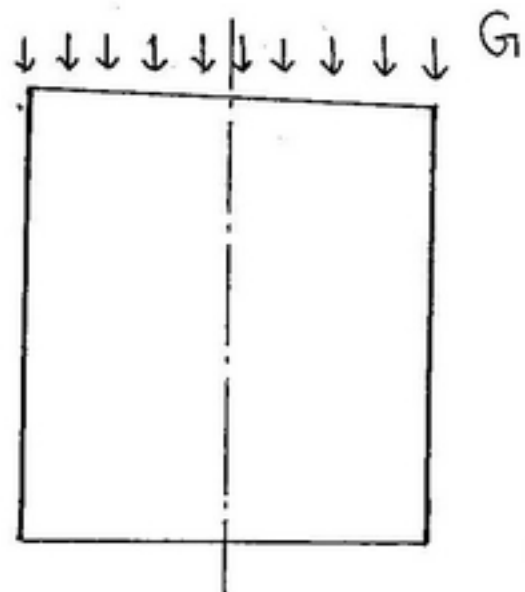
$$P_r = \gamma h$$

$$N_\theta = -P_r \cdot a$$

$$N_s = 0$$

$$dr = -G \leftarrow \text{حمل مرکز}$$

$$dr = -W \leftarrow \text{o.w.t}$$



$$P_r = \gamma h \longrightarrow \text{water load}$$

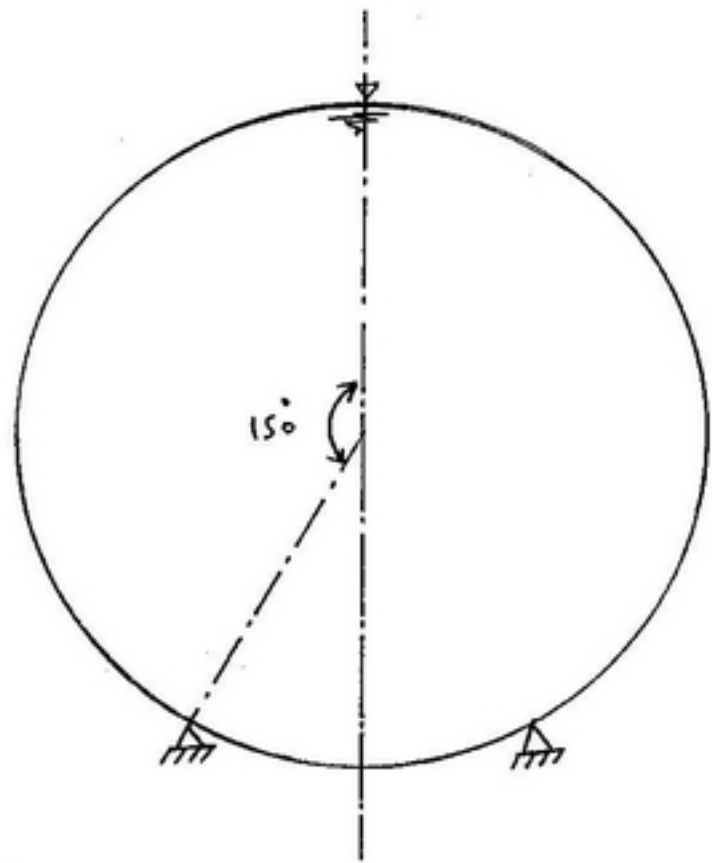
### Example ①

$$a = 5.0 \text{ m}$$

$$\gamma = 1 \text{ t/m}^3$$

Required

$$N_\phi, N_\theta$$

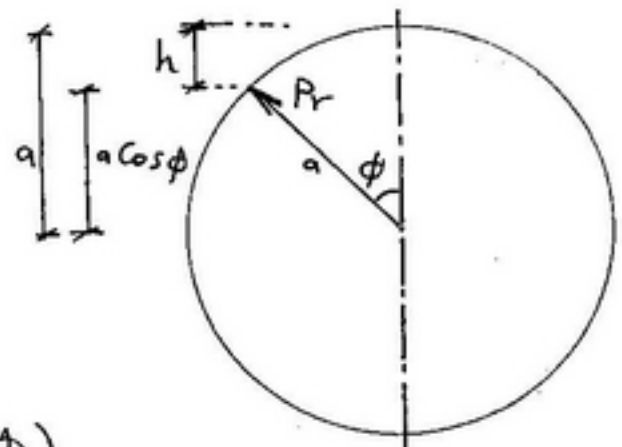


———— SOL ———

$$h = a - a \cos \phi$$

$$P_r = -\gamma h$$

$$P_\phi = 0.0$$



$$\therefore P_r = -\gamma (a - a \cos \phi)$$

$$= -1 [5 - 5 \cos \phi] = 5 \cos \phi - 5$$

$$= -5 [1 - \cos \phi]$$

$$N\phi = \frac{-1}{r_2 \cdot \sin^2 \phi} \int r_1 \cdot r_2 [P_r \cos \phi + \cancel{P_\phi \cdot \sin \phi}] \sin \phi \cdot d\phi$$

$$= \frac{-a}{\sin^2 \phi} \int -5 [1 - \cos \phi] \cos \phi \sin \phi \, d\phi$$

$$= \frac{25}{\sin^2 \phi} \int \cos \phi \sin \phi - \cos^2 \phi \cdot \sin \phi \, d\phi$$

$$= \frac{25}{\sin^2 \phi} \left[ -\frac{\cos^2 \phi}{2} + \frac{\cos^3 \phi}{3} + C \right]$$

$$\boxed{N\phi = \frac{25}{6 \sin^2 \phi} [-3 \cos^2 \phi + 2 \cos^3 \phi + C]} \rightarrow \textcircled{1}$$

$$N_\theta = -P_r \cdot a - N\phi$$

$$\boxed{N_\theta = -5 [-5 (1 - \cos \phi)] - N\phi} \rightarrow \textcircled{2}$$

For part ①

$$\text{at } \phi = 0 \dots \Rightarrow N\phi = 1 \dots$$

$$\Rightarrow -3 \cos^2 0 + 2 \cos^3 0 + C = 0 \dots$$

$$\Rightarrow \boxed{C = 1 \dots}$$

$$\therefore \boxed{\begin{aligned} N\phi &= \frac{25}{6 \sin^2 \phi} [-3 \cos^2 \phi + 2 \cos^3 \phi + 1] \\ N\theta &= -5 [-5(1 - \cos \phi)] - N\phi \end{aligned}}$$

$$\boxed{\text{at } \phi = 0 \dots}$$

$$N\phi = \frac{25}{6 \cdot \sin^2 0} [-3 \cos^2 0 + 2 \cos^3 0 + 1]$$

$$= \frac{0}{0}$$

by l'Hopital

$$\lim_{\phi \rightarrow 0} = \frac{25}{6} \cdot \frac{6 \cos \phi \sin \phi - 6 \cos^2 \phi \sin \phi}{2 \cos \phi \cdot \sin \phi}$$

$$= \frac{25}{6} \cdot \frac{6 - 6 \cos \phi}{2}$$



$$= \frac{25}{6} \times \frac{6-6}{2} = 0.0$$

$$N_{\theta} = 25 [1-1] - 0 = 0.0$$

|              |   |
|--------------|---|
| $\phi$       | 0 |
| $N_{\phi}$   | 0 |
| $N_{\theta}$ | 0 |

at  $\phi = 90$

$$N_{\phi} = \frac{25}{6} [-3 \times 0 + 2 \times 0 + 1]$$

$$= 4.167$$

$$N_{\theta} = 25 [1 - \cos 90] - 4.167$$

$$= 21.8$$

|              |       |
|--------------|-------|
| $\phi$       | 90    |
| $N_{\phi}$   | 4.167 |
| $N_{\theta}$ | 21.8  |

at  $\phi = 150$  support reaction

$$\Rightarrow N\phi = \frac{25}{6 \sin^2 150} [-3 \cos^2 150 + 2 \cos^3 150 + 1]$$

$$= -42$$

$$N\theta = 25 [1 - \cos 150] - (-42)$$

$$= 88.6$$

| $\phi$    | 150  |
|-----------|------|
| $N\phi$   | -42  |
| $N\theta$ | 88.6 |

Part II

Boundary Conditions

$$\text{at } \phi = 180 \Rightarrow N\phi = 0$$

$$3 \cos^2 180 - 2 \cos^3 180 + C = 0$$

$$\Rightarrow \boxed{C = 5}$$

$$\Rightarrow \left\{ \begin{array}{l} N\phi = \frac{25}{6 \sin^2 \phi} [-3 \cos^2 \phi + 2 \cos^3 \phi + 5] \\ N\theta = 25(1 - \cos \phi) - N\phi \end{array} \right\} \Rightarrow \text{II}$$

at  $\phi = 150$  support here

$$N\phi = 24.18$$

$$N\theta = 22.5$$

| $\phi$    | 150   |
|-----------|-------|
| $N\phi$   | 24.18 |
| $N\theta$ | 22.5  |

at  $\phi = 180$

$$N\phi = \frac{25}{6 \sin^2 180} [-3 \cos^2 180 + 2 \cos^3 180 + 5]$$

$$= \frac{25 [-3 - 2 + 5]}{0} = \frac{0}{0}$$

By using l'hopital

$$= \lim_{\phi \rightarrow 180} \frac{\frac{25}{6} [6 \cos \phi \sin \phi - 6 \cos^2 \phi \sin \phi]}{2 \cos \phi \sin \phi}$$

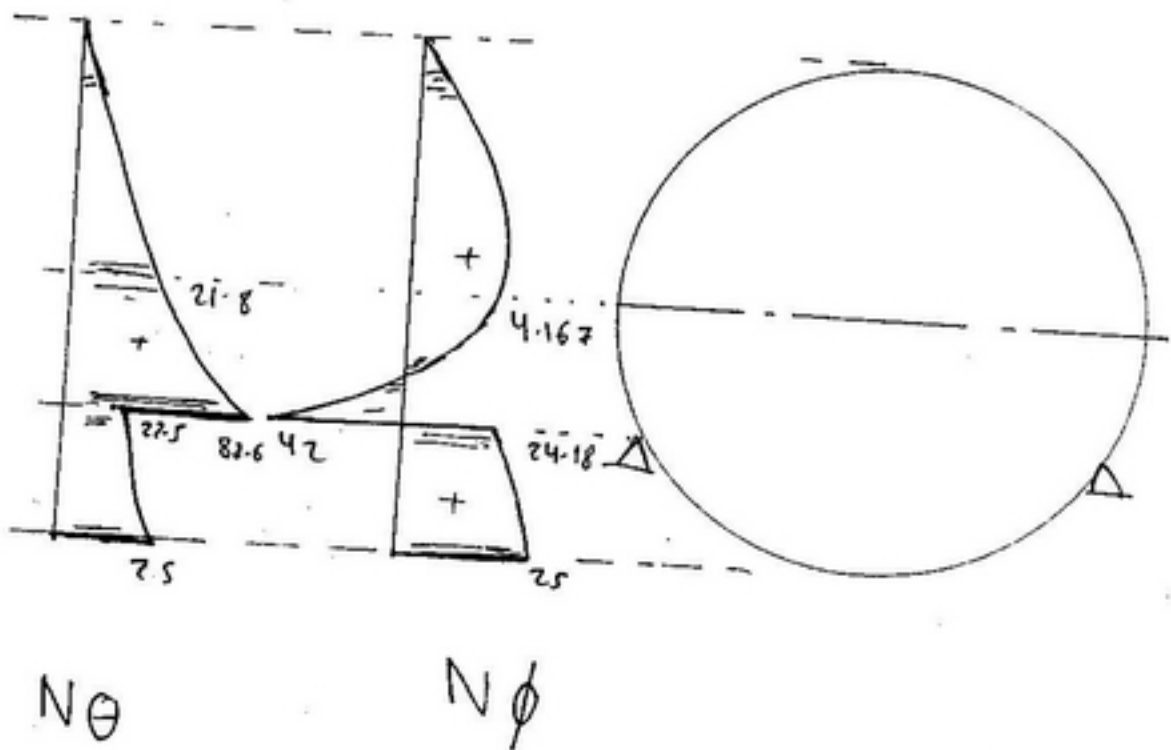
$$= \lim_{\phi \rightarrow 180} \frac{\frac{25}{6} \cdot \frac{6 - 6 \cos 180}{2}}{6 \times 2} = \frac{25 (5+6)}{6 \times 2}$$

$$\therefore N_{\phi} = 25$$

$$N_{\theta} = 25 [1 - \cos 180] - 25$$

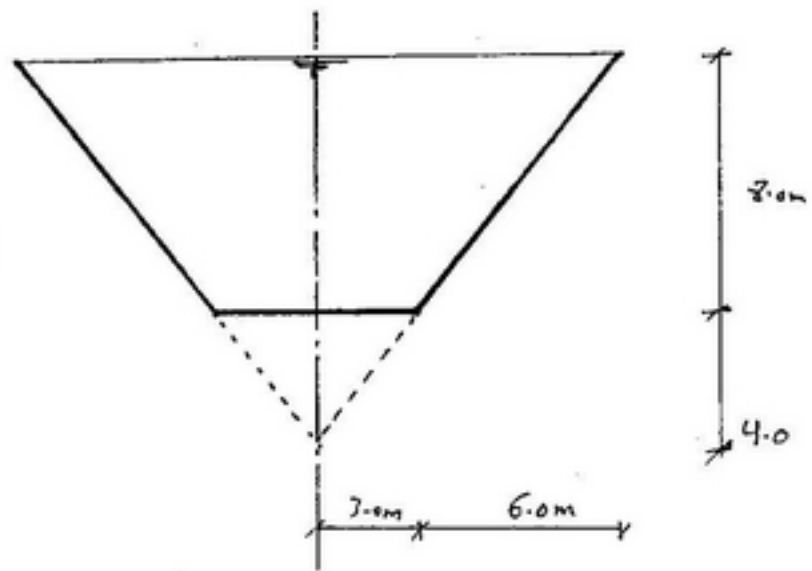
$$= 25 \times 2 - 25 = 25$$

| $\phi$       | 180 |
|--------------|-----|
| $N_{\phi}$   | 25  |
| $N_{\theta}$ | 25  |

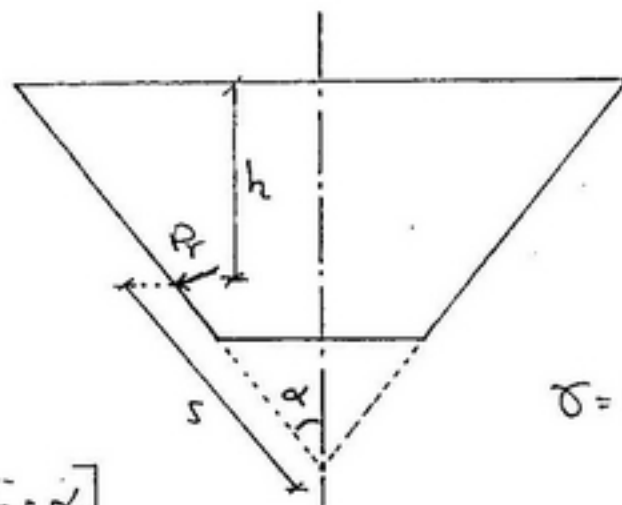


## Example ②

draw  $N_s, N_\theta$   
diagram



— sol —



$$h = 12 - s \cos \alpha$$

$$P_r = -\gamma h$$

$$= -\gamma [12 - s' \cos \alpha]$$

$$= \gamma (s' \cos \alpha - 12)$$

$$= \boxed{s' \cos \alpha - 12}$$

$$P_s = 0 \dots$$

$$\gamma = 1.0$$

$$\alpha = 36.87^\circ$$

$$\phi = 90^\circ - \alpha = 53.13^\circ$$

$$N_s = \frac{-1}{s \cdot \sin \phi} \int (P_r \cos \phi + P_s \sin \phi) s' ds$$



$$\therefore N_s = \frac{-1}{s \cdot \sin \phi} \int (s \cos \alpha - 12) \cos \phi \, s \, ds$$

$$= \frac{-1}{s \cdot \sin \phi} \int (s^2 \cos \alpha \cos \phi - 12 s \cos \phi) \, ds$$

$$= \frac{-1}{s \cdot \sin \phi} \left[ \frac{s^3}{3} \cos \alpha \cos \phi - \frac{12 s^2}{2} \cos \phi + C \right]$$

B.C

$$\text{at } s' = 15 \Rightarrow N_s = 0.0$$

$$\begin{cases} \phi = 53.13 \\ \alpha = 36.86 \end{cases}$$

$$\frac{(15)^3}{3} \cos(36.87) \cos(53.13) - \frac{12(15)^2}{2} \cos(53.13) + C = 0.0$$

$$\Rightarrow \boxed{C = +270}$$

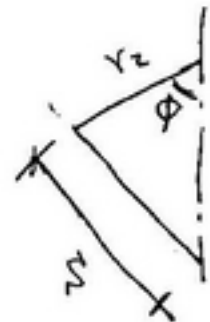
$$\therefore N_{s'} = \frac{-1}{s' \cdot \sin \phi} \left[ \frac{s'^3}{3} \times 0.48 - \frac{12 s'^2}{2} \times 0.6 + 270 \right]$$

$$\boxed{N_{s'} = \frac{-1}{s'} \left[ 0.2 s'^3 - 4.5 s'^2 + 337.5 \right]}$$

$$\frac{N_s}{r_2} + \frac{N_\theta}{r_2} = -P_r$$

$$P_r = s' \cdot \cos \alpha - 12$$

$$r_2 = \frac{s'}{\tan \phi}$$

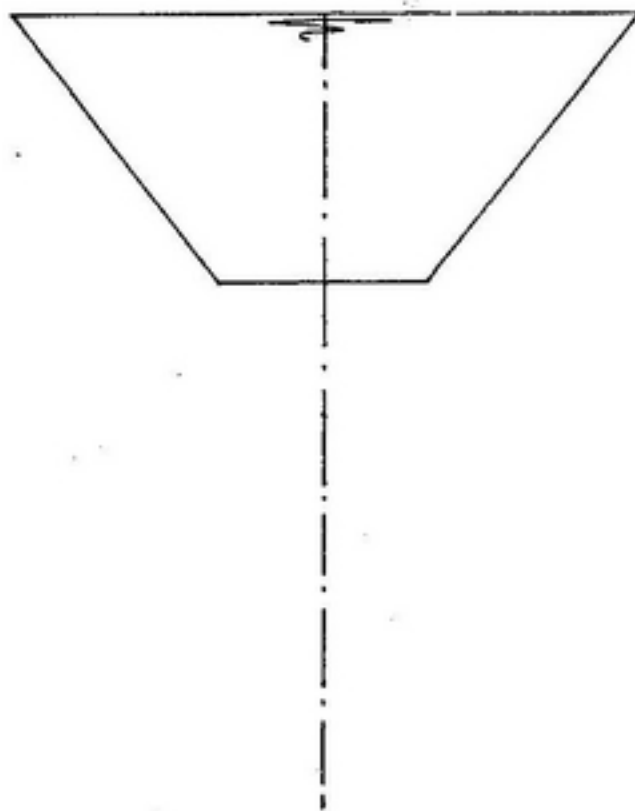
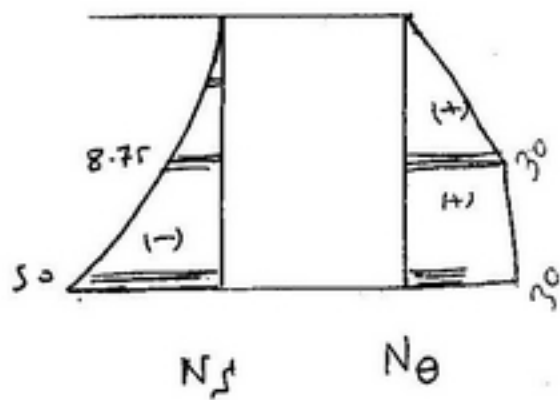


$$\tan \phi = \frac{s'}{r_2}$$

$$\begin{aligned} N_\theta &= - (s' \cdot \cos \alpha - 12) \left( \frac{s'}{\tan \phi} \right) \\ &= - (s' \times 0.8 - 12) \left( \frac{s'}{1.33} \right) \end{aligned}$$

$$N_\theta = -0.6 s'^2 + 9.0 s'$$

|            |    |       |       |
|------------|----|-------|-------|
| $s'$       | 15 | 10    | 5     |
| $N_{s'}$   | 0  | -8.75 | -50   |
| $N_\theta$ | 0  | +30.0 | +30.0 |



13.

Faculty of engineering  
3rd year

# structural analysis

No ( )

## Shells

### 1] spherical.

$$r_1 = r_2 = a$$

$$N_\phi = \frac{-1}{r_2 \cdot \sin \phi} \int r_1 r_2 [P_r \cos \phi + P_\phi \sin \phi] \sin \phi d\phi$$

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = -P_r$$

#### (i) dead load (g)

$$P_r = g \cos \phi$$

$$P_\phi = g \sin \phi$$

تم التقويف من الجاذبية

#### (ii) water load

$$P_r = \gamma h$$

$$P_\phi = 0$$

تم التقويف من الجاذبية

### 2] Conical

$$r_1 = \infty$$

$$d = \sqrt{r}$$

$$r_2 = \frac{s}{\tan \phi}$$

$$\phi = \sqrt{r}$$

$$N_s = \frac{-1}{s \cdot \sin \phi} \int (P_r \cos \phi + P_\phi \sin \phi) s ds$$

$$\frac{N_s}{r_1} + \frac{N_\theta}{r_2} = -P_r \Rightarrow N_\theta = -P_r \cdot r_2$$

(1)



(i) dead load (g)

$$P_r = g \cos \phi$$

$$P_s = g \sin \phi$$

← التعويض في معادلاته

(ii) water Load

$$P_r = \gamma h$$

$$P_s = 0.0$$

← التعويض في معادلاته

[3] Cylindrical shell

water only

$$N_s = 0.0$$

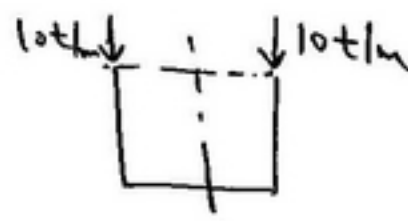
$$N_\theta = -P_r \cdot a$$

$$P_r = \gamma h$$

$N_s = 0$  في لو كان نصف الكرة  
في cylinder

$$N_s = -10$$

مثال



## Live Load

$$N\phi = \frac{W}{2\pi r_2 \cdot \sin^2 \phi}$$

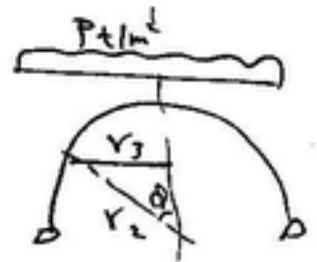
W → عبارة عن حمل مركّز  
لو لم يكن مركّز يتم تركيزه الى صيغة بالية

①

$$W = P \times \pi r_3^2$$

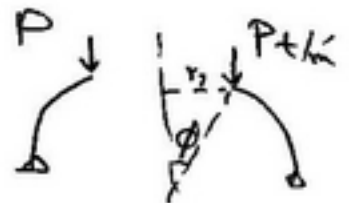
$$r_3 = r_2 \sin \phi$$

$$W = P \pi r_2^2 \sin^2 \phi$$

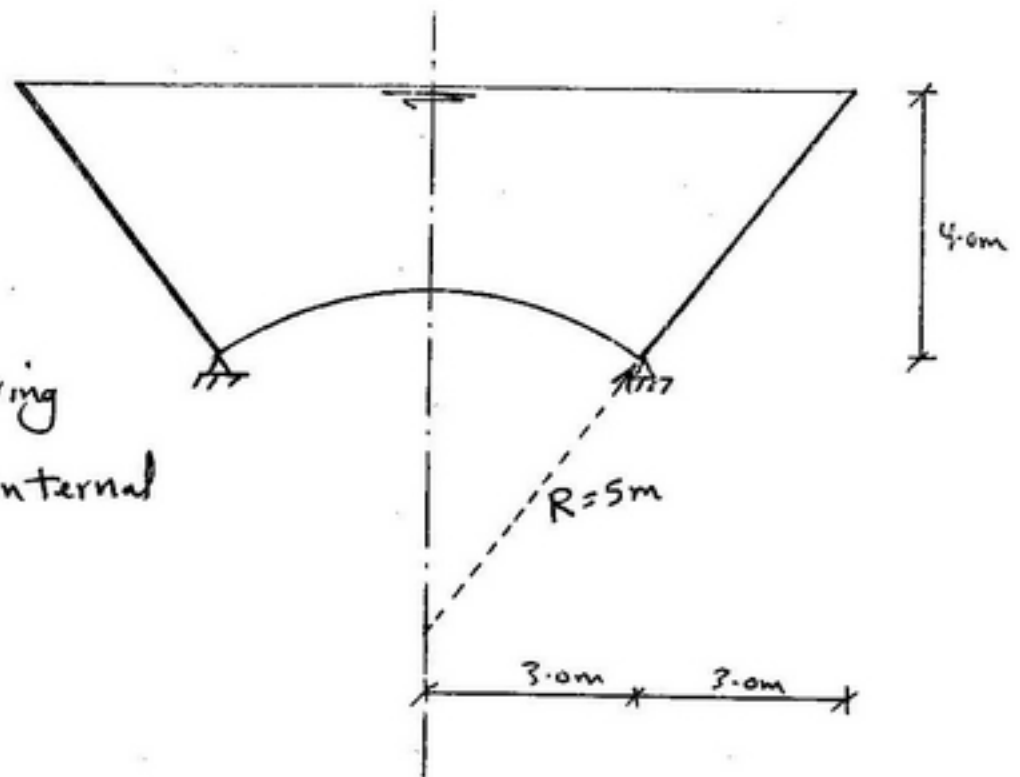


②  $W = P \times (2\pi r_3)$

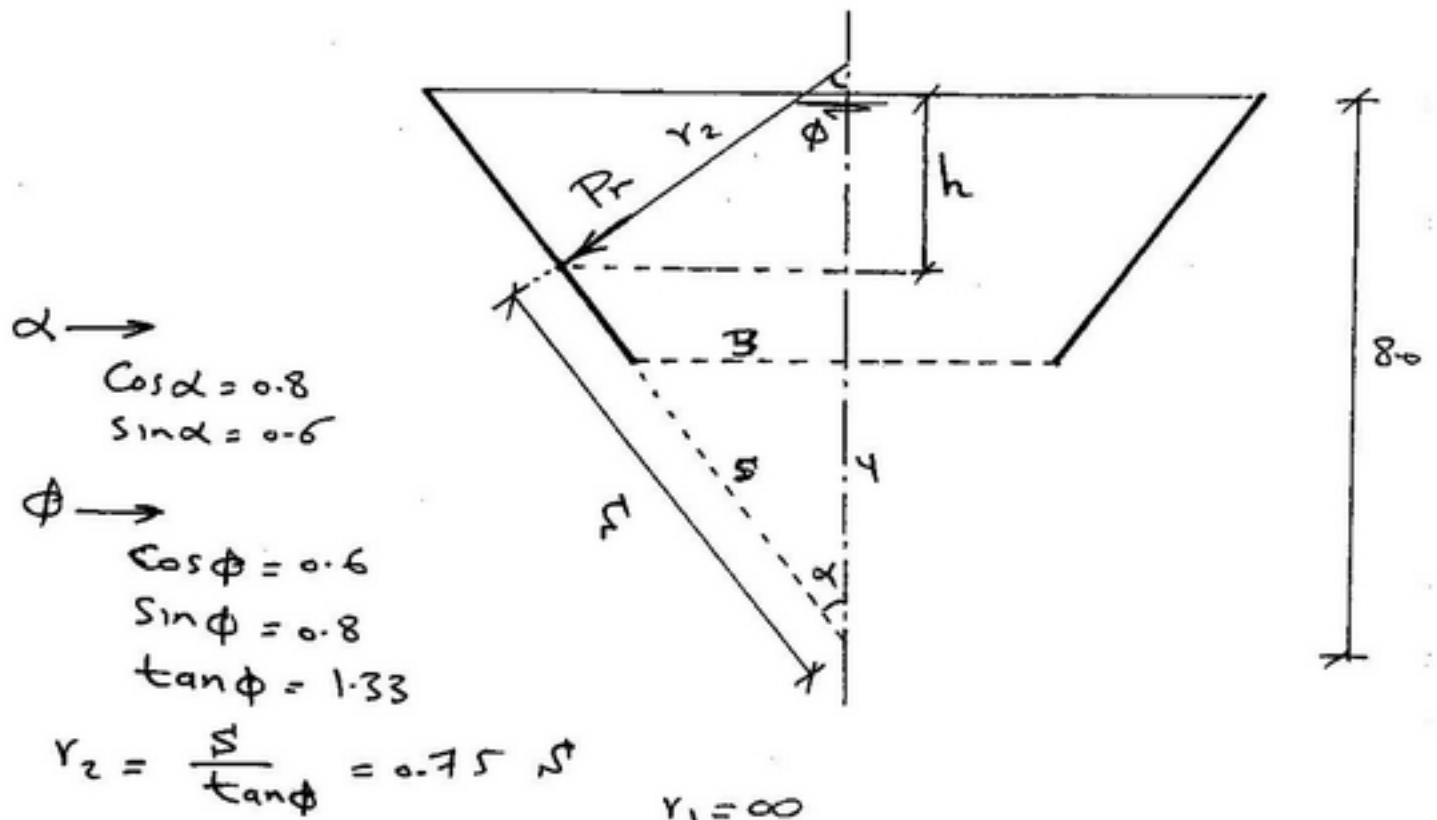
$$W = P \times 2\pi r_2 \sin \phi$$



For the following  
tank Find Internal  
Forces.



———— Sol ————



(14)

$$h = 8 - S' \cos \alpha$$

$$h = 8 - 0.8 S'$$

$$P_r = -\gamma h$$

$$P_r = 0.8 S' - 8$$

$$P_s = 0.0$$

$$N_s = \frac{-1}{S' [0.8]} * \int (0.8 S' - 8) (0.6) S' \cdot dS'$$

$$= \frac{-0.75}{S'} \int (0.8 S'^2 - 8 S') dS'$$

$$= \frac{-0.75}{S'} \left[ \frac{0.8 S'^3}{3} - 4 S'^2 + C \right]$$

$$\text{at } S' = 10 \longrightarrow \int = 0.0$$

منزاعی نقطہ  
خروج  
S=10

$$\frac{0.8(1000)}{3} - 4(100) + C = 0.0$$

$$C = 133.3$$

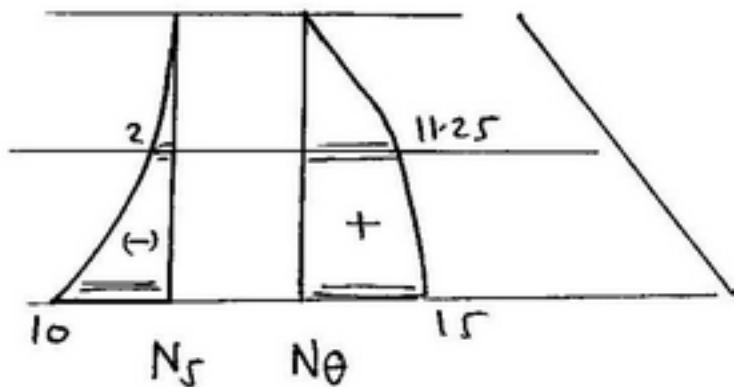
$$\therefore N_s = \frac{-0.75}{S'} \left[ \frac{0.8 S'^3}{3} - 4 S'^2 + 133.33 \right]$$

$$N_{\theta} = -P_r \cdot r_2$$

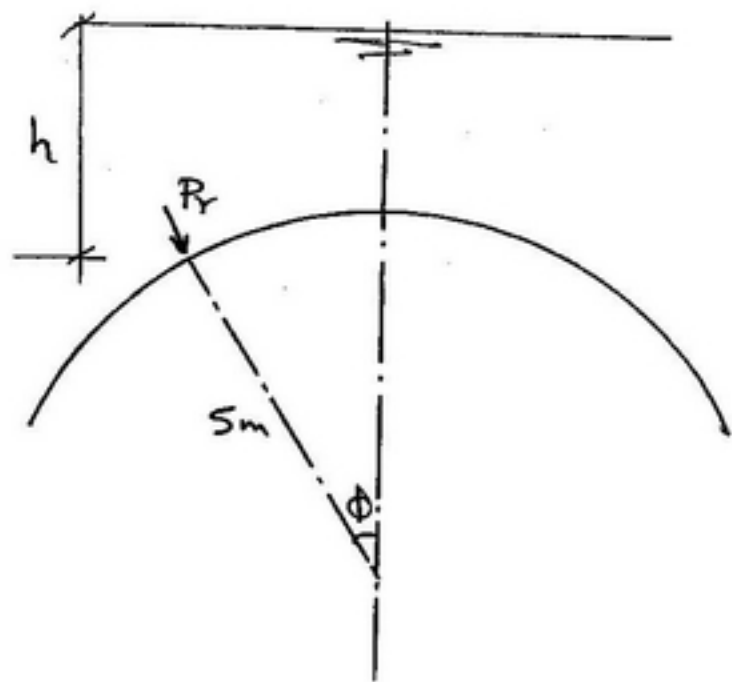
$$= (8 - 0.8s)(0.75s)$$

$$N_{\theta} = 6s - 0.6s^2$$

| s              | 5   | 7.5   | 10 |
|----------------|-----|-------|----|
| N <sub>s</sub> | -10 | -2    | 0  |
| N <sub>θ</sub> | 15  | 11.25 | 0  |







$$r_1 = r_2 = 5$$

$$P_r = \gamma h = h$$

$$P_r = 8 - 5 \cos \phi$$

$$P_\phi = 0 \dots$$

$$N_\phi = \frac{-1}{5 \cdot \sin^2 \phi} \int 5^2 \cdot [8 - 5 \cos \phi] \cos \phi \cdot \sin \phi d\phi$$

$$= \frac{-5}{\sin^2 \phi} \int [8 \cos \phi \cdot \sin \phi - 5 \cos^2 \phi \cdot \sin \phi] d\phi$$

$$= \frac{-5}{\sin^2 \phi} \left[ -4 \cos^2 \phi + \frac{5}{3} \cos^3 \phi + C \right]$$

$$\text{at } \phi = 0 \rightarrow \int = 0$$

$$\Rightarrow -4 + \frac{5}{3} + C = 0$$

$$\Rightarrow C = 2.33$$

$$\therefore N\phi = \frac{-5}{\sin^2 \phi} \left[ -4\cos^2 \phi + \frac{5}{3}\cos^3 \phi + 2.33 \right]$$

$$\frac{N\phi}{5} + \frac{N\theta}{5} = -(8 - 5\cos \phi)$$

$$N\theta = (-40 + 25\cos \phi) - N\phi$$

$$\underline{\underline{\text{at } \phi = 0}}$$

$$N\phi = \frac{0}{0}$$

By using l'Hopital

$$\lim_{\phi \rightarrow 0} \frac{-5}{\sin^2 \phi} \left[ -4\cos^2 \phi + \frac{5}{3}\cos^3 \phi + 2.33 \right]$$

$$\lim_{\phi \rightarrow 0} \frac{-5}{2 \sin \phi \cos \phi} \left[ + 4(2 \cos \phi \sin \phi) - \frac{5}{3} (3 \cos^2 \phi \sin \phi) \right]$$

$$\lim_{\phi \rightarrow 0} \frac{-5}{2} [8 - 5 \cos \phi]$$

$$= \frac{-5}{2} [8 - 5] = -7.5$$

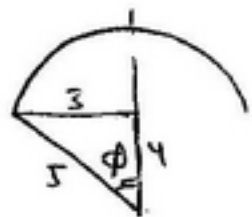
$$\therefore N_{\phi} = -7.5$$

$$\therefore N_{\theta} = (-40 + 25) - (-7.5) = -7.5$$

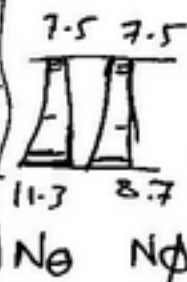
$$\text{At } \phi = 36.867$$

$$N_{\phi} = -8.7$$

$$N_{\theta} = -11.3$$



|              |      |        |
|--------------|------|--------|
| $\phi$       | 0.0  | 36.867 |
| $N_{\phi}$   | -7.5 | -8.7   |
| $N_{\theta}$ | -7.5 | -11.3  |



## Ring beam

$$36.867 = \phi \quad \text{آخريتي من الجدران}$$

$$8.7 = N\phi$$

$$\left\{ \begin{array}{l} \text{الافتقار} \rightarrow N\phi \cos \phi = 8.7 \cos 36.87 = 6.96 \\ \text{الرأس} \rightarrow N\phi \sin \phi = 8.7 \sin 36.87 = 5.22 \end{array} \right.$$

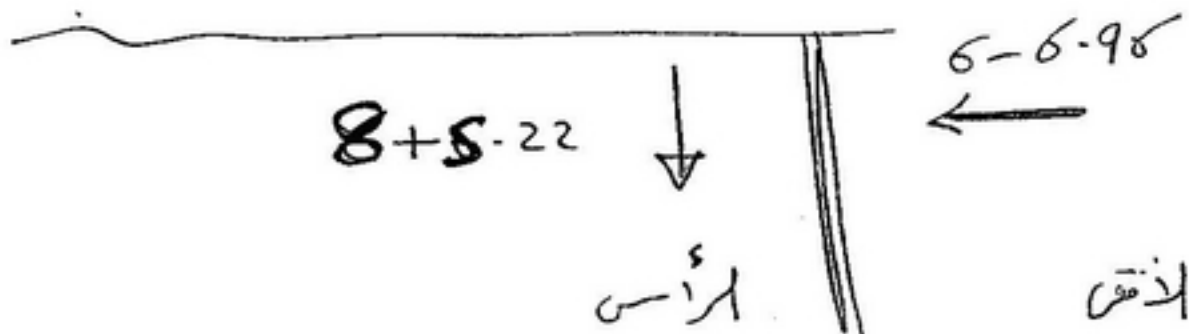


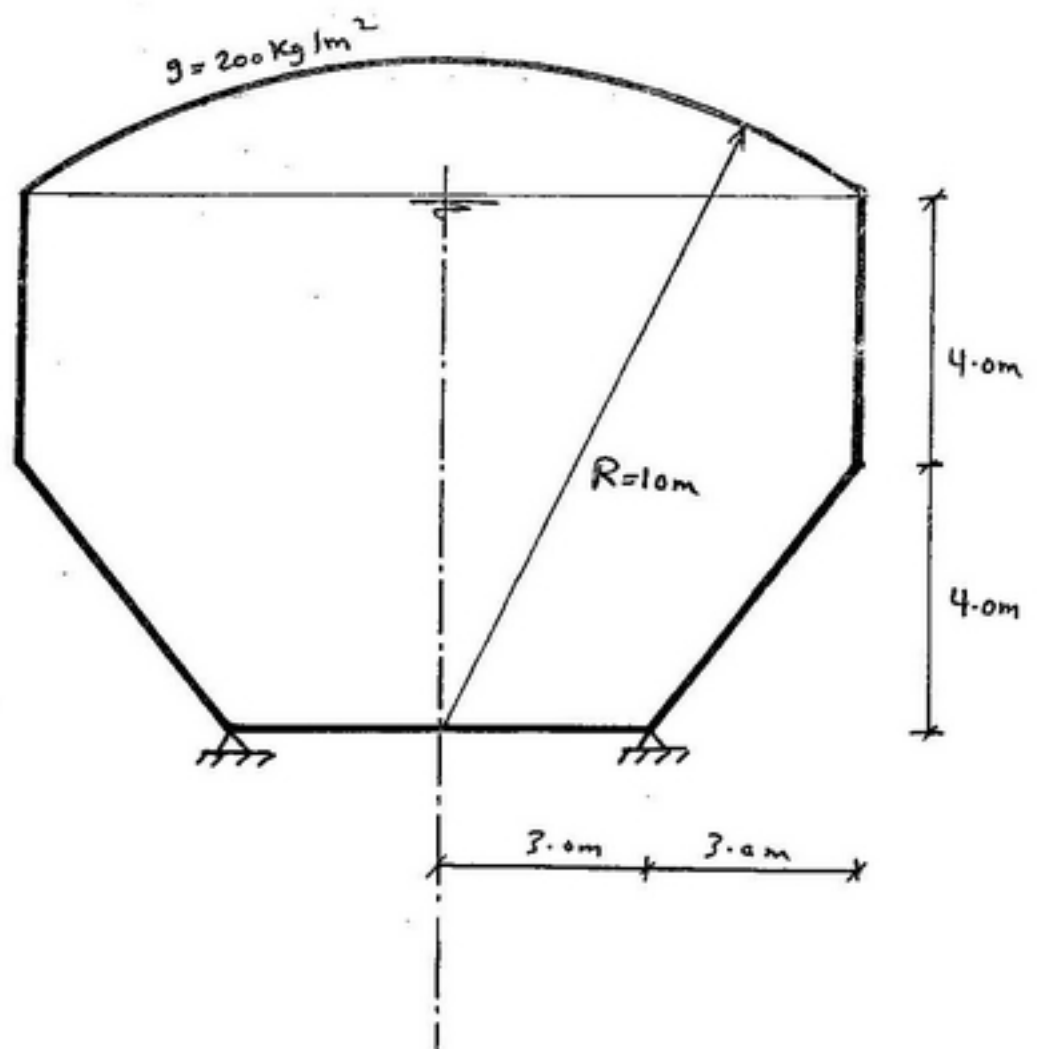
Conical ، لكنه يوجد قوس آخر

$$\left( \begin{array}{l} \text{القوس من الدعامات} \\ \text{تحت} \end{array} \right) \left( \begin{array}{l} 10 = N_s \\ 53.13^\circ = \phi \end{array} \right. \quad \text{آخريتي}$$

$$\text{الافتقار} \quad N_s \cos \phi = 6$$

$$\text{الرأس} \quad N_s \sin \phi = 8$$





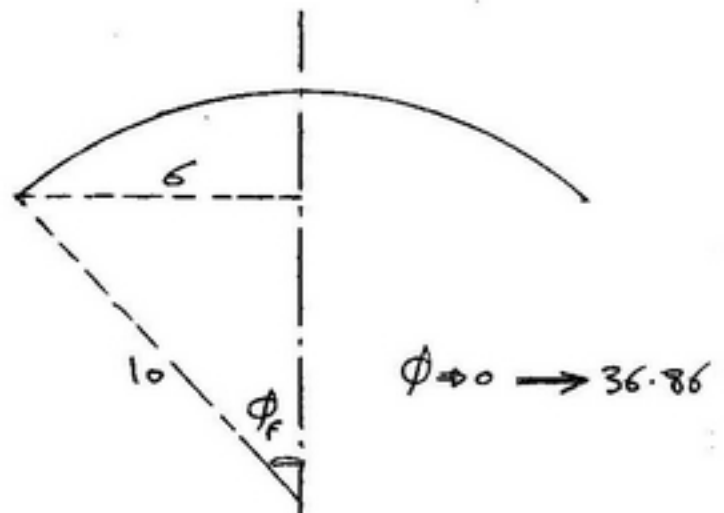
— Sol —

III spherical part → dead load ( $g = 0.2 \text{ t/m}^2$ )

$$P_r = 0.2 \cos \phi$$

$$P_\phi = 0.2 \sin \phi$$

$$r_1 = r_2 = 10$$



$$N\phi = \frac{-1}{10 \sin^2\phi} \int 10^2 * [0.2 \cos^2\phi + 0.2 \sin^2\phi] \sin\phi d\phi$$

$$= \frac{-10 * 0.2}{\sin^2\phi} \int \underbrace{[\cos^2\phi + \sin^2\phi]}_{=1.0} \sin\phi d\phi$$

$$= \frac{-2}{\sin^2\phi} \int \sin\phi d\phi.$$

$$= \frac{-2}{\sin^2\phi} [-\cos\phi + C]$$

$$\text{at } \phi = 0.0 \longrightarrow \int \sim = 0.0$$

$$-\cos(0) + C = 0.0$$

$$C = 1.0$$

$$\therefore N\phi = \frac{-2}{\sin^2\phi} [1 - \cos\phi]$$

$$\sin^2\phi = (1 - \cos\phi)(1 + \cos\phi)$$

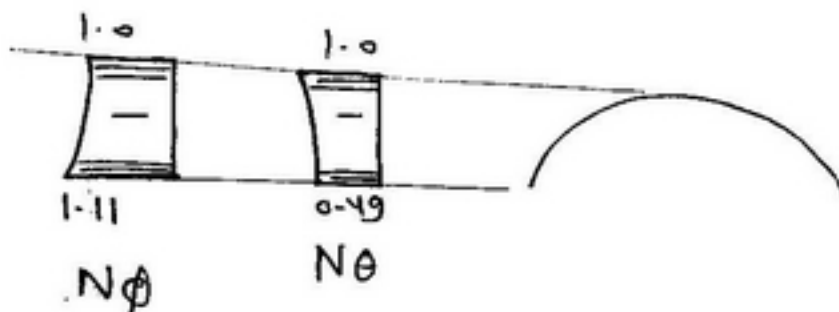
$$N\phi = \frac{-2}{(1 + \cos\phi)}$$

$$\frac{N\phi}{10} + \frac{N\theta}{10} = -(0.2 \cos\phi)$$

$$N\phi = -2 \cos\phi + \frac{2}{1 + \cos\phi}$$



|           |    |       |
|-----------|----|-------|
| $\phi$    | 0  | 36.86 |
| $N\phi$   | -1 | -1.11 |
| $N\theta$ | -1 | -0.49 |



## 2) Cylindrical Part

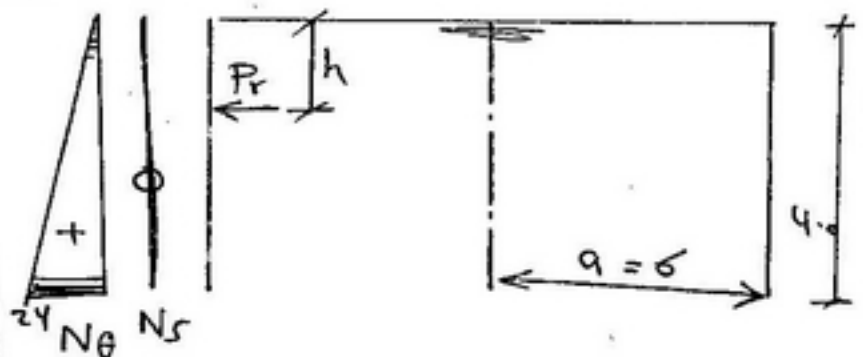
$$P_r = -\gamma h$$

$$= -h$$

$$P_s = 0.0$$

$$N_s = 0.0$$

$$N_\theta = -P_r a = 6h$$



$\rightarrow$  لا يوجد للـ  $N_s$  تأثير من  $N_\theta$  لأن  $\phi = 0$   
 لا يوجد للـ  $N_s$  تأثير من  $N_\theta$  لأن  $\phi = 0$   
 $(N_\phi \cdot \sin \phi)$  sphere لا يوجد تأثير من  $N_\theta$  لأن  $\phi = 0$   
 $N_s = -[N_\phi \sin \phi] = [-1.11 \times 0.6] = 0.66$

3) Conical shell

Try

اعمل كالمثل

Faculty of Engineering  
3<sup>rd</sup> civil year

"8"  
9/10  
10/10

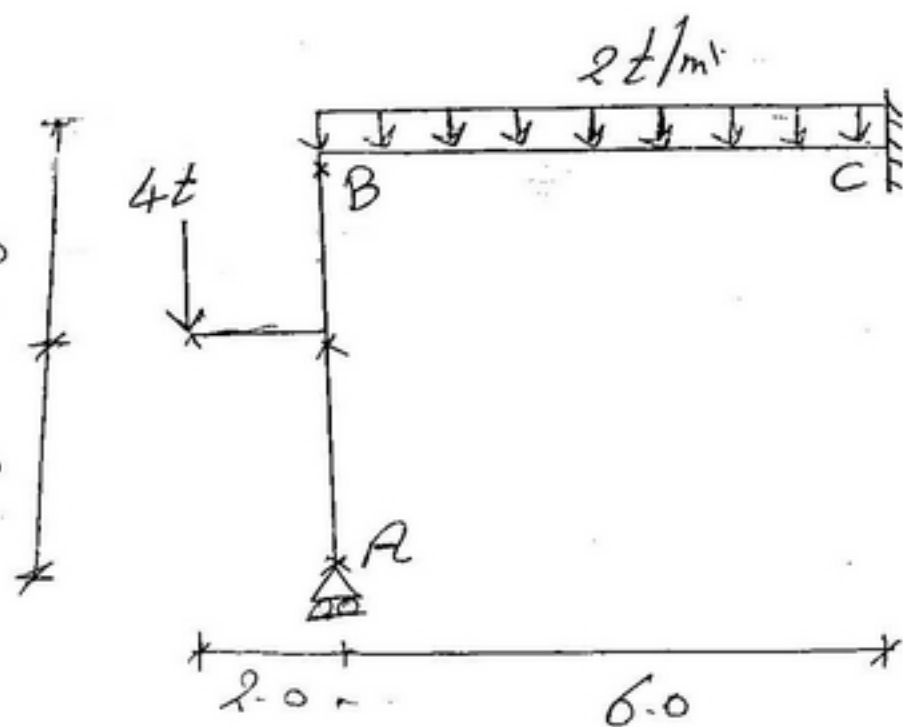
# Revision Mid Term

## *Structure Analysis*

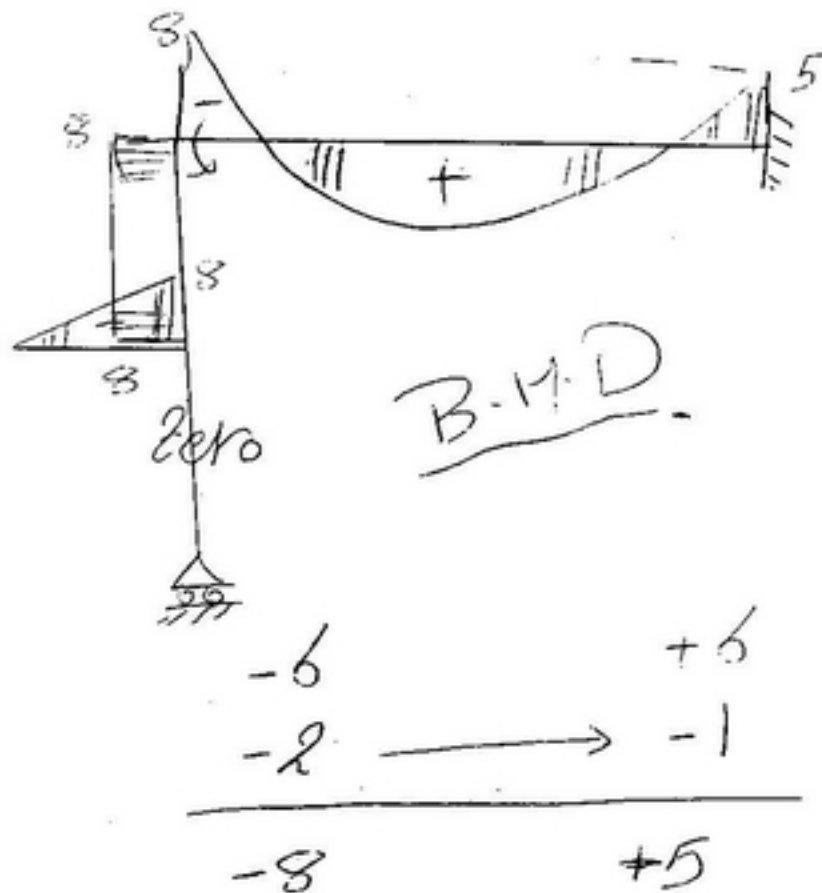
### Part "1"

Ex:

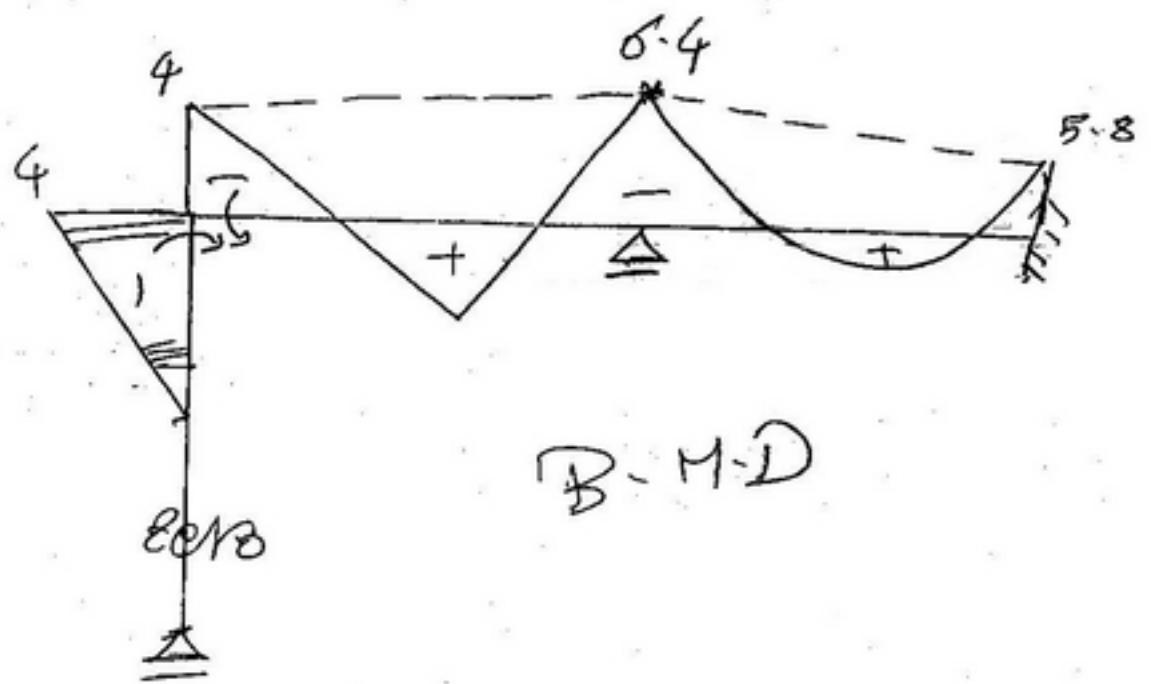
Draw  
N-F-D 2.0  
S-F-D  
B-M-D 2.0



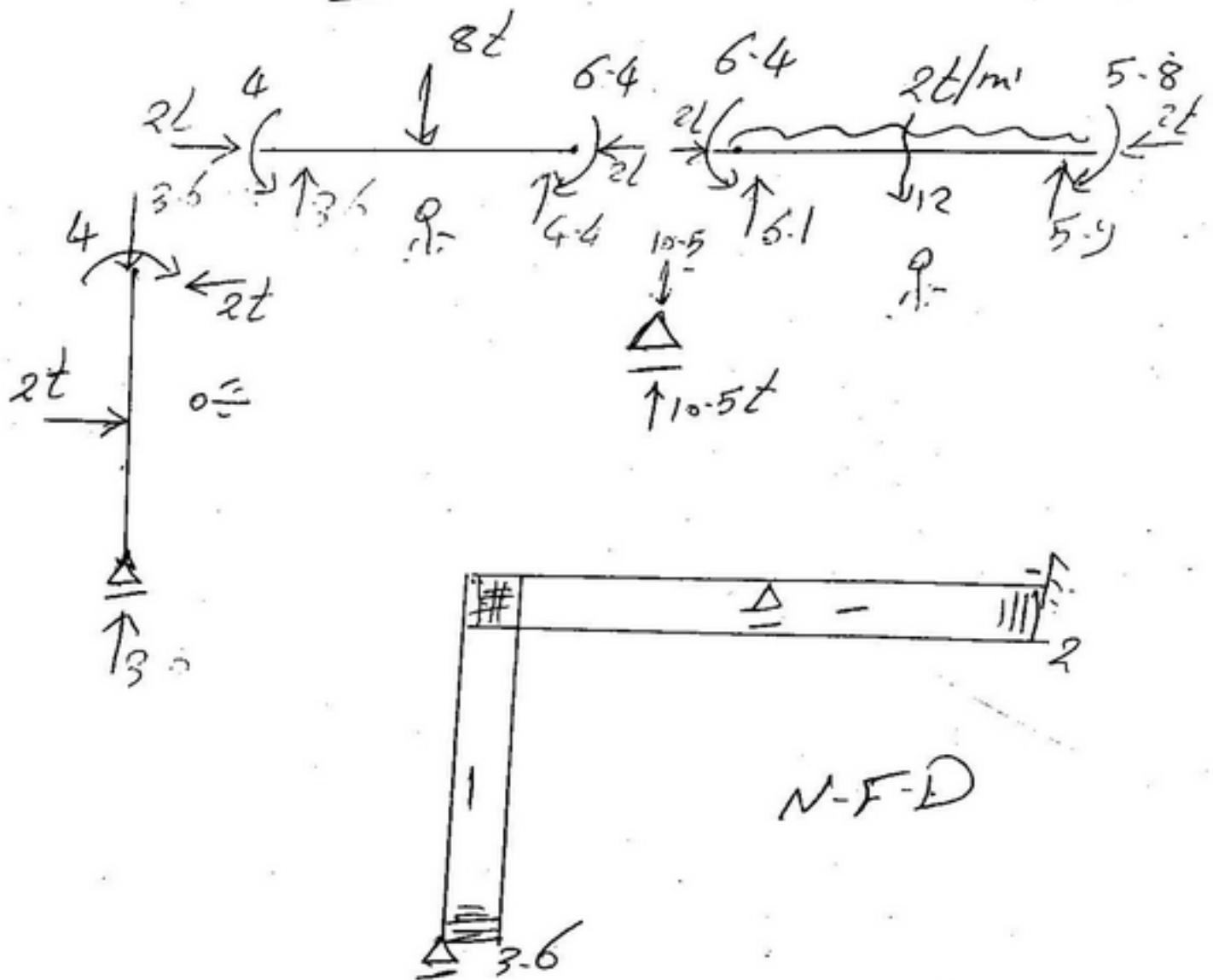
\* solution :





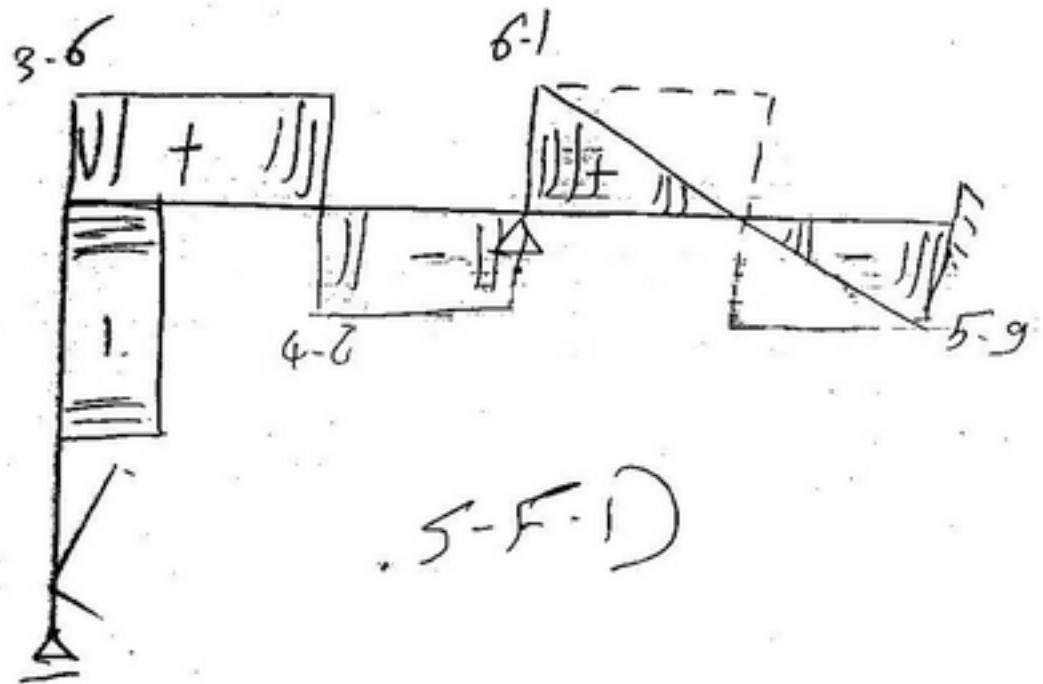


B-M-D



N-F-D





EX:- Draw S-F-D & B-M-D :-

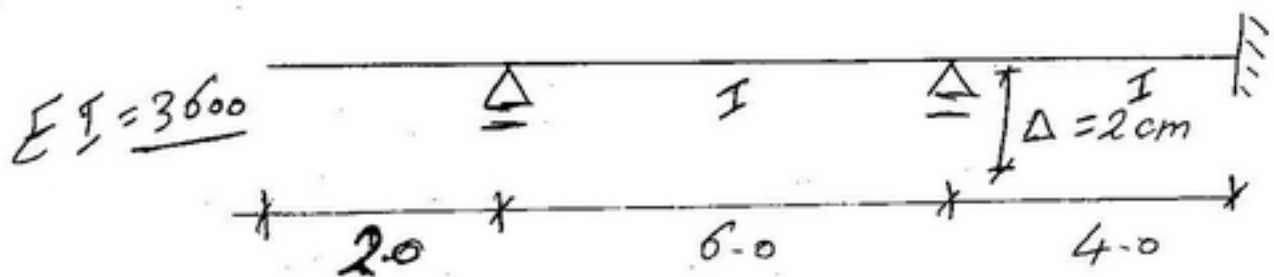
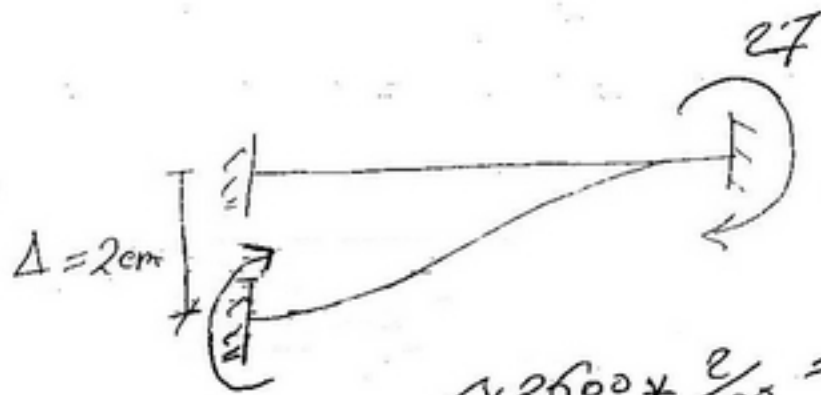


Diagram of a beam of length  $L = 6m$ , fixed at the right end and free at the left end. A vertical displacement  $\Delta = 2cm$  is shown at the right end. The beam is labeled "L = 6m" and "Δ = 2cm".

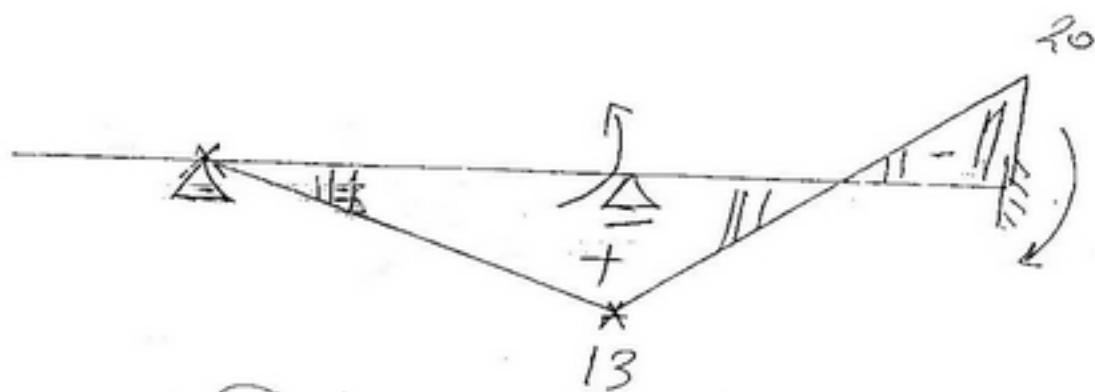
$$\frac{6EF}{L^2} \Delta = \frac{6 \times 3600}{36} \times \frac{2}{100} = 12$$



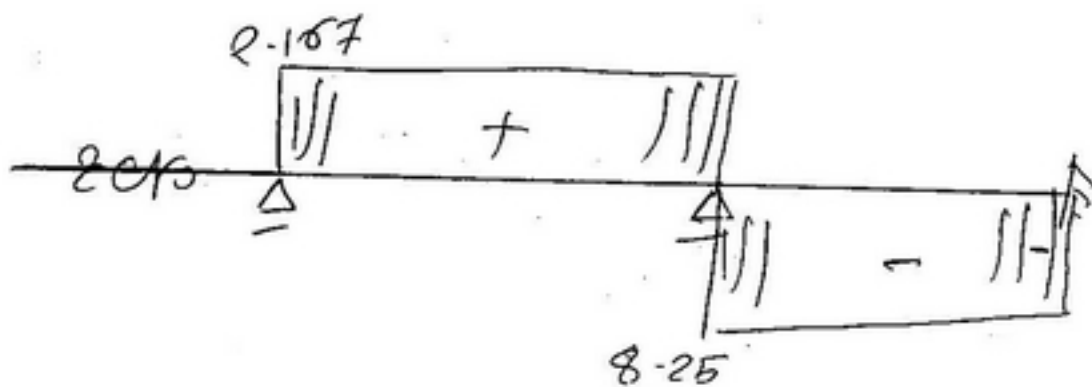
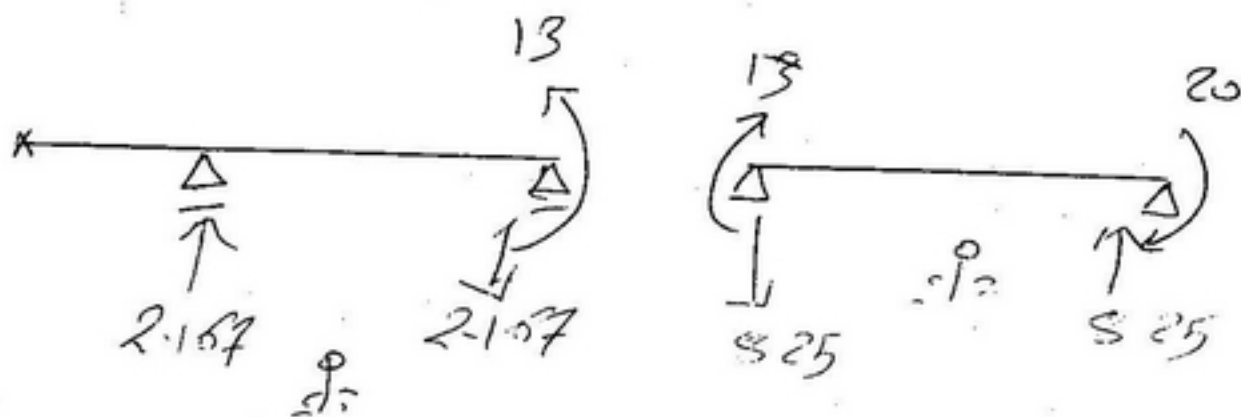
$$\frac{6EI}{L^2} \Delta = \frac{6 \times 3600}{16} \times \frac{2}{100} = 27$$

~~21~~  
-21

|                  |   |     |     |
|------------------|---|-----|-----|
|                  |   |     |     |
| R-5              |   |     |     |
| DF               |   |     |     |
| FEM <sub>0</sub> | 0 | -12 | +27 |
|                  |   | +12 |     |
| FEM              | 0 | 0   | 27  |
| D-M              | 0 | 0   | 0   |
| C-M              | 0 | 0   | 0   |
| D-M              | 0 | 0   | 20  |
|                  |   | -13 | +13 |



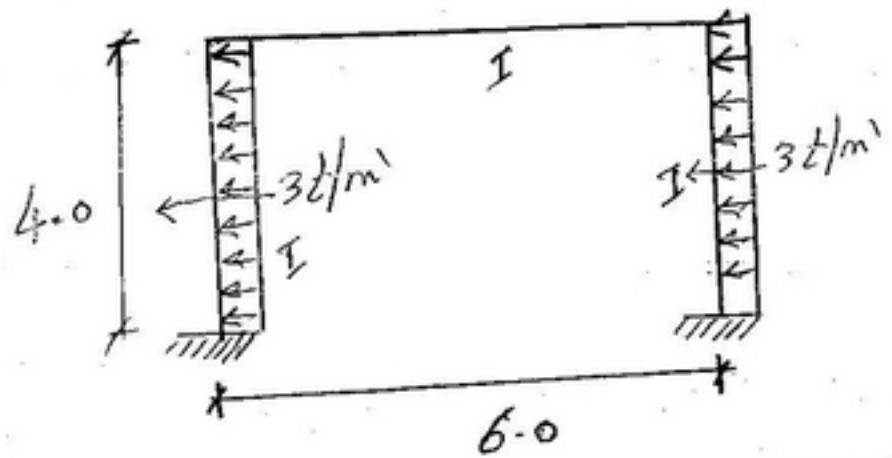
B.M.D



S.F.D

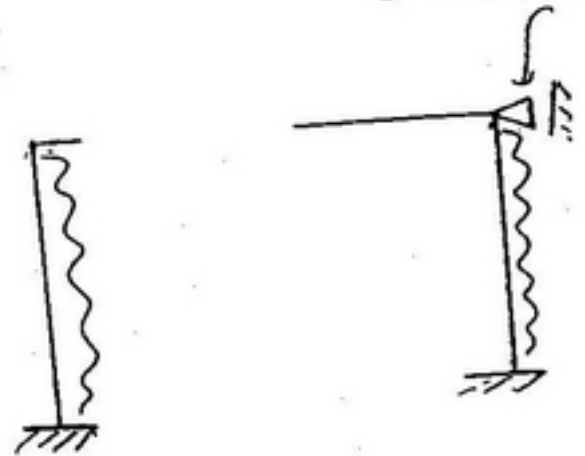
EX:

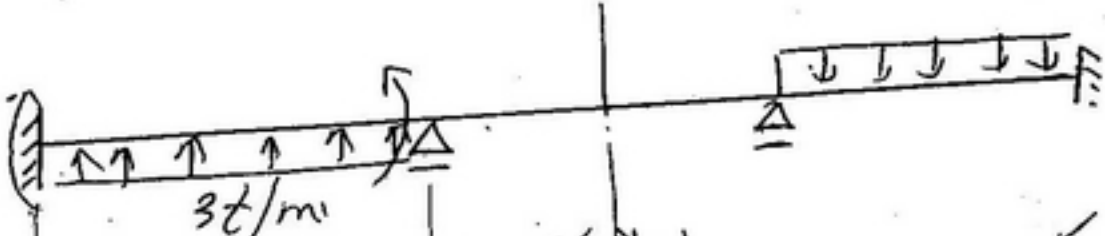
DRAW N-F-D  
SFD  
B-M-D

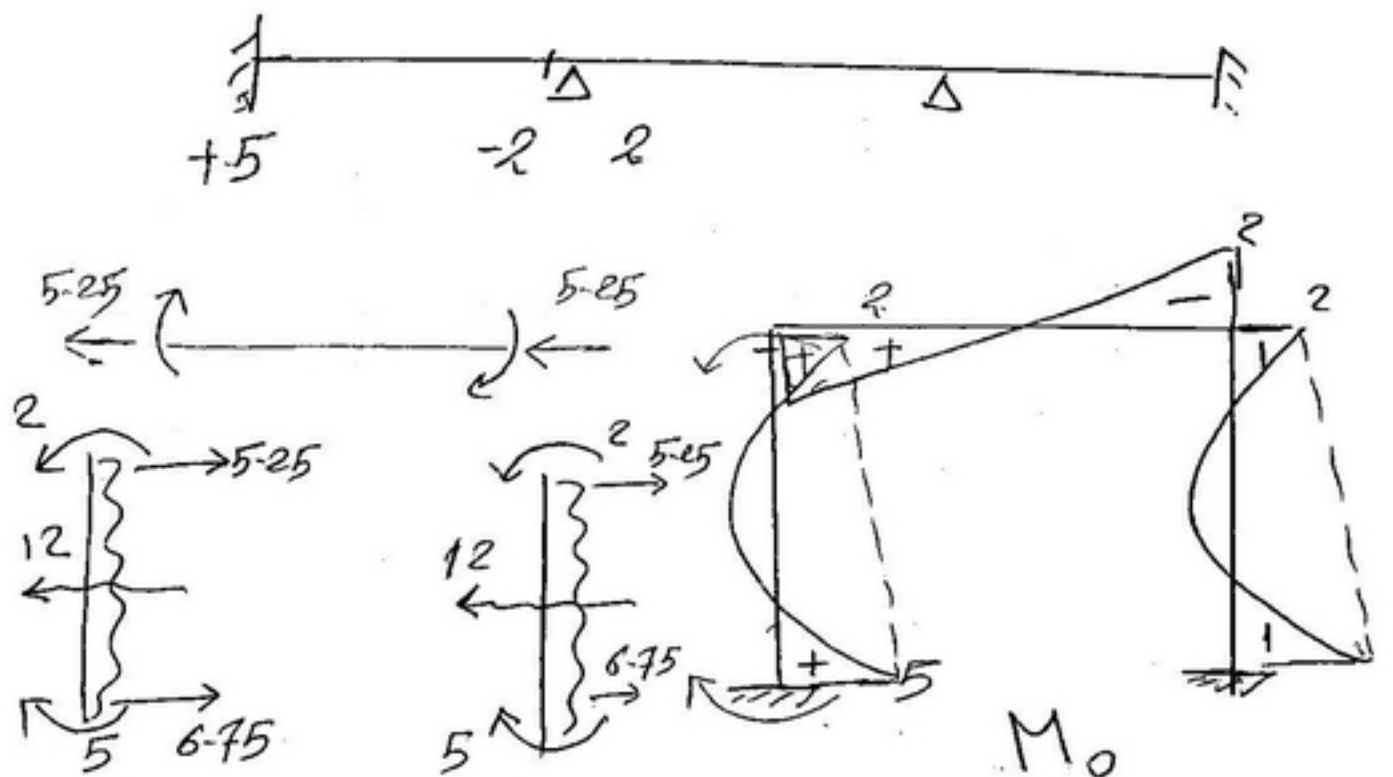


\* solution:

رکیزه‌های



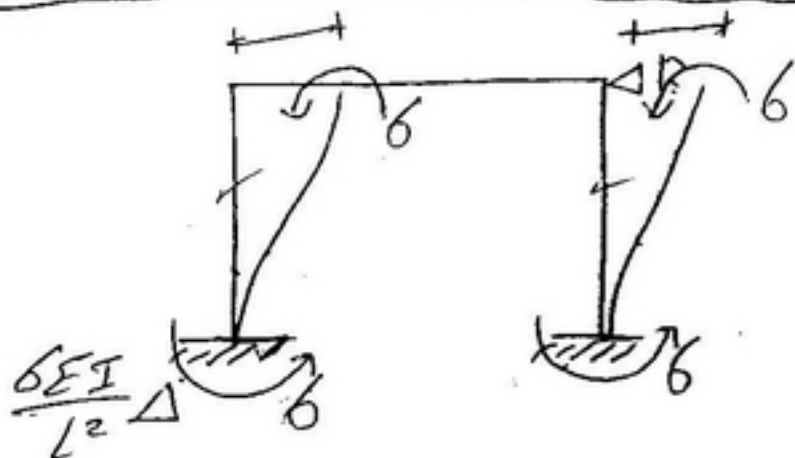
|       |                                                                                      |                                                |
|-------|--------------------------------------------------------------------------------------|------------------------------------------------|
|       |  |                                                |
| R-S   | $\frac{1}{4}$                                                                        | $\frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$ |
| D-F   | 0.5                                                                                  | 0.5                                            |
| F-E-M | +4                                                                                   | -4                                             |
| D-M   | 0                                                                                    | 2                                              |
| C-o-M | 1                                                                                    | 0                                              |
| D-M   | 0                                                                                    | 0                                              |



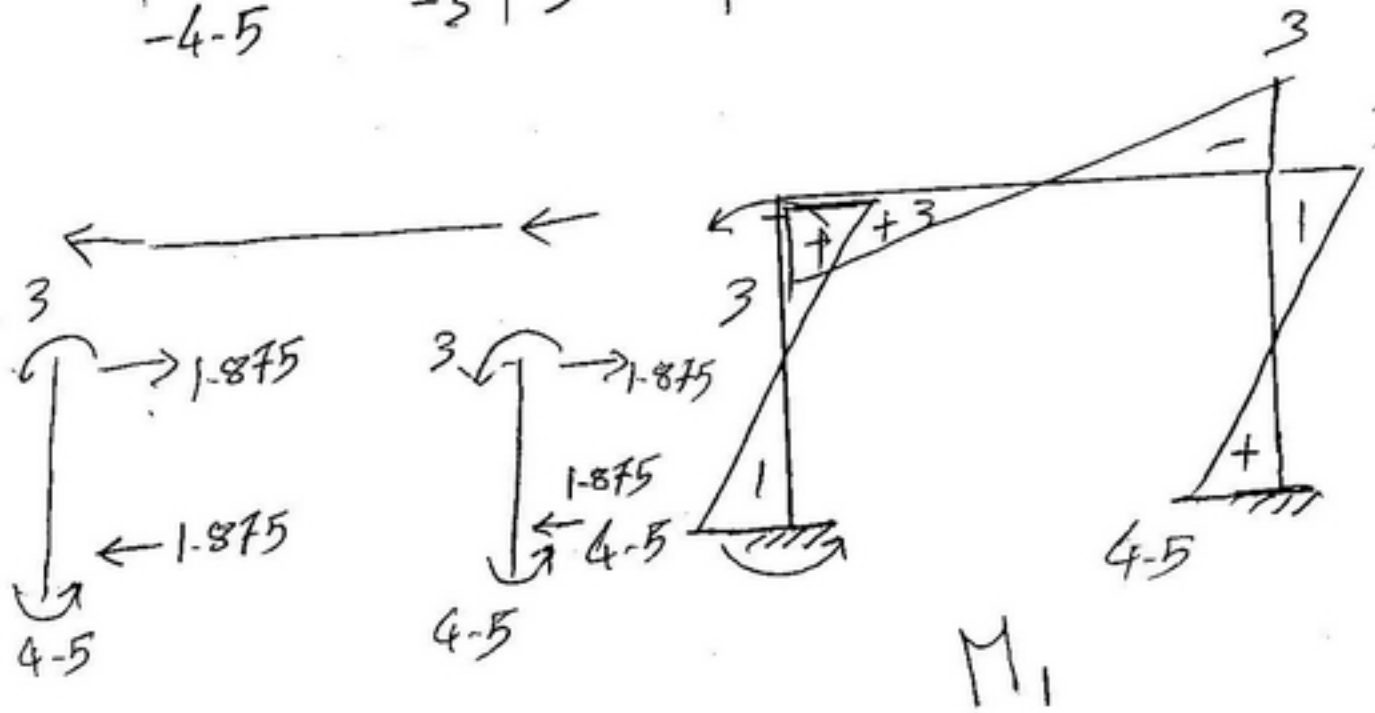
$$R_0 = 10.5 \rightarrow$$

التي تهدف الى ايجاد وحل في الكيفية

$$EI\Delta = 16$$



|      |      |     |     |  |
|------|------|-----|-----|--|
| D-F  | 1    | 0.5 | 0.5 |  |
| FE-M | -6   | -6  | 0   |  |
| D-M  | 0    | 3   | 3   |  |
| Co-M | 1.5  | 0   | 0   |  |
| D-M  | 0    | 0   | 0   |  |
|      | -4.5 | -3  | 3   |  |



$\leftarrow 1.875$ 
 $\leftarrow 1.875$ 
 $\leftarrow 1.875$ 
 $\rightarrow R_1 = 3.75$



$$R_0 + \Delta R_1 = 0$$

$$10.5 + \Delta * 3.75 = 0$$

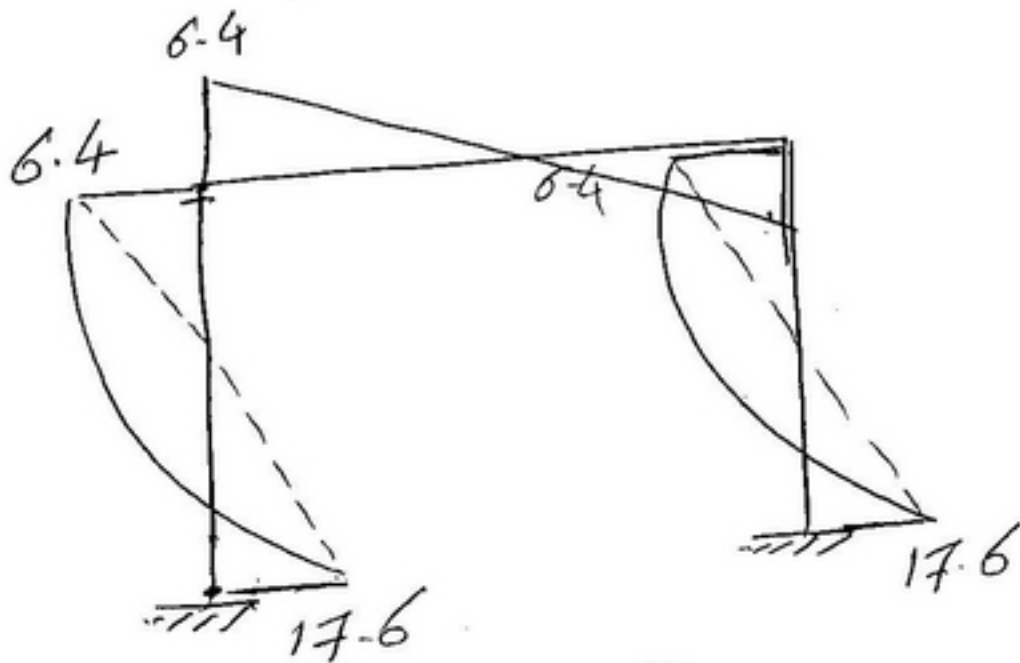
$$\Delta = -2.8$$

$$M_{final} = M_0 + \Delta M_1$$

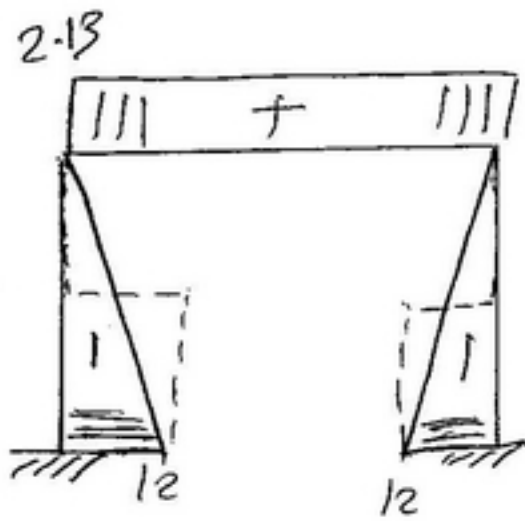
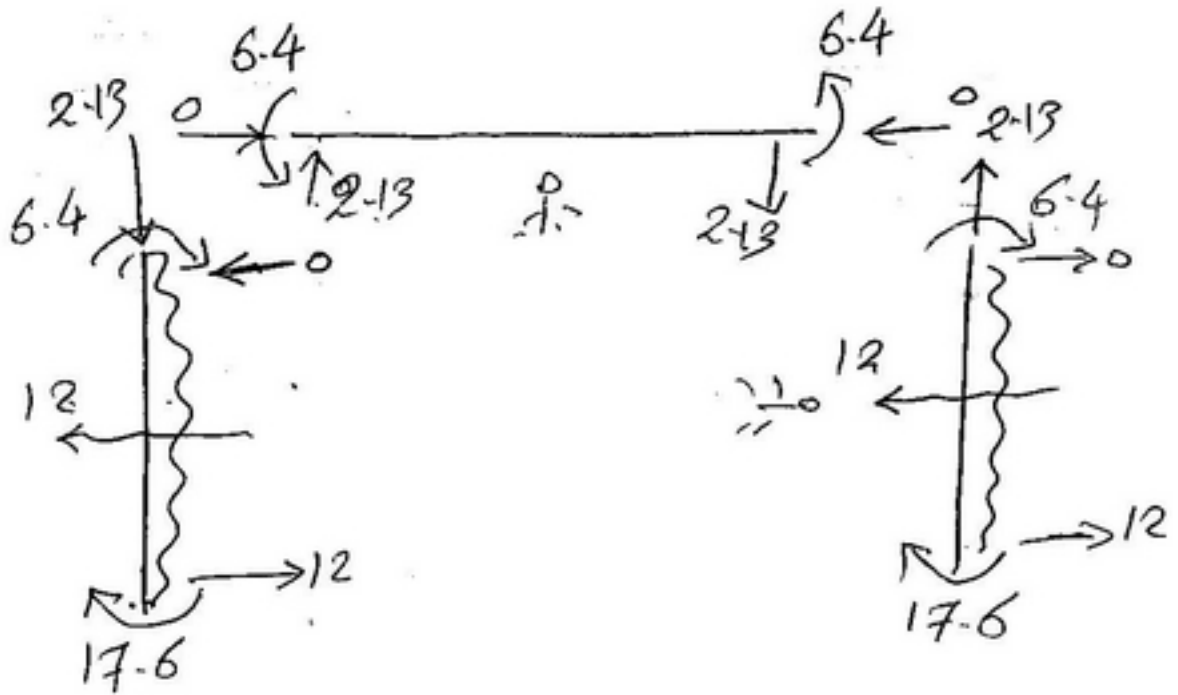
قاعدة  
الإشارة العادية

$$= M_0 - 2.8 M_1$$

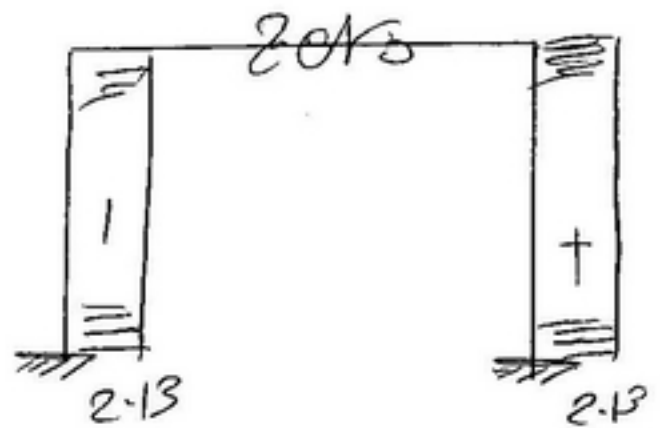
-ve | +ve |



B.M.D

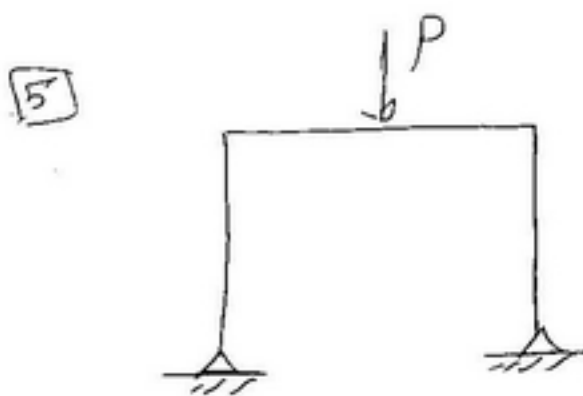
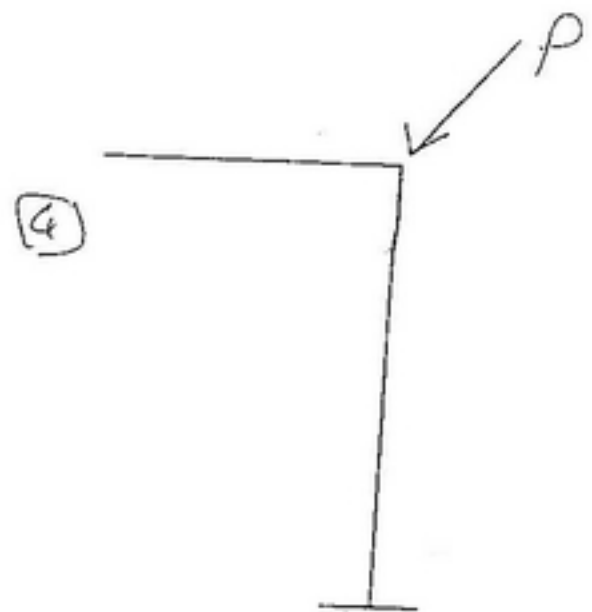
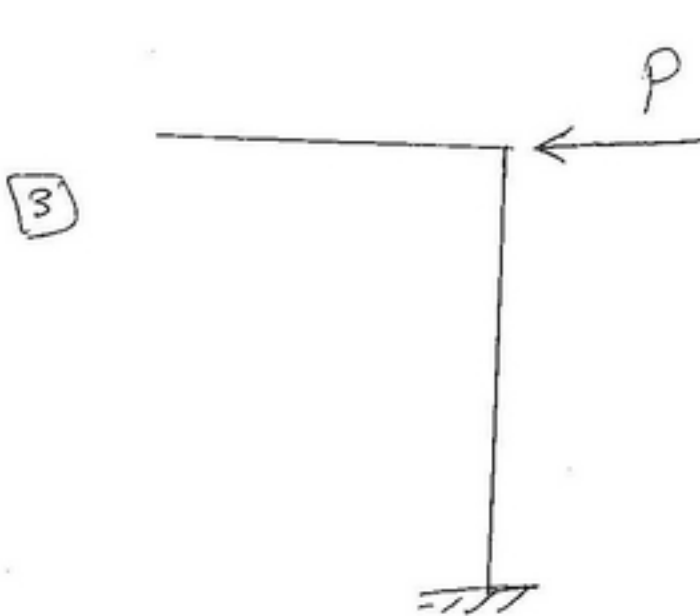
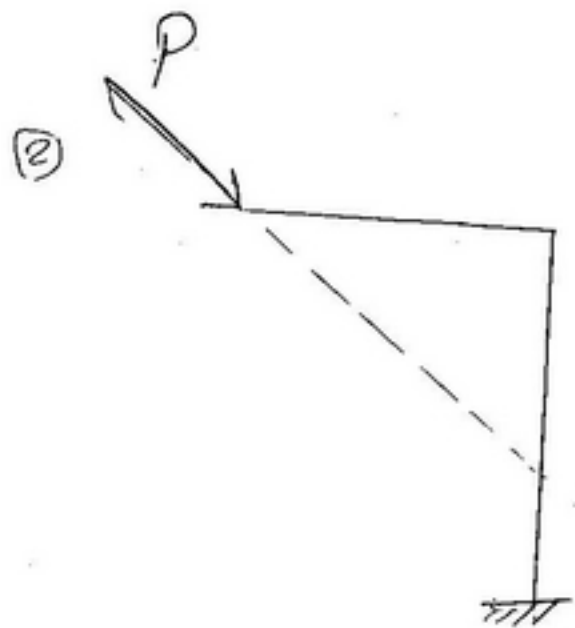
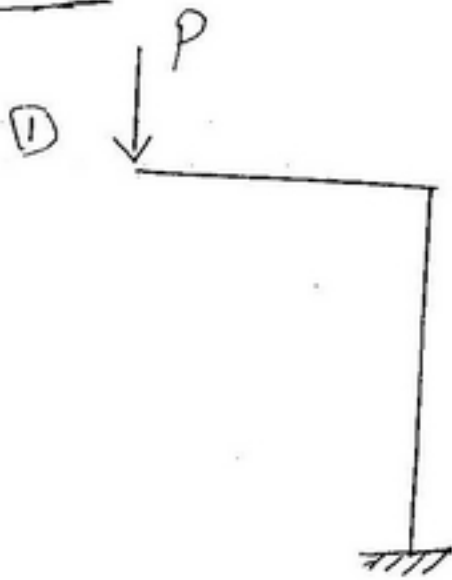


S-F-D



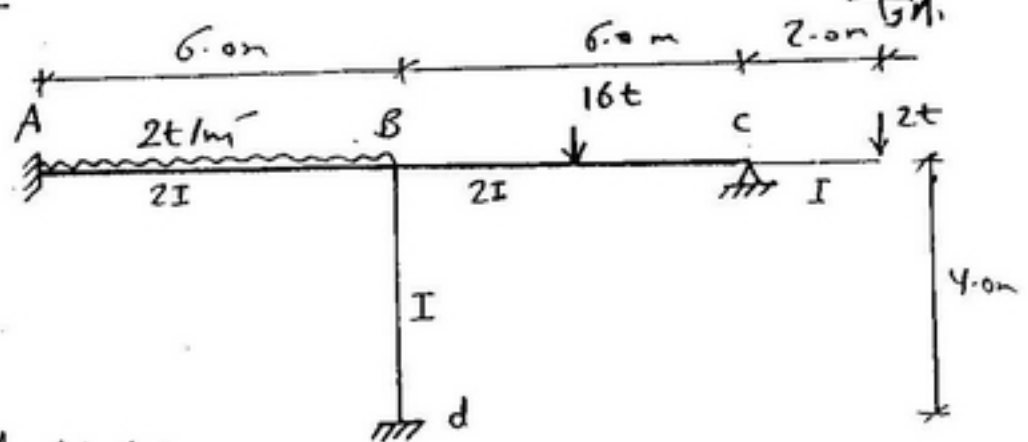
N-F-D

Ex: HW



Midterm 2006

"6"



using moment dist<sup>n</sup>  
draw B.M.D, S.F.D

———— Sol ————

R.s

$\overrightarrow{AB} : \overrightarrow{BC} : \overrightarrow{BD}$

$$\frac{2}{6} : (\frac{3}{4}) \times \frac{2}{6} : \frac{1}{4}$$

$$\frac{1}{3} : \frac{1}{4} : \frac{1}{4}$$

|   |   |   |   |   |
|---|---|---|---|---|
| 4 | : | 3 | : | 3 |
|---|---|---|---|---|

\* 12

D.F

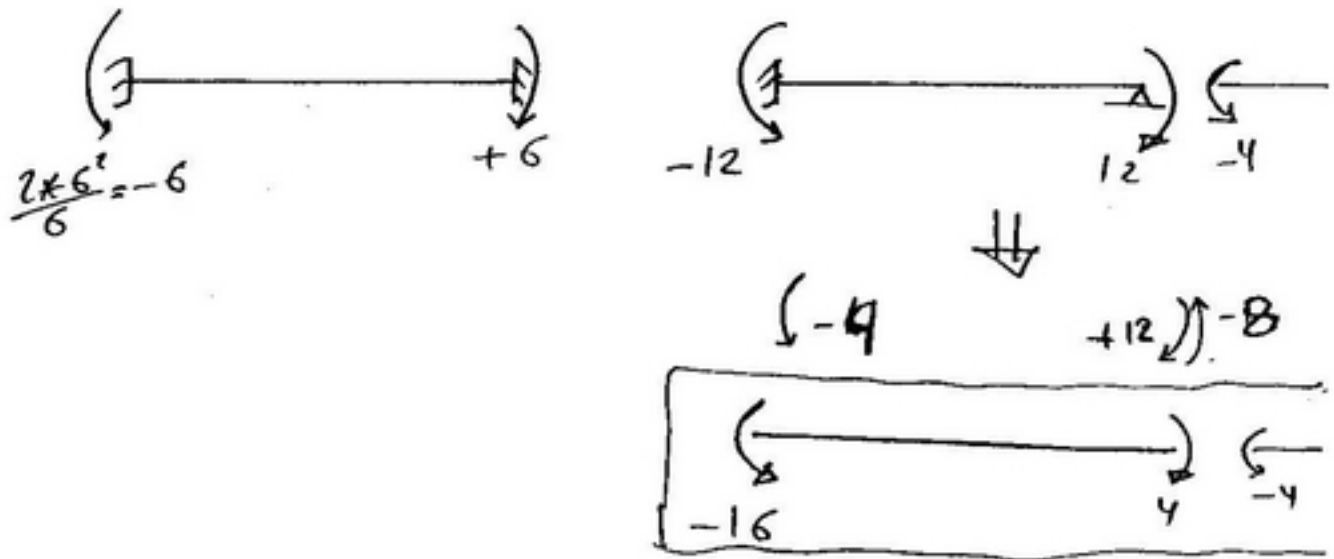
$$\frac{4}{3+3+4} : \frac{3}{10} : \frac{3}{10}$$

$$0.4 : 0.3 : 0.3$$

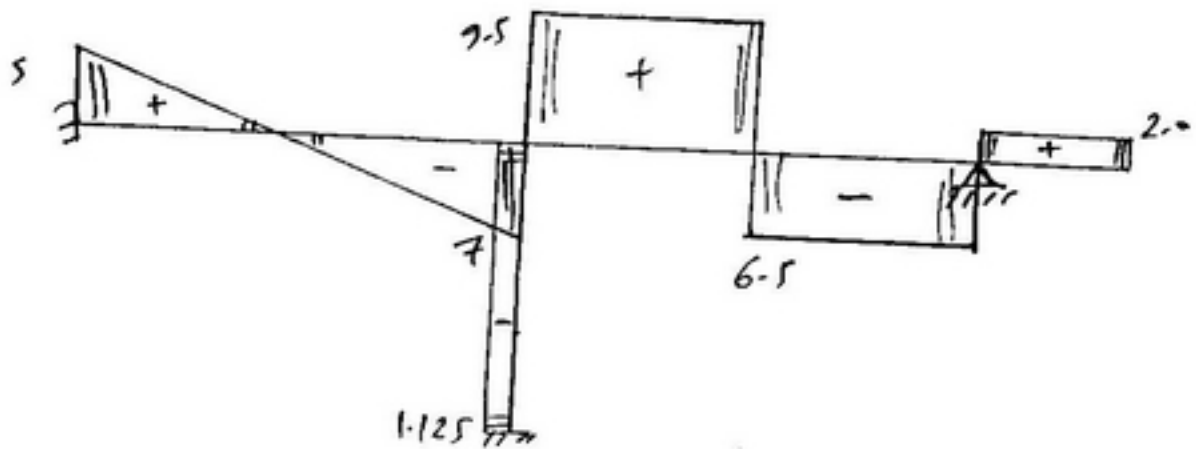
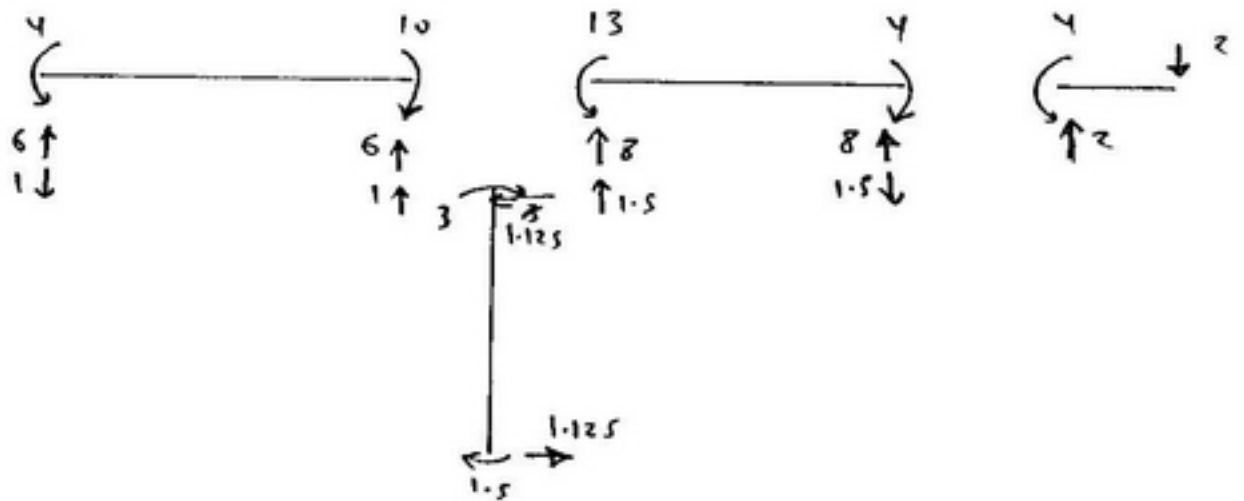
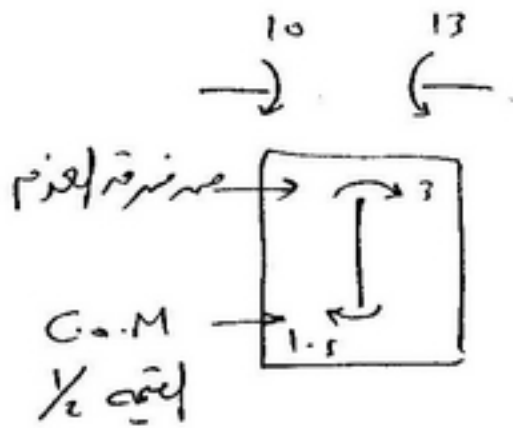
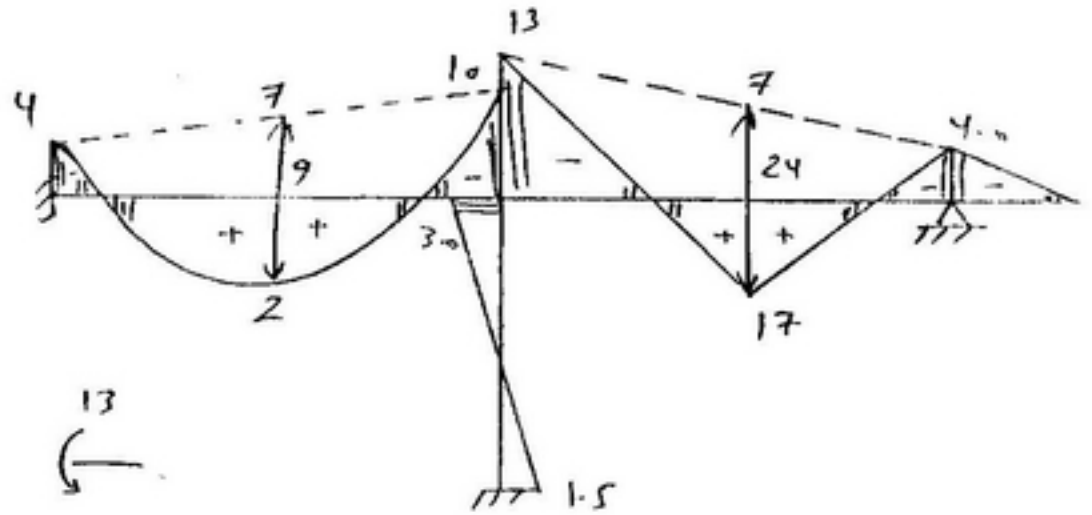
1

موزون العزم على (A) و (B)

## Fixed End moment

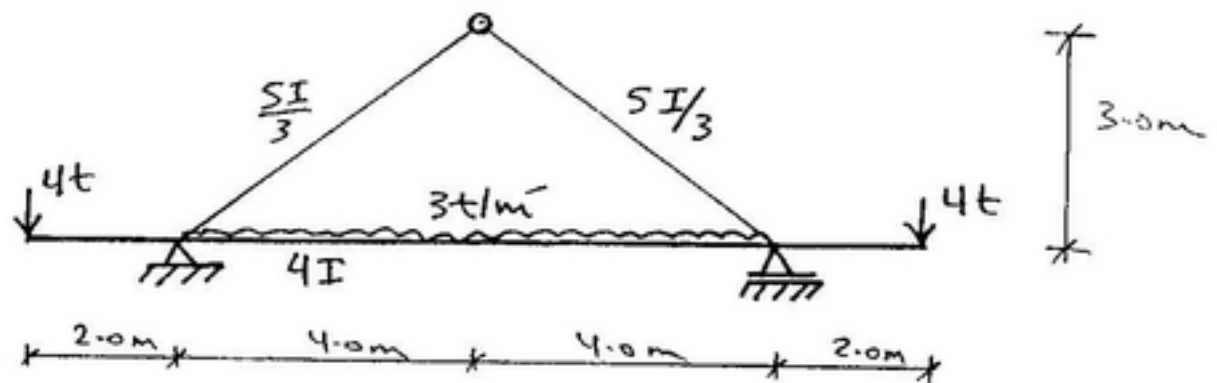


|       |    | 0.4 | 0.3 |    |
|-------|----|-----|-----|----|
| F.E.M | -6 | +6  | -16 | +4 |
| D.M   | 0  | +4  | +3  | 0  |
| C.O.M | 2  | 0   | 0   | 0  |
| D.M   | 0  | 0   | 0   | 0  |
| F.M   | -4 | 10  | -13 | 4  |



3 S.F.D



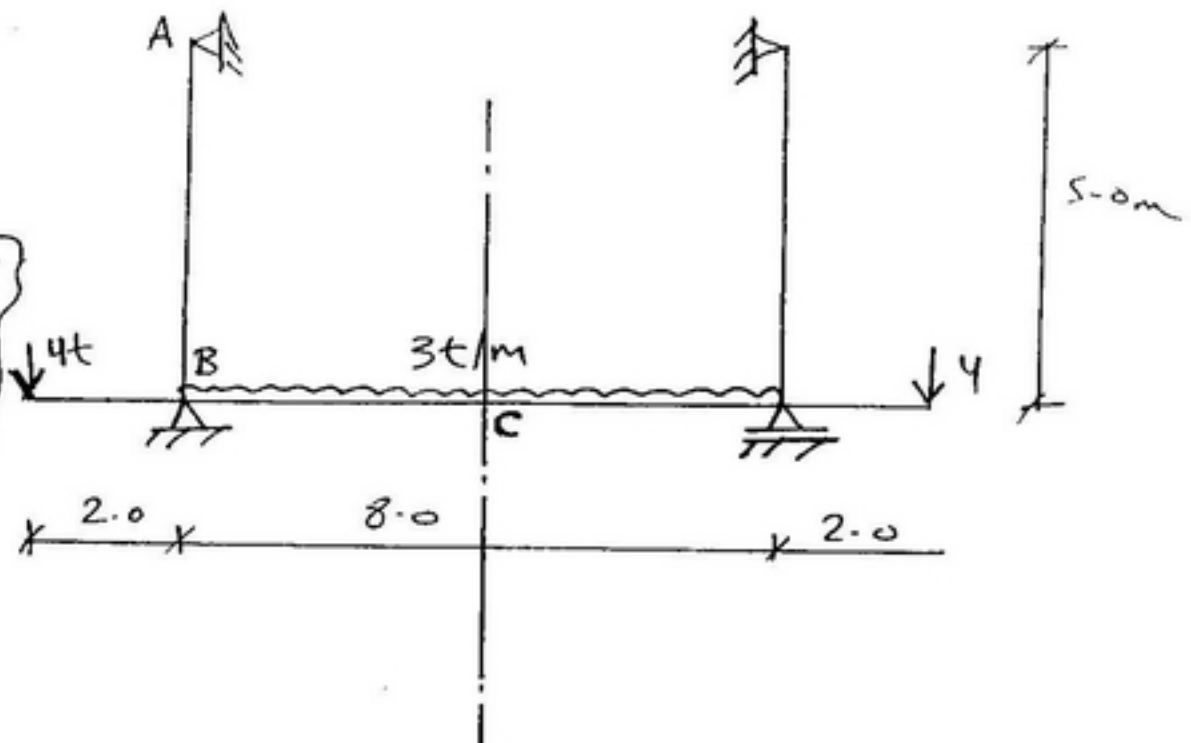


For the following shape  
draw B.M.D, S.F.D and P.N.f.D

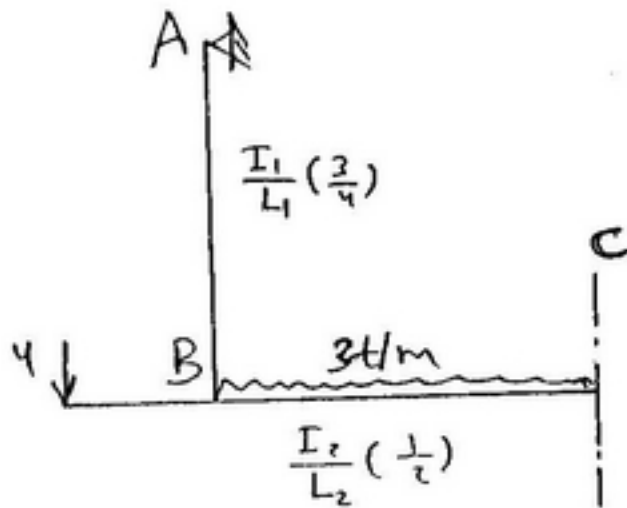
— Sol —

على شكل  
الهيكل

منطقة  
Intermed  
في  
Fixed



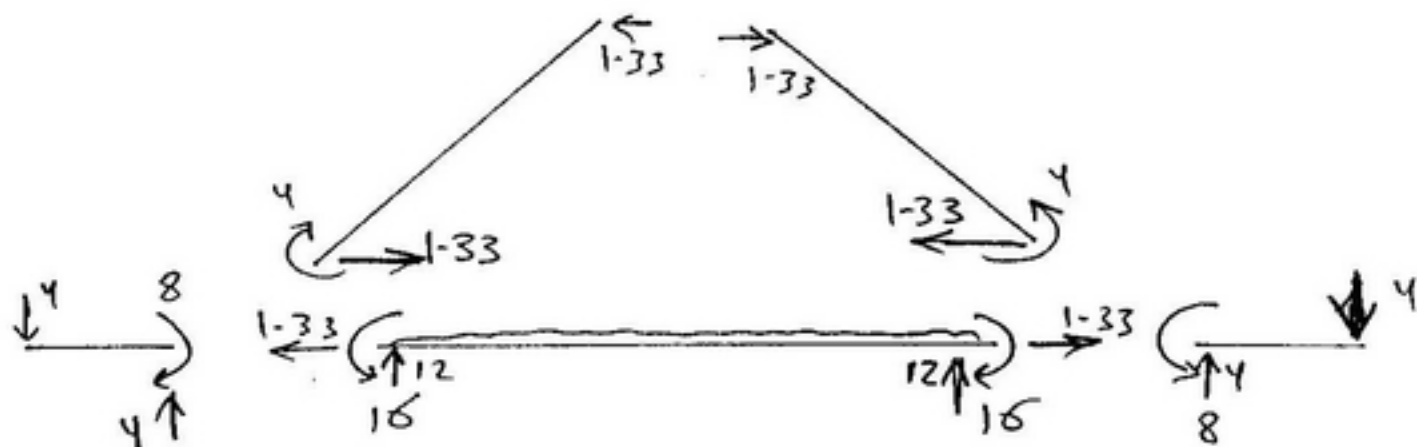
خط  
4



$$\left( \frac{5/3}{5} \times \frac{3}{4} \right) = \frac{1}{4}$$

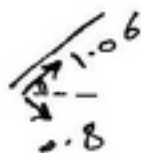
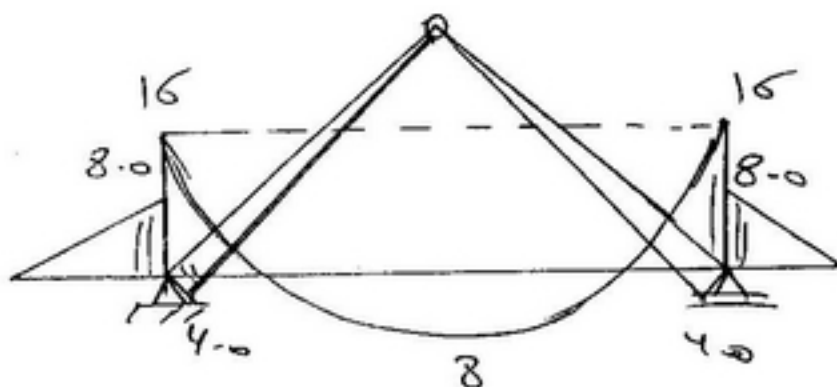
$$+8 \quad -16 \quad \frac{1}{2} \times \frac{4}{8} = \frac{1}{4}$$

|       | A                 | B                                         | C   |
|-------|-------------------|-------------------------------------------|-----|
| R-5   | $\frac{1}{4} = 1$ | $\frac{1}{4} = 1$                         |     |
| D-f   |                   | $\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$ |     |
| F.E.M | 0                 | 0                                         | -16 |
| D.M   | 0                 | +4                                        | +4  |
| C.O.M | 0                 | 0                                         | 0   |
| D.M   | 0                 | 0                                         | 0   |
| P.M   | 0                 | +4                                        | -12 |

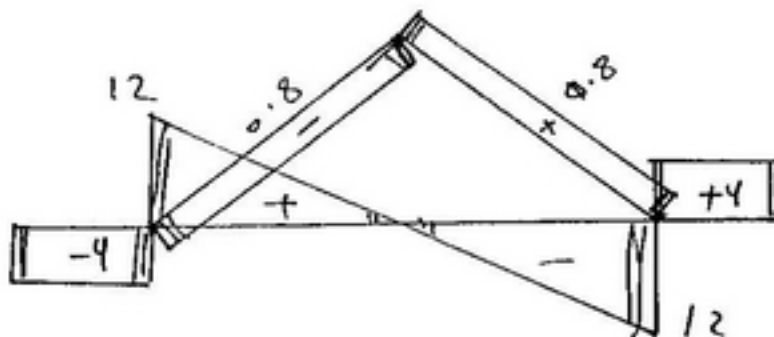


Support II Reaction  $\leftarrow 8 + 16 \rightarrow$

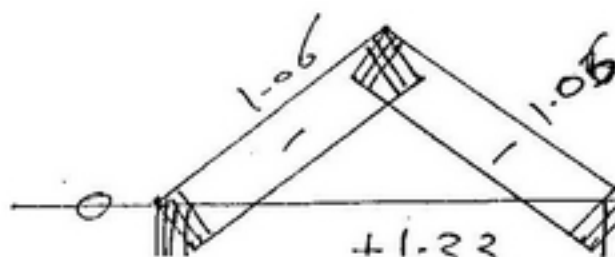
B.M.D



S-f.D



N.f.D

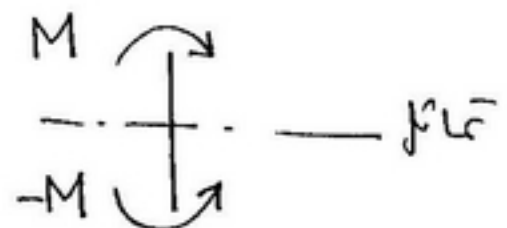
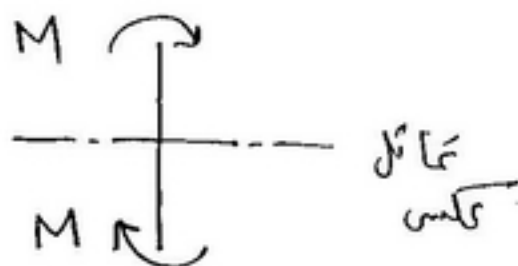
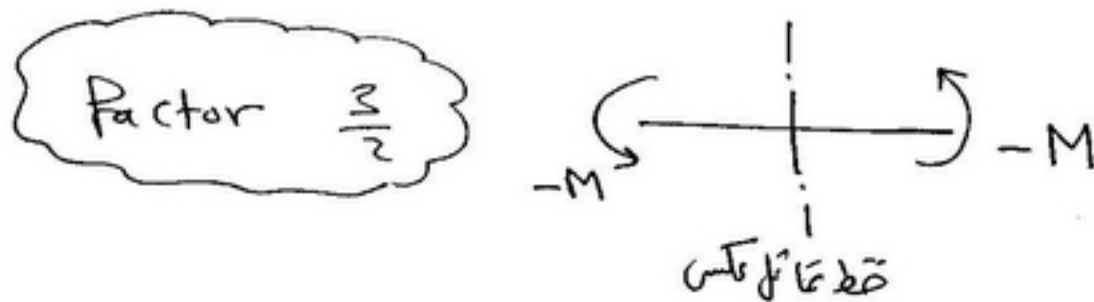


## قالب مغلق

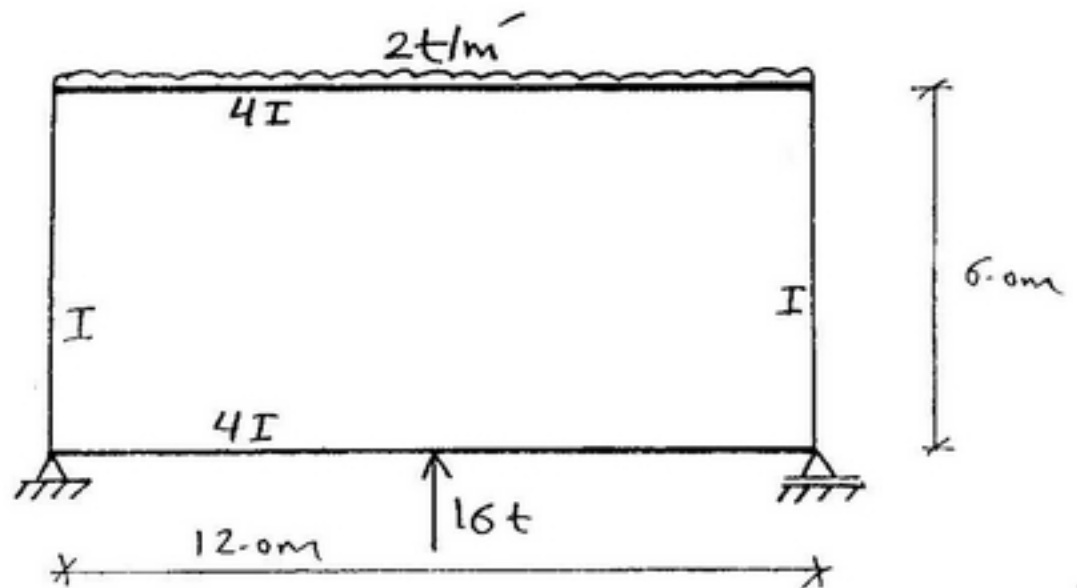
\* إذا وجدنا قيمة معينة، وحال  
نفسية إيجاب، ونعكس الإشارة،  
يكون خط الرأس (خط عمودي)  
خط عمودي



\* إذا كانت نفس الإشارة، نفس القيمة يكون خط  
الرأس خط (عمودي)

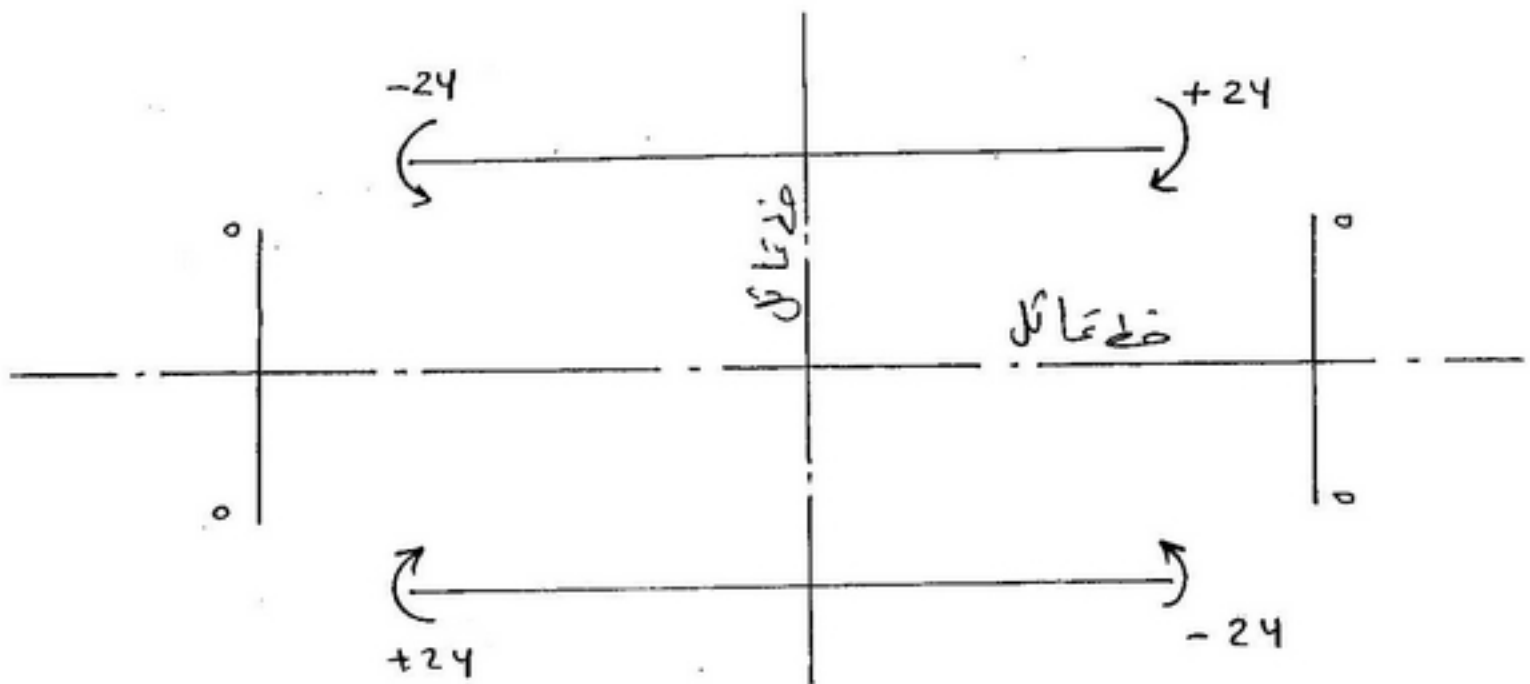


نفس الكلام



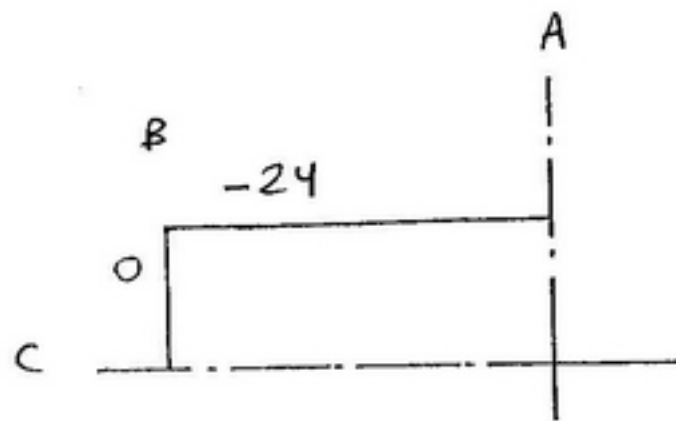
draw N.F.D, S.F.D, B.M.D

— 502 —

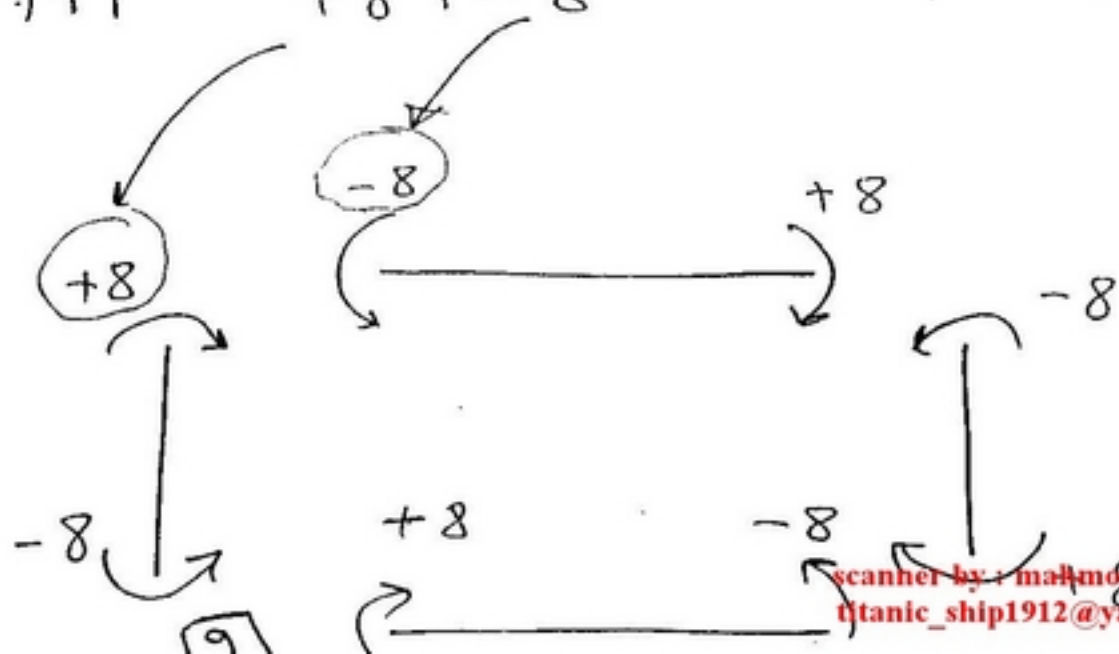


خط عمائل  
فوقه -M  
تحت +M  
خط عمائل

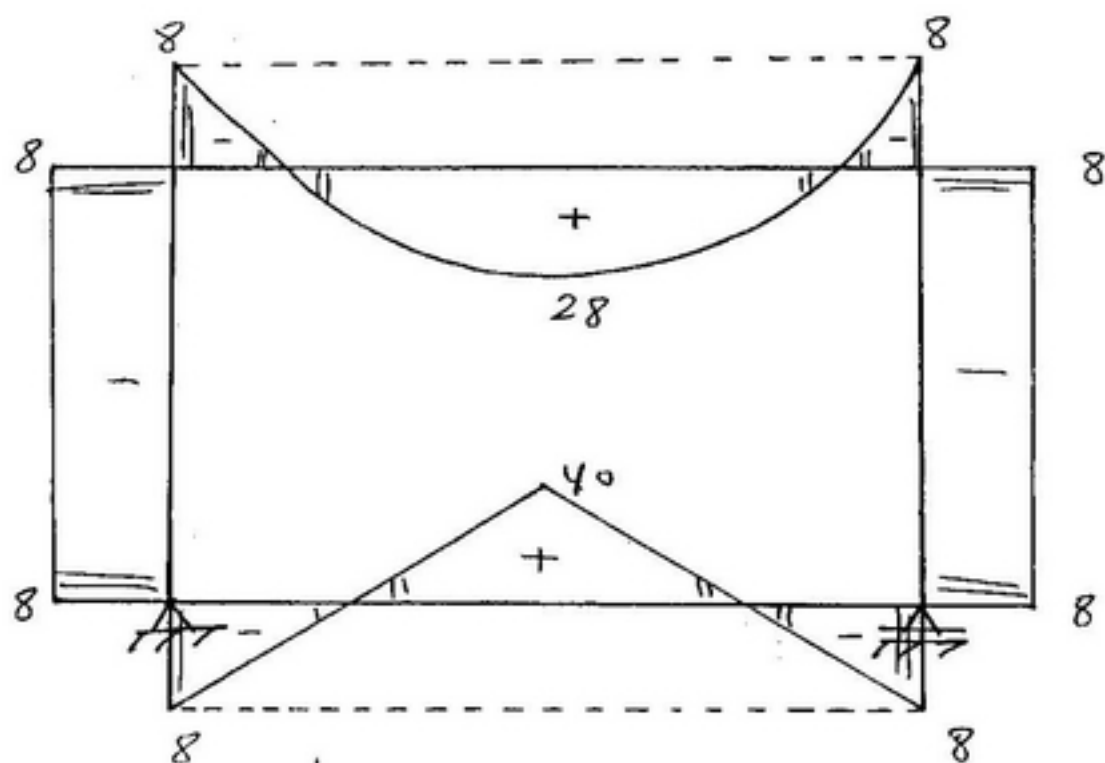
خط عمائل  
شمال -M  
جنوب +M  
scanner by mahmoud ashraf  
titanic\_ship12@yahoo.com



|       | C                                                           | B                                                            | A |
|-------|-------------------------------------------------------------|--------------------------------------------------------------|---|
| R.s   | $\left(\frac{1}{2}\right) \times \frac{1}{6} = \frac{1}{6}$ | $\left(\frac{1}{2}\right) \times \frac{4}{12} = \frac{1}{3}$ |   |
| D.f   | $\frac{1}{3}$                                               | $\frac{2}{3}$                                                |   |
| F.E.M | 0                                                           | -24                                                          |   |
| D.M   | +8                                                          | +16                                                          |   |
| C.o.M | +0                                                          | 0                                                            |   |
| D.M   | 0                                                           | 0                                                            |   |
| F.M   | +8                                                          | -8                                                           |   |







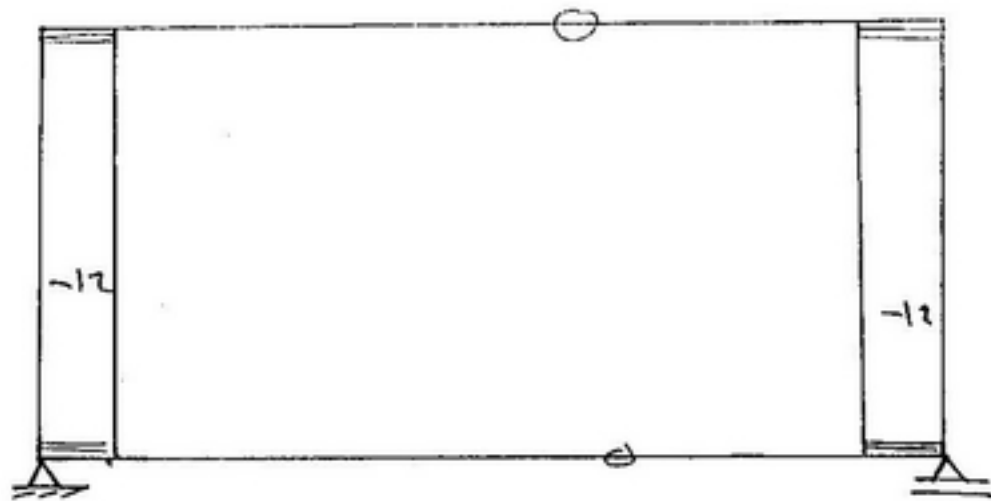
①  
 خازن لسته تم  
 تويل 12 امية  
 كصفة لل  
 مباشرة  
 12



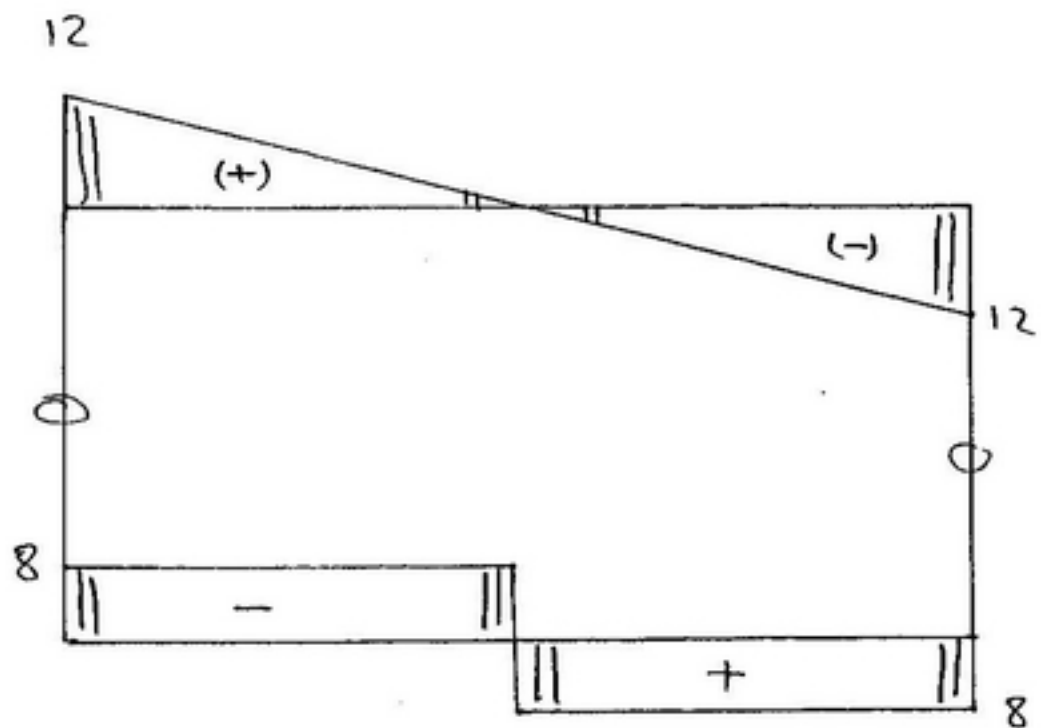
②  
 لا تراهم  
 12



③  
 افنة سب  
 لقنة سب  
 support



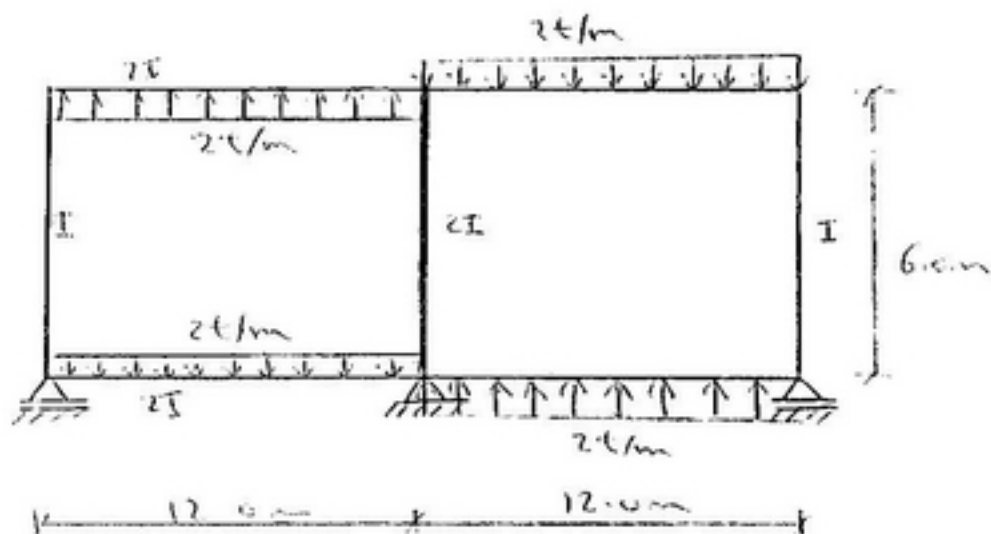
N.F.D



S.F.D



final 2004, 08

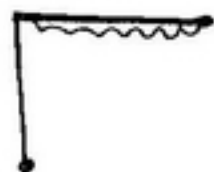
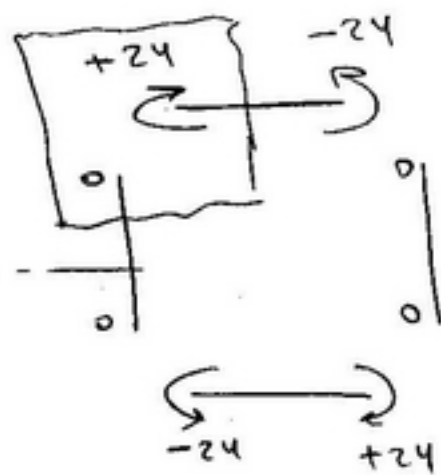
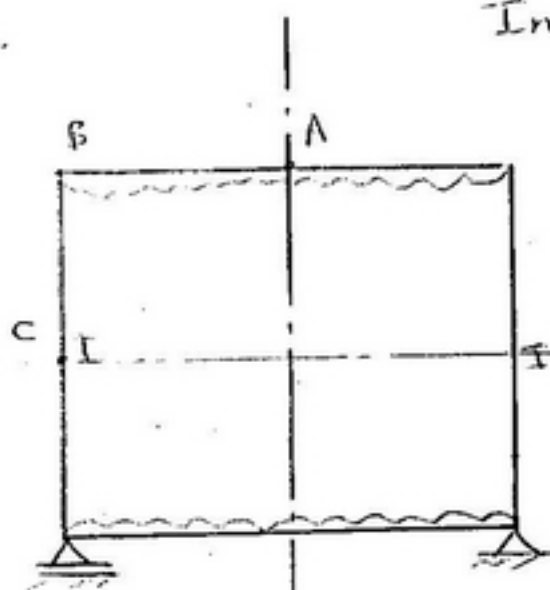


— Sol —

هذا كذا ما تم عكس به الـ 2t في جميع اركان المبنى ، مع ان هذا المبنى هو

Inertia . ١٥٠٠

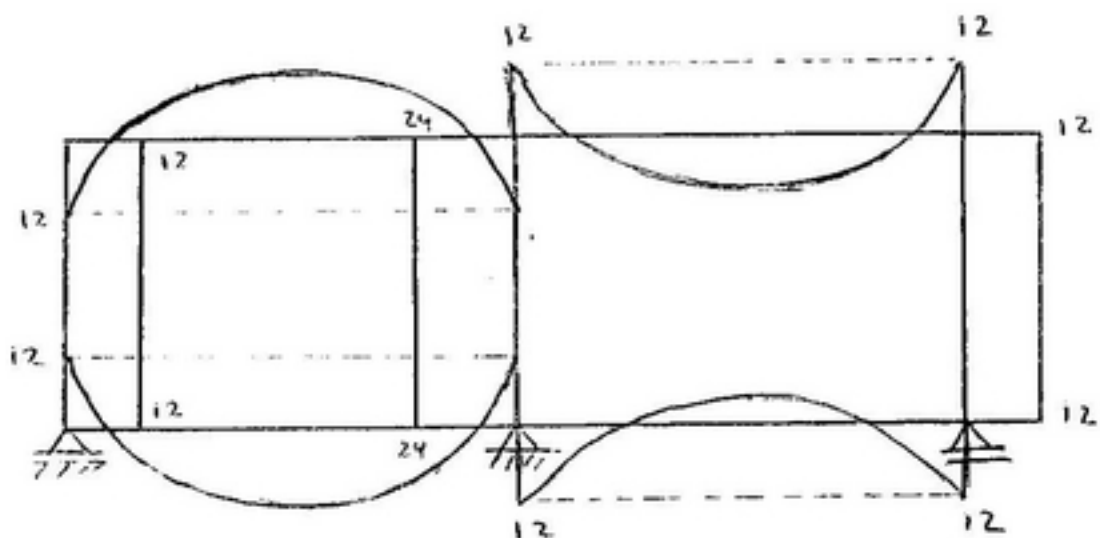
١٥٠٠



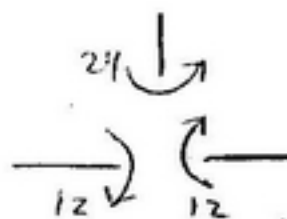
١١٢

مع هذا المبنى نقطة على

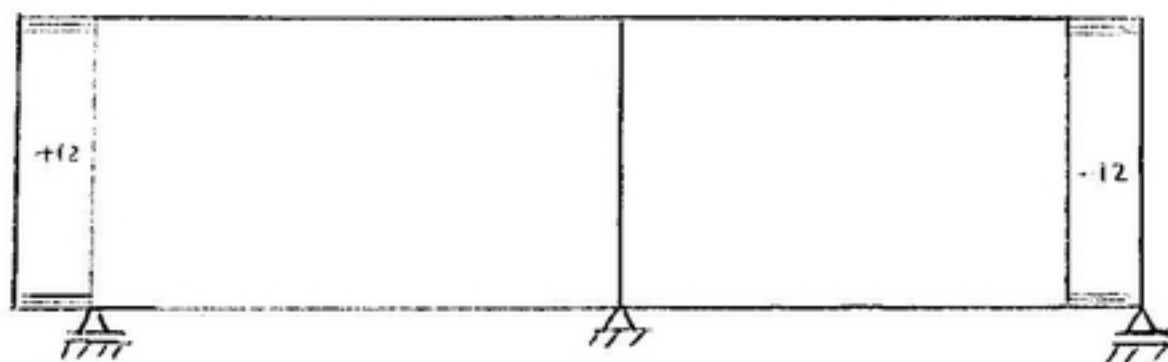
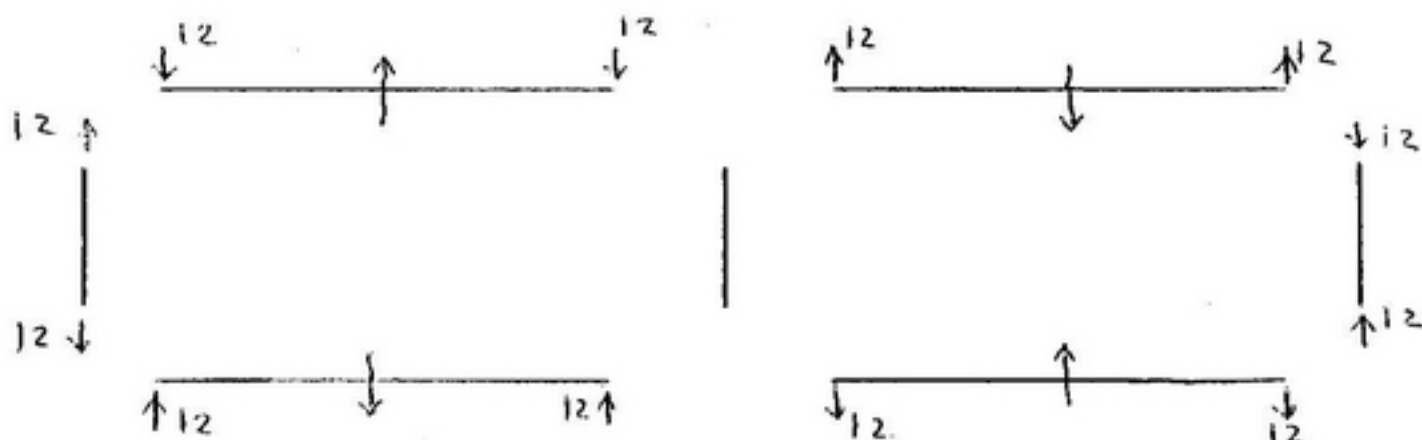
|       |                                      |     |                                       |
|-------|--------------------------------------|-----|---------------------------------------|
|       | C                                    | B   | A                                     |
| R.S   | $\frac{1}{2} \times \frac{1}{6} = 1$ |     | $\frac{1}{2} \times \frac{2}{12} = 1$ |
| D.F   | 0.5                                  |     | 0.5                                   |
| F.C.M | 0                                    | +24 |                                       |
| DM    | -12                                  | -12 |                                       |
| F.M   | -12                                  | 12  |                                       |



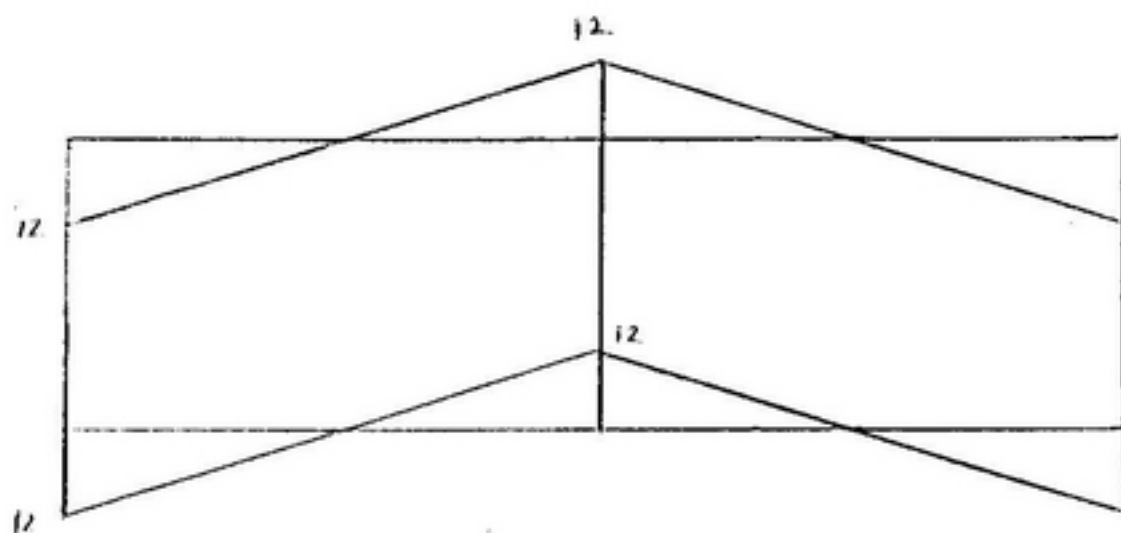
B.M.D



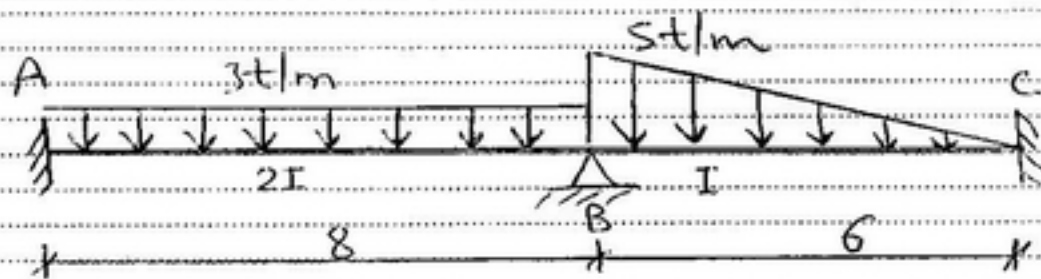
13



N.F.D



S.F.D

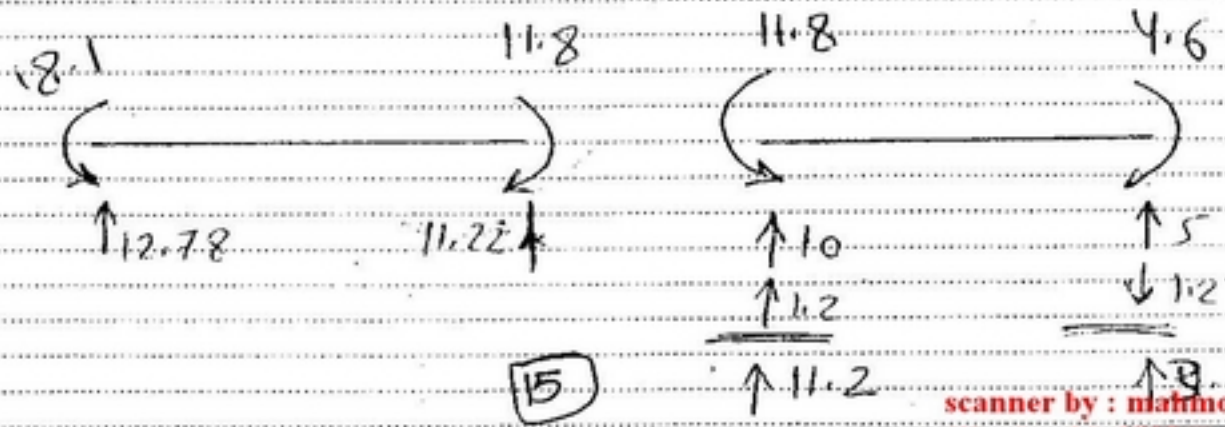


Solution

$$\frac{2}{8} = \frac{1}{4} = 3$$

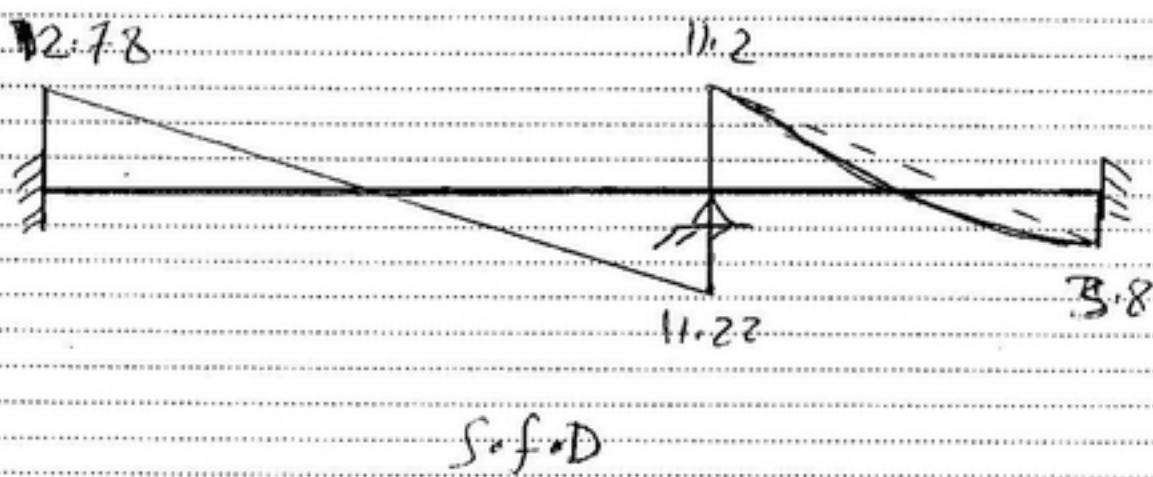
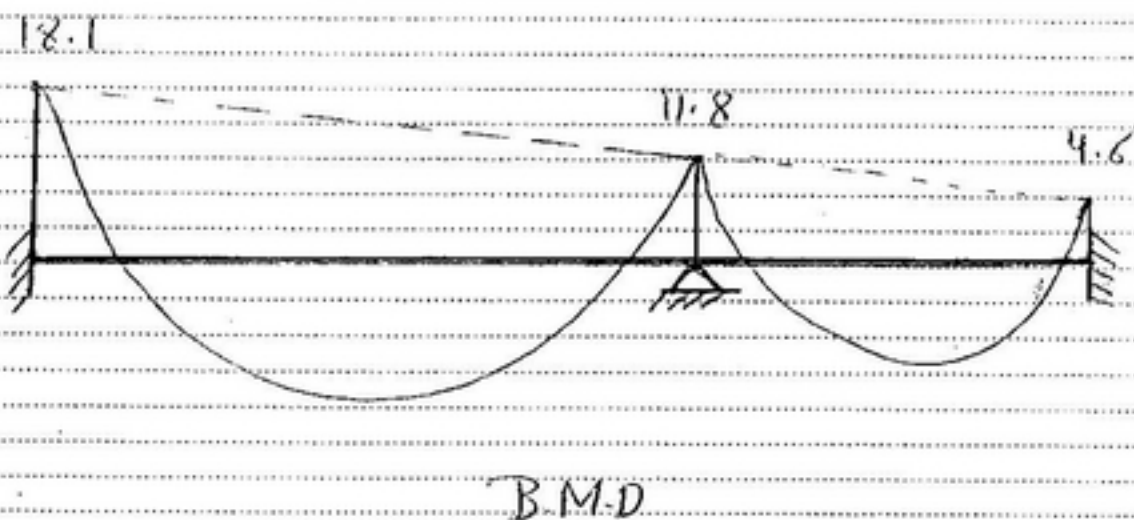
$$\frac{1}{6} = 2$$

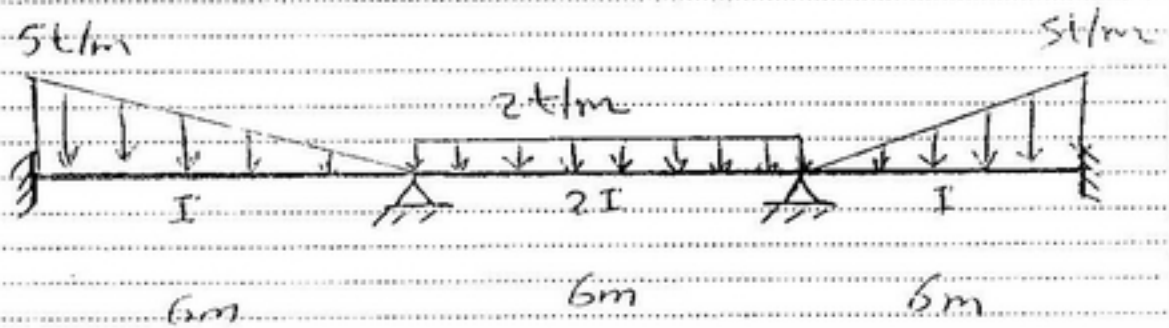
|          |       |      |       |      |
|----------|-------|------|-------|------|
|          |       | 0.6  | 0.4   |      |
| F.E.M    | -16   | +16  | -9    | +6   |
| D.M      | 0     | -4.2 | -2.8  | 0    |
| Cross M  | -2.1  | 0    | 0     | -1.4 |
| D.M      | 0     | 0    | 0     | 0    |
| $\sum M$ | -18.1 | 11.8 | -11.8 | 4.6  |



(15)







FEM



در این صورت که در این صورت

از FEM → استفاده می‌کنیم

**Problem No. (5) [ 12 Degrees] (12 minute)**

For the plate shown in Fig. (5), draw the bending moment diagrams ( $M_r$ ,  $M_\theta$ ), where  $\mu=0$

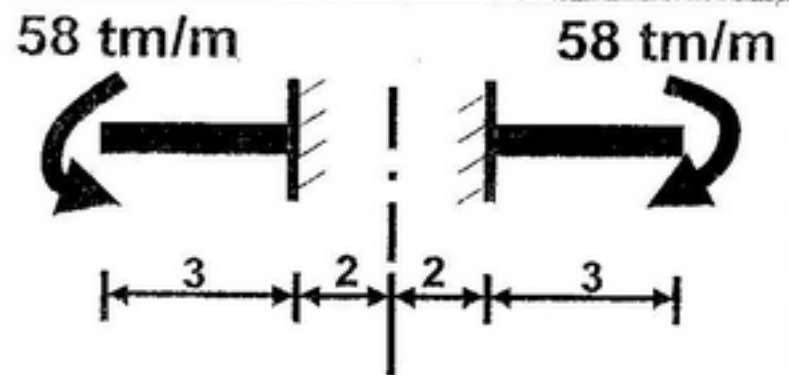


Fig. (5)

**Problem No. (6) [ 8 Degrees] (8 minute)**

Calculate and draw the internal forces diagrams for the concrete conical shell shown in Fig. (6) due to own weight where the expected thickness is 8 cm and the cover weight is  $100 \text{ kg/m}^2$

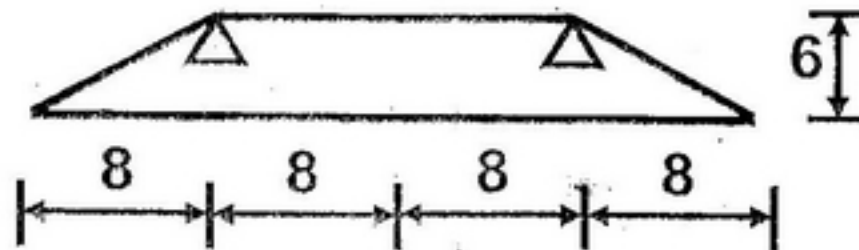


Fig. (6)

**Problem No. (7) [ 6 Degrees] (1 minute)**

Draw the B.M.D for the frame shown in Fig. (7)

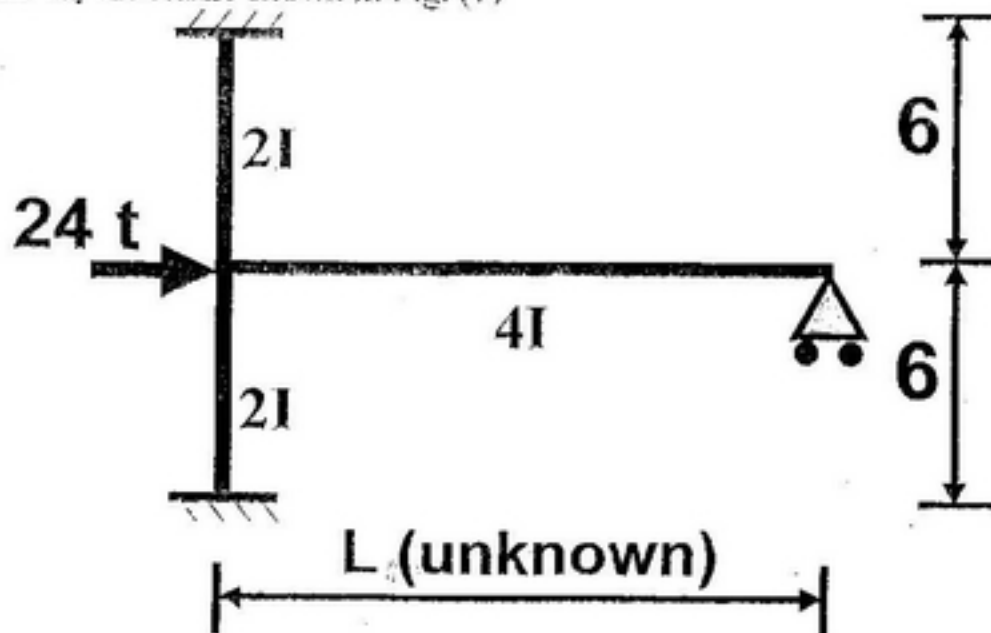


Fig. (7)

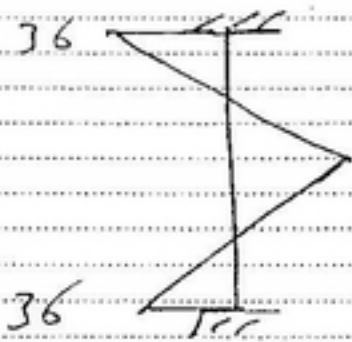
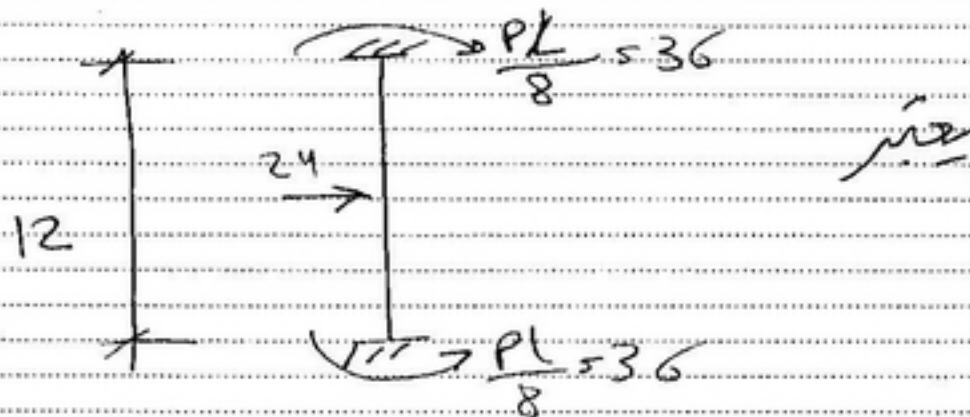
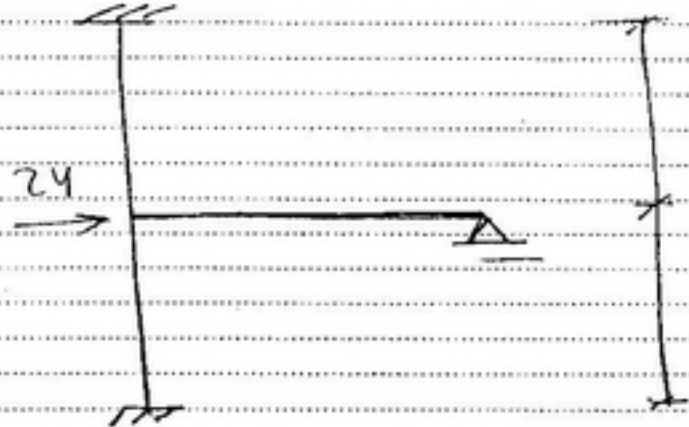
With best wishes

Assist Prof. Atef Eraky

Assist Prof. Eman El-Shamy

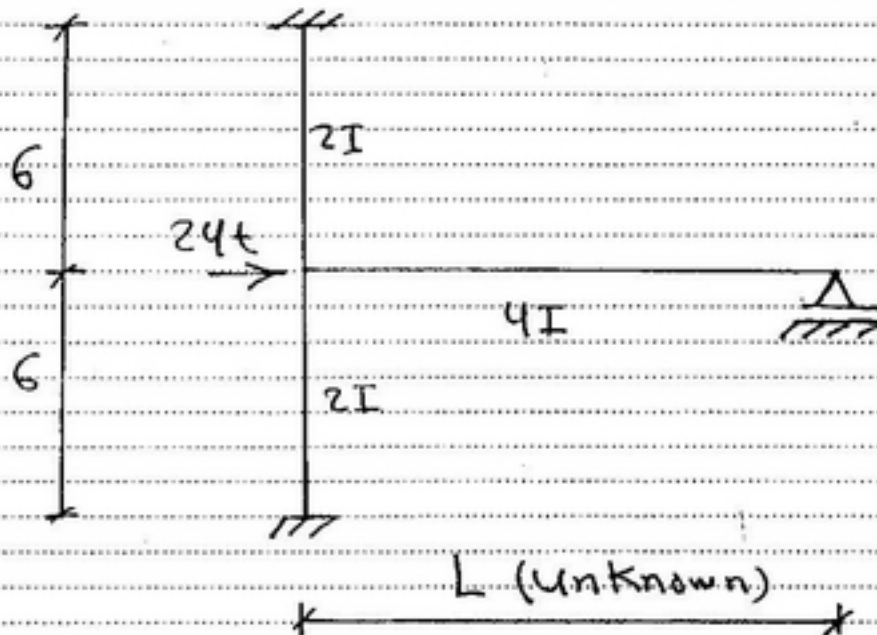


Prob(7) 2009



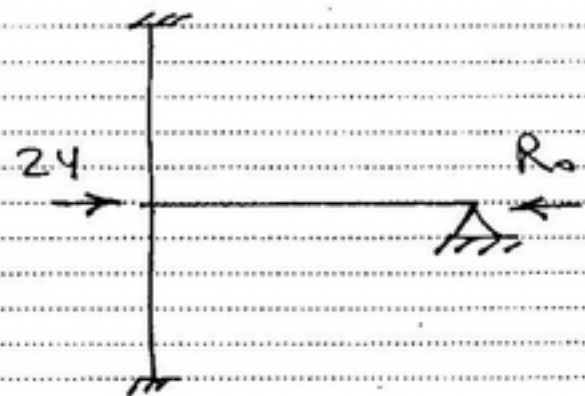
حل آفر

prob (7) 2009



Solution

(i) without



$$M_0 = \text{Zero}$$

$$R_0 = 24$$

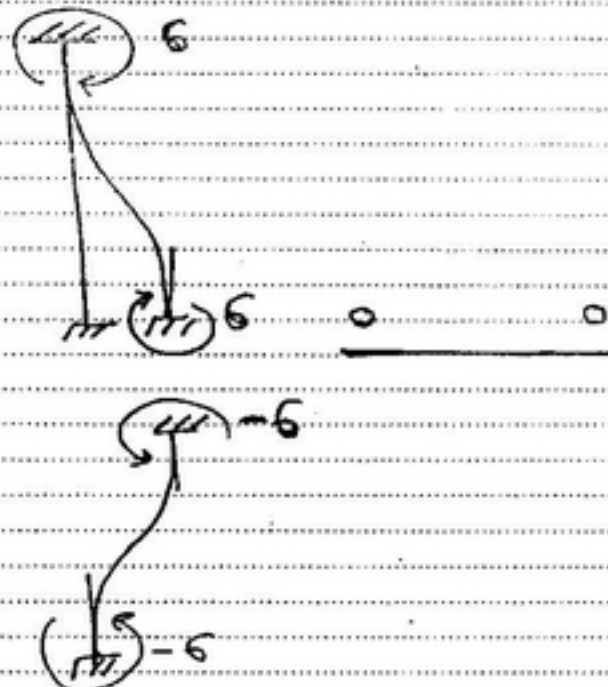


(ii) with sway

$$M_1 = \frac{6EI}{L^2}$$

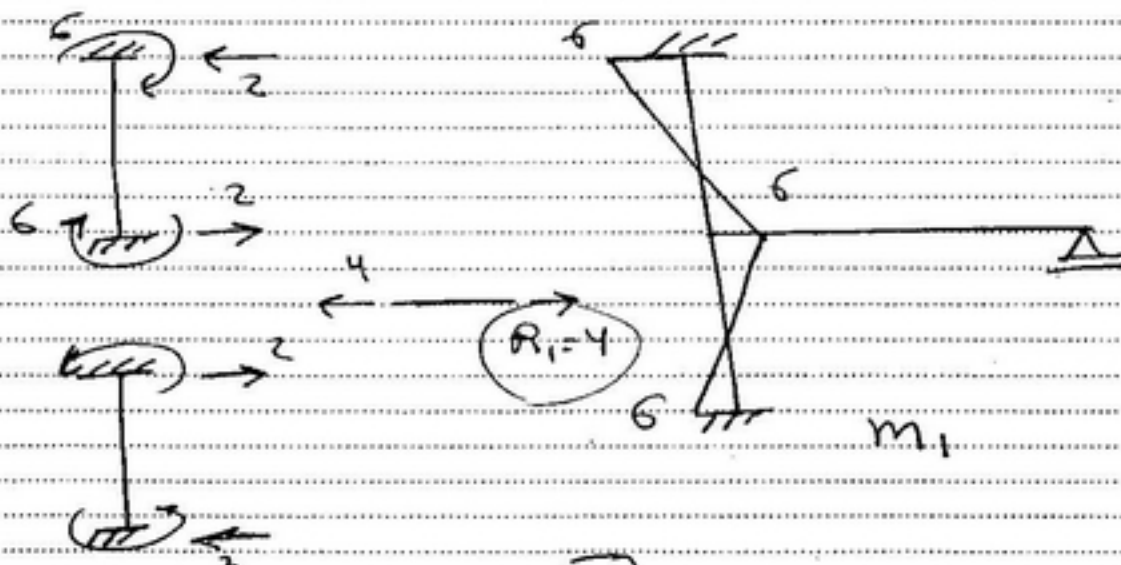
assume  $EI = 36$

∴  $M_1 = 6$



Zero =  $(6+)(-6)$  रजत डी ४

f.e.m. जलन डी ४



(21)



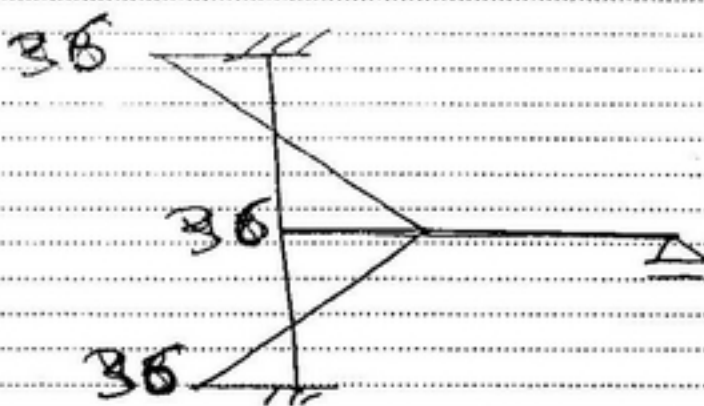
$$R_0 + \Delta R_1 = 0 \dots$$

$$24 + \Delta \times (-4) = 0 \dots$$

$$\Delta = 6$$

$$\therefore m_f = m_0 + 6 m_1$$

$$m_f = 6 m_1$$



~~Final Revision~~  
**3rd civil year**

"14"  
P.K.

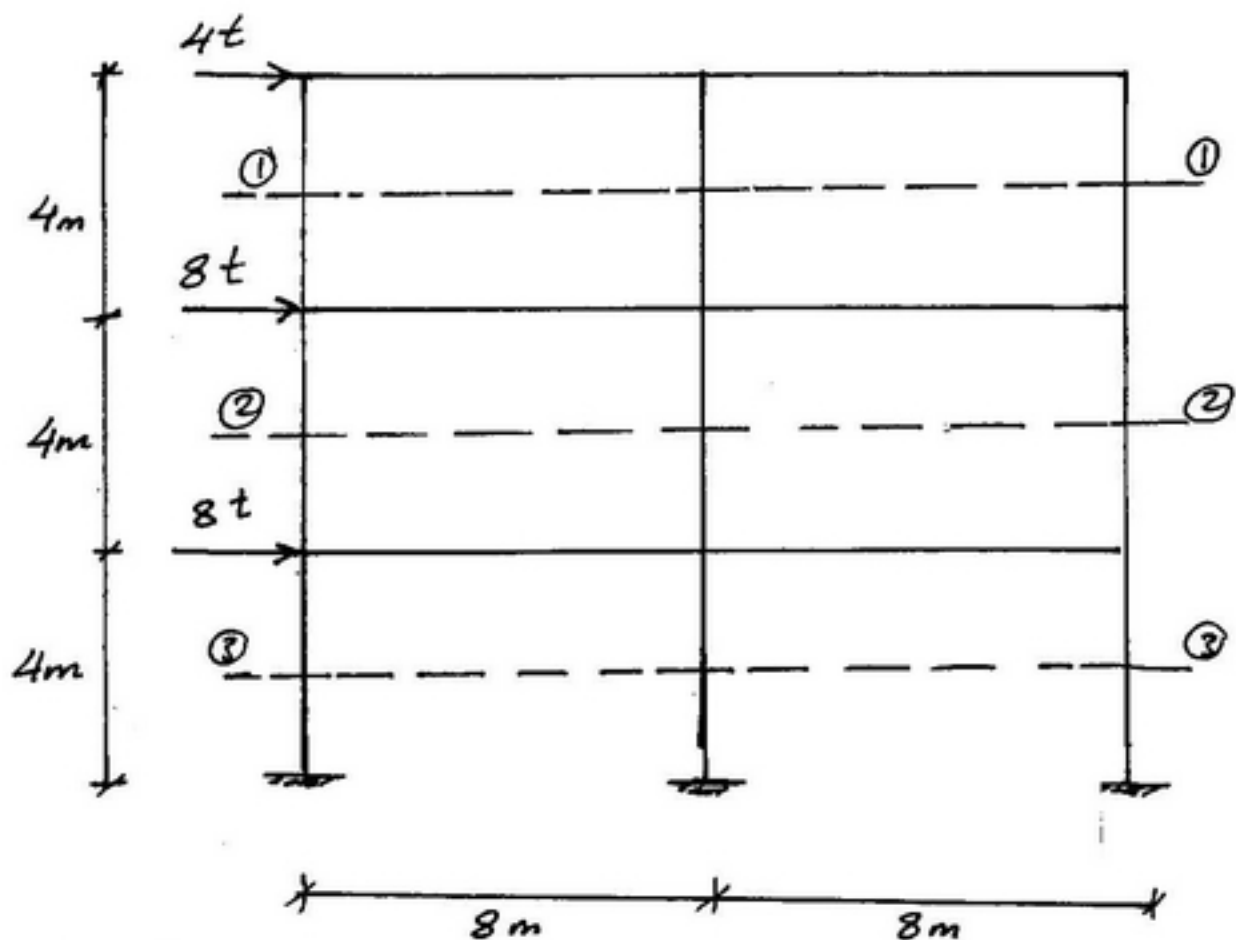
**structural Analysis**

- Revision - II -

- Approximate methods -

• Ex①: Final (2005)

- Using portal frame method draw N.F.D, S.F.D, B.M.D due to given loads.



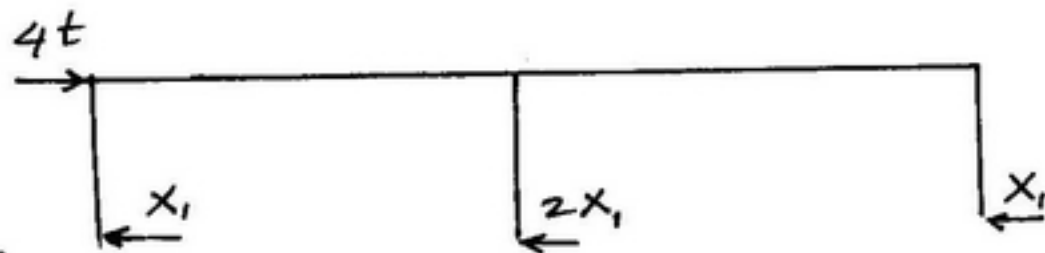
Sol/g

Level ①:

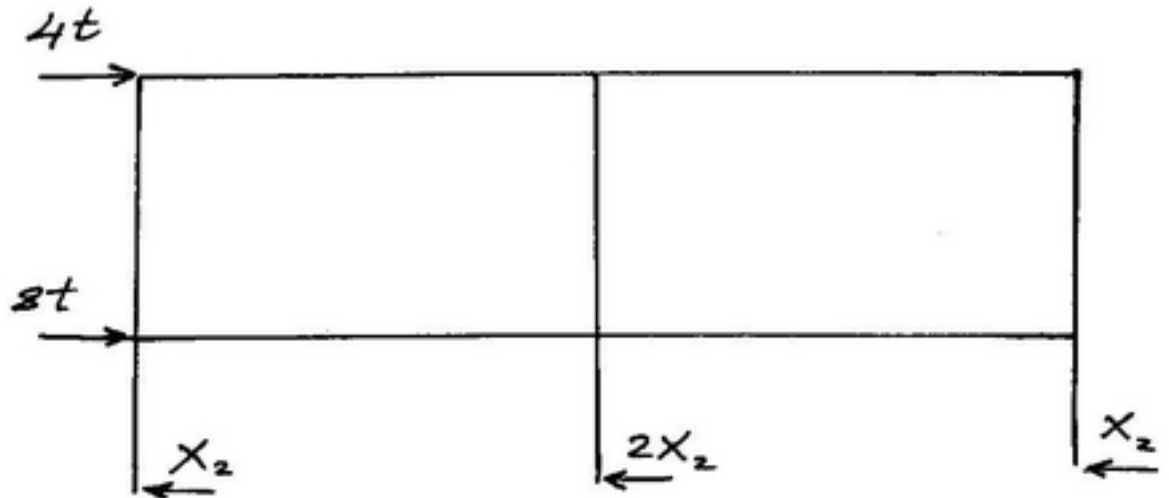
$$\sum f_x = 0.0 \rightarrow \oplus$$

$$4 - X - 2X - X = 0.0$$

$$\therefore X = 1 \text{ ton}$$



Level ②:

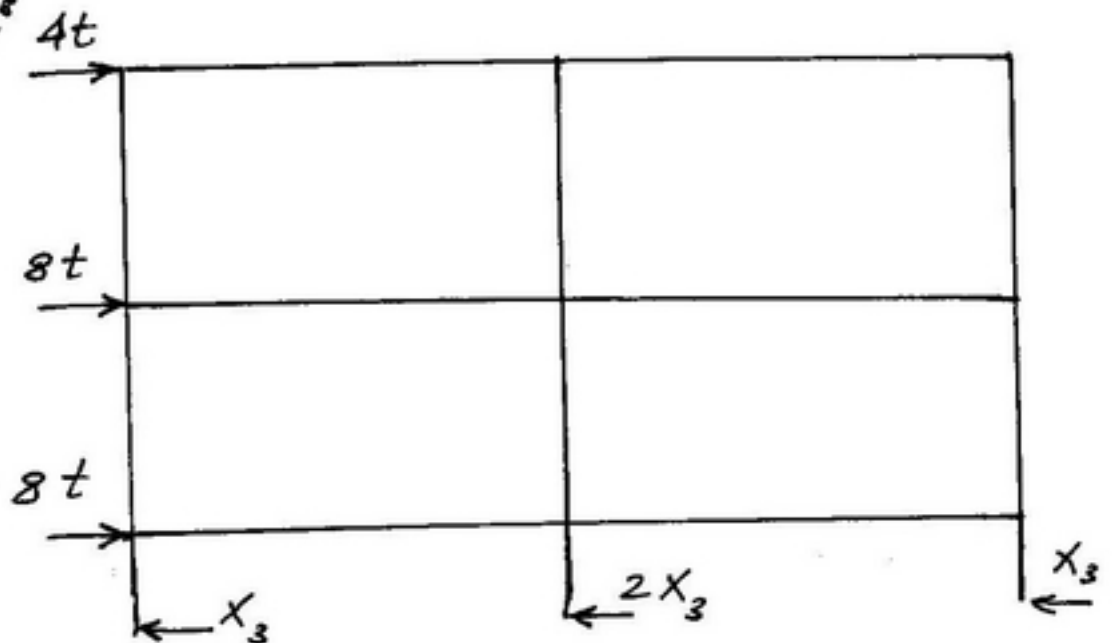


$$\sum f_x = 0 \quad \oplus \rightarrow$$

$$4 + 8 - X_2 - 2X_2 - X_2 = 0$$

$$\therefore X_2 = 3 \text{ ton}$$

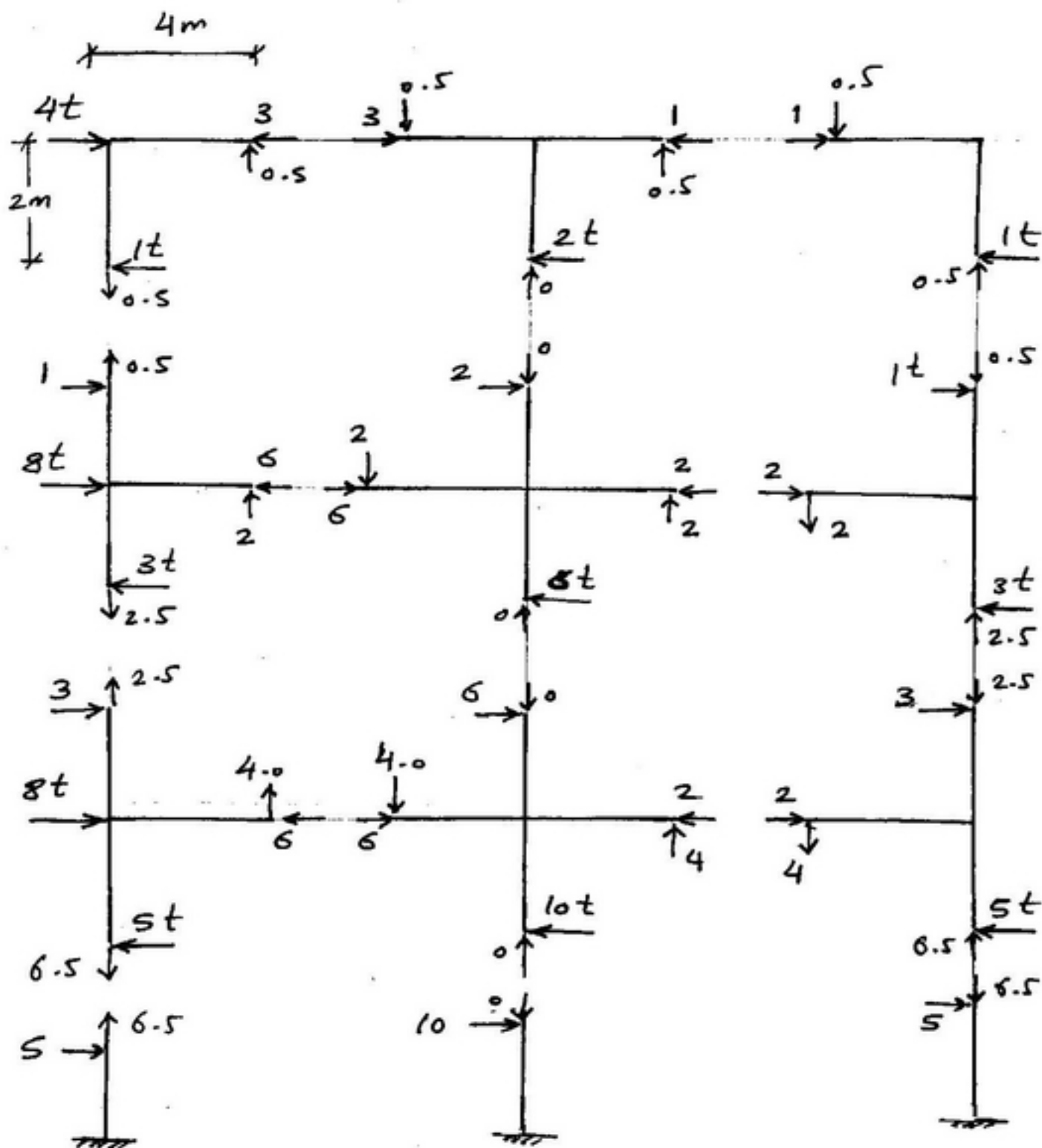
Level ③:



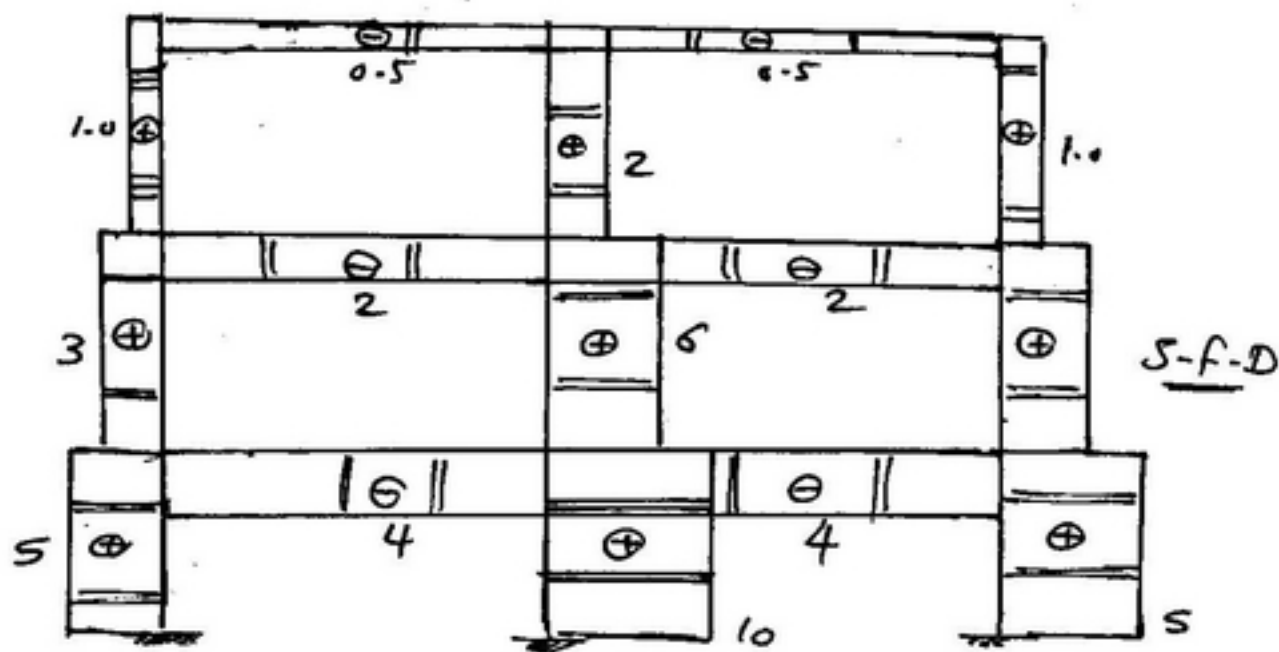
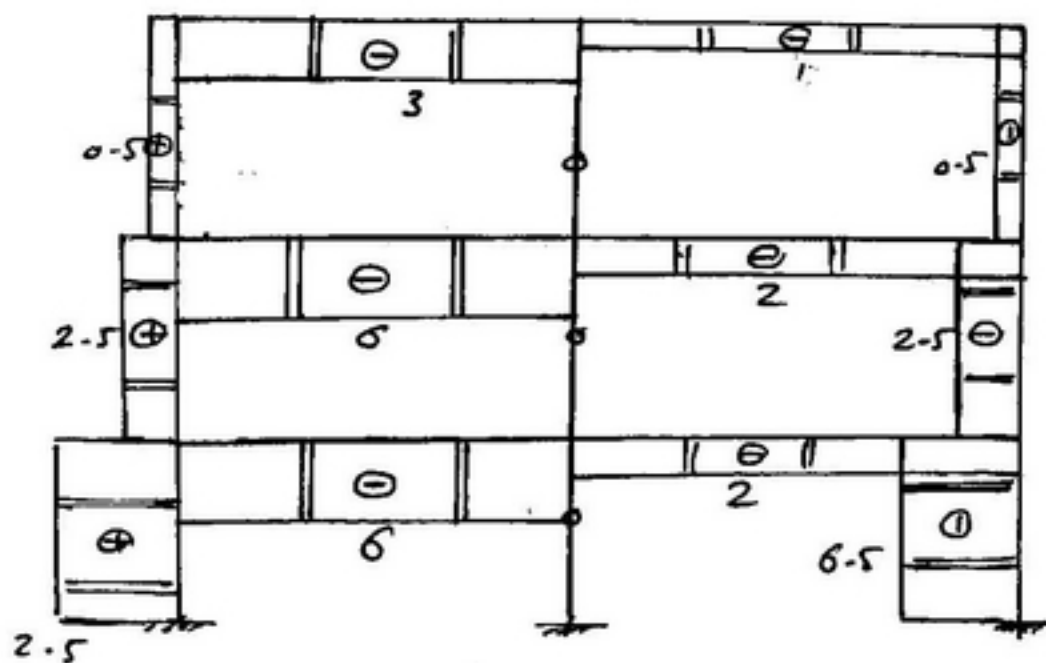
$$\sum f_x = 0 \quad \oplus \rightarrow$$

$$4 + 8 + 8 - 4X_3 = 0$$

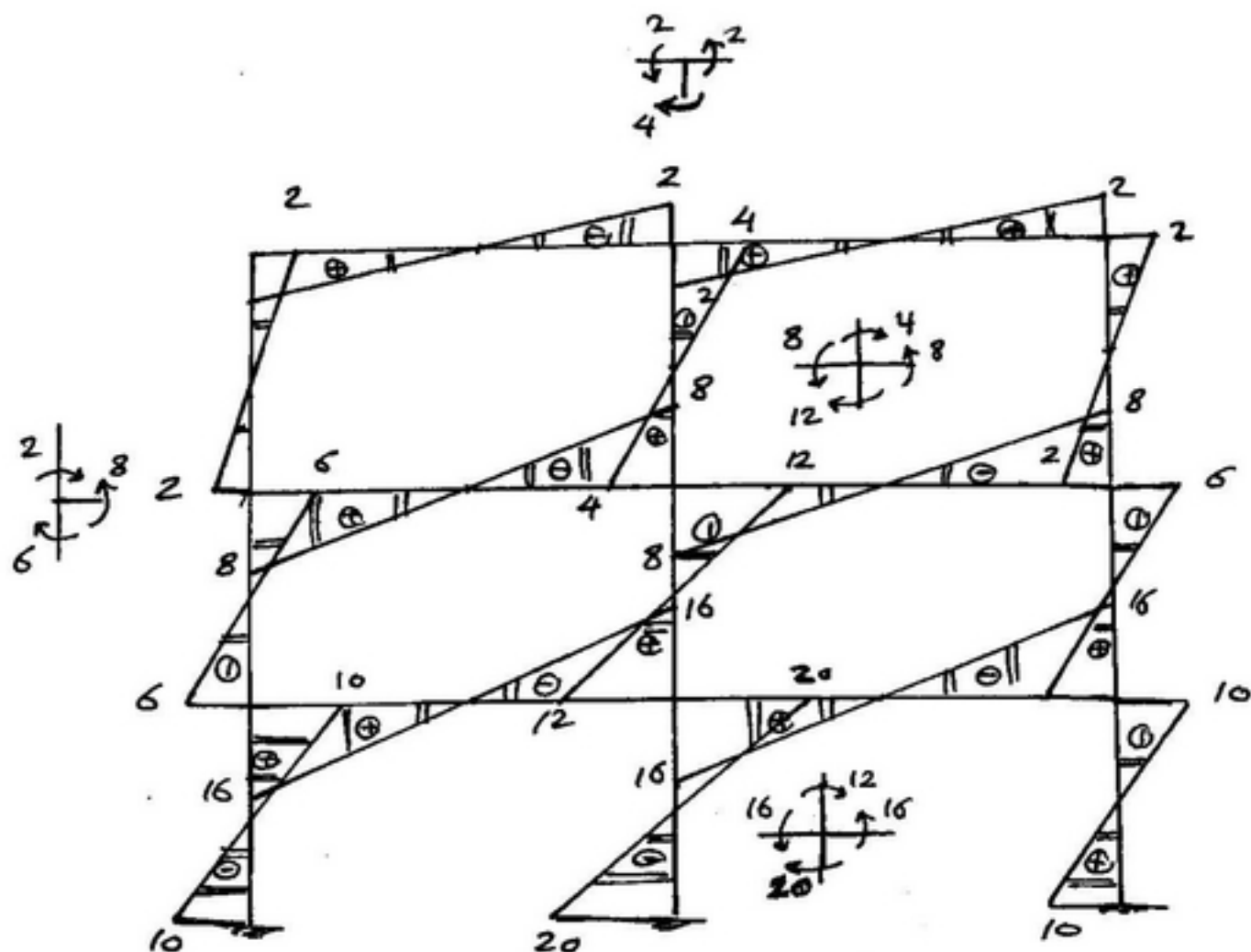
$$\therefore X_3 = 5 \text{ ton}$$





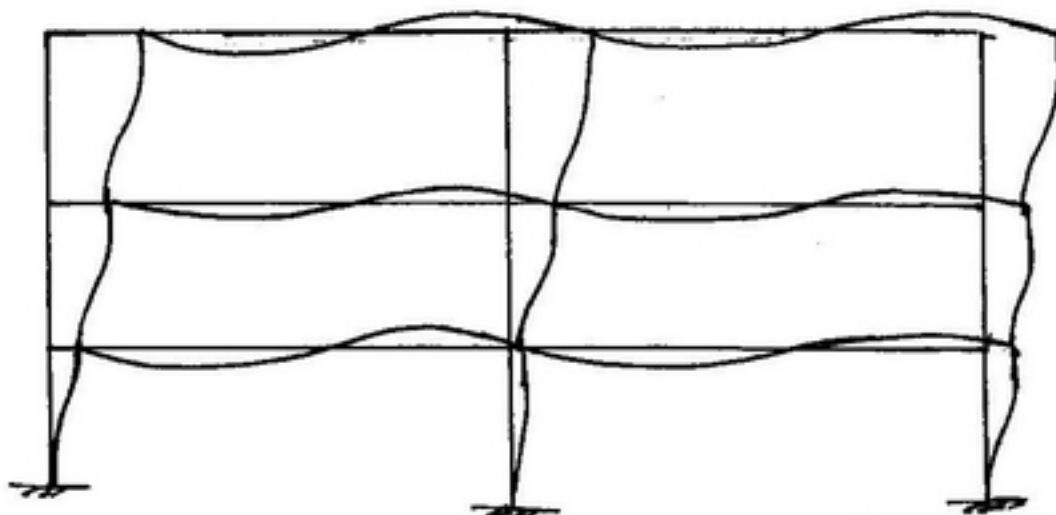




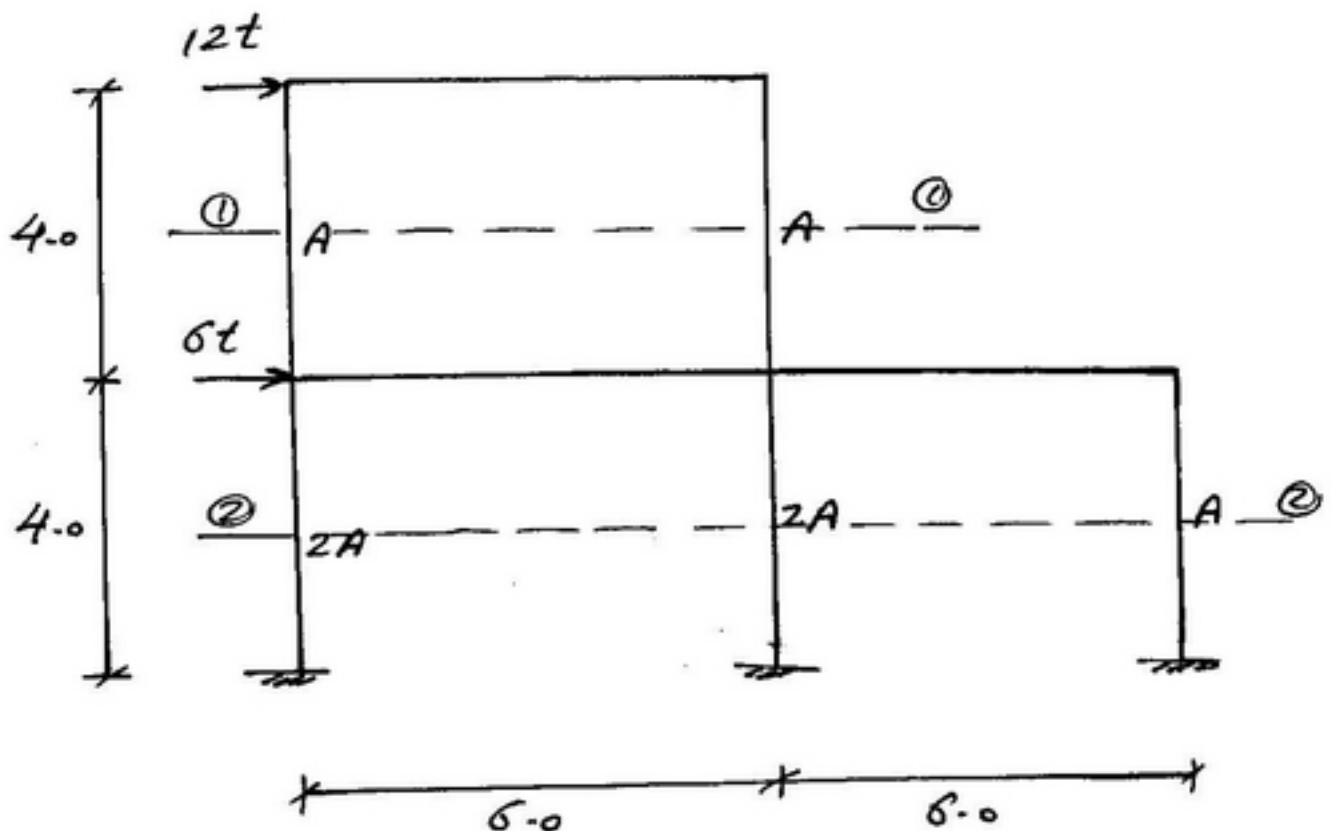


B.M.D

Elastic curve:



Ex③: using cantilever method draw N.f.D, S.f.D, B.M.D for the given frame.



Sol:

Level ①:

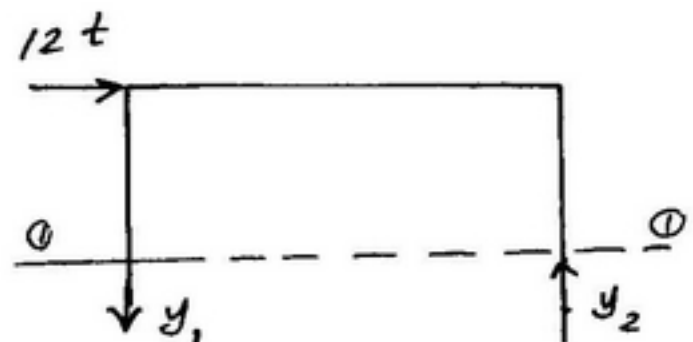
$$1 - \sum M_{y_2} = 0 \dots \oplus$$

$$12 \times 2 - y_1 \times 6 = 0 \dots$$

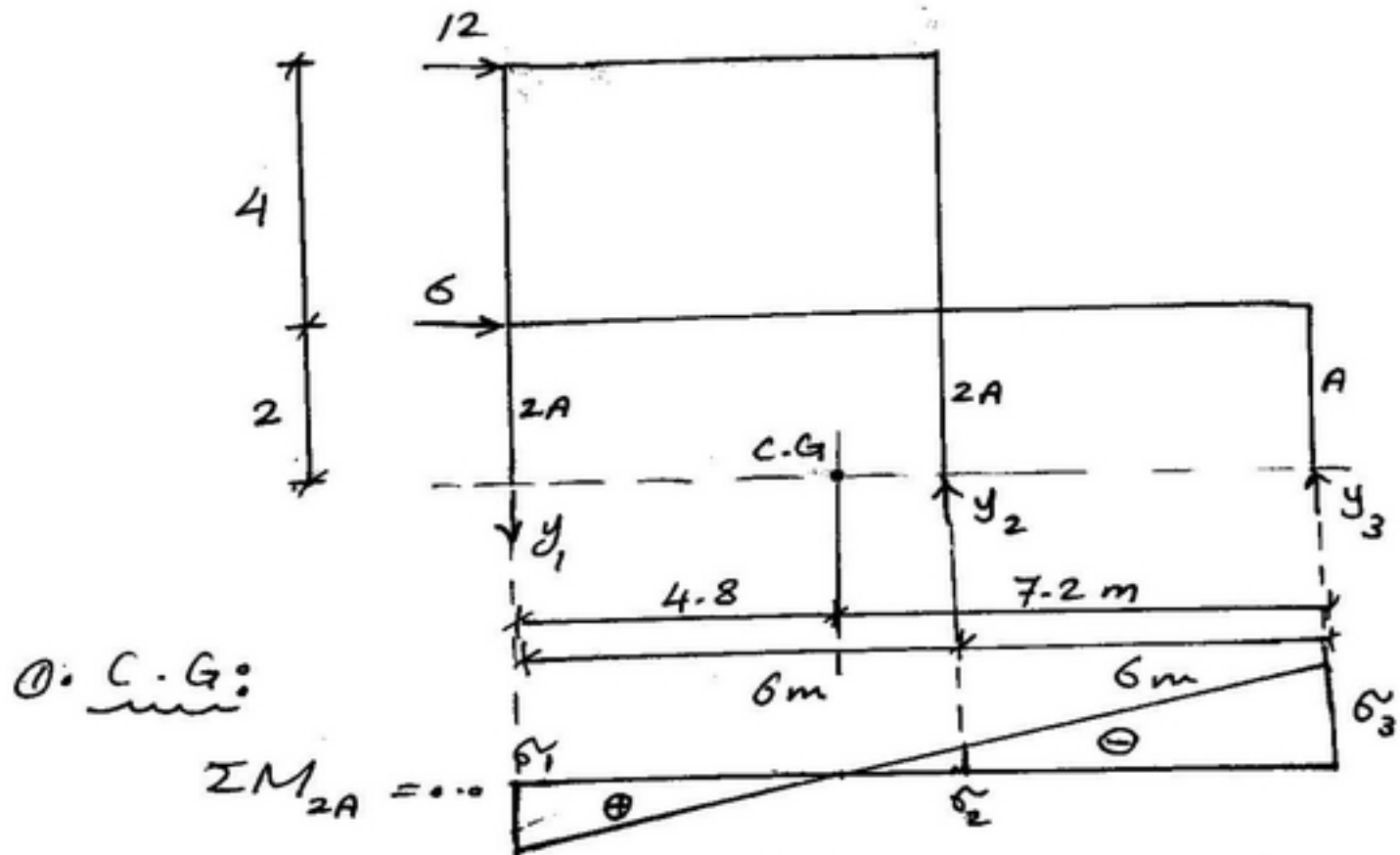
$$\therefore y_1 = 4 \text{ ton}$$

$$2 - \sum H_y = 0 \dots \uparrow \oplus$$

$$y_2 = y_1 = 4 \text{ ton}$$



Level ②:



①. C.G.:

$$\sum M_{2A} = \dots$$

$$X_c = \frac{\sum A_i \cdot X_i}{\sum A} = \frac{2A \cdot 6 + A \cdot 12}{2A + 2A + A} = 4.8 \text{ m}$$

$$1 - \sum M_{y_3} = \dots \quad \oplus \curvearrowright$$

$$\therefore 12 \cdot 6 + 6 \cdot 2 - y_1 \cdot 12 + y_2 \cdot 6 = \dots$$

$$-12 y_1 + 6 y_2 = -84 \rightarrow \textcircled{1}$$

$$2 - \sum P_y = \dots \quad \uparrow \oplus$$

$$-y_1 + y_2 + y_3 = \dots \rightarrow \textcircled{2}$$

• Δ Δ Δ ~ ( ) ~ .

$$\frac{\delta_1}{4.8} = \frac{\delta_2}{1.2} = \frac{\delta_3}{7.2}$$

$$\therefore \frac{y_1}{2A \times 4.8} = \frac{y_2}{2A \times 1.2} = \frac{y_3}{7.2 \times A}$$

$$\frac{y_1}{9.6} = \frac{y_2}{2.4} = \frac{y_3}{7.2}$$

$$\therefore y_1 = 4 y_2 = 1.33 y_3 \rightarrow \textcircled{3}$$

• from ③ in ①:

$$-12 y_1 + 6 y_2 = -84$$

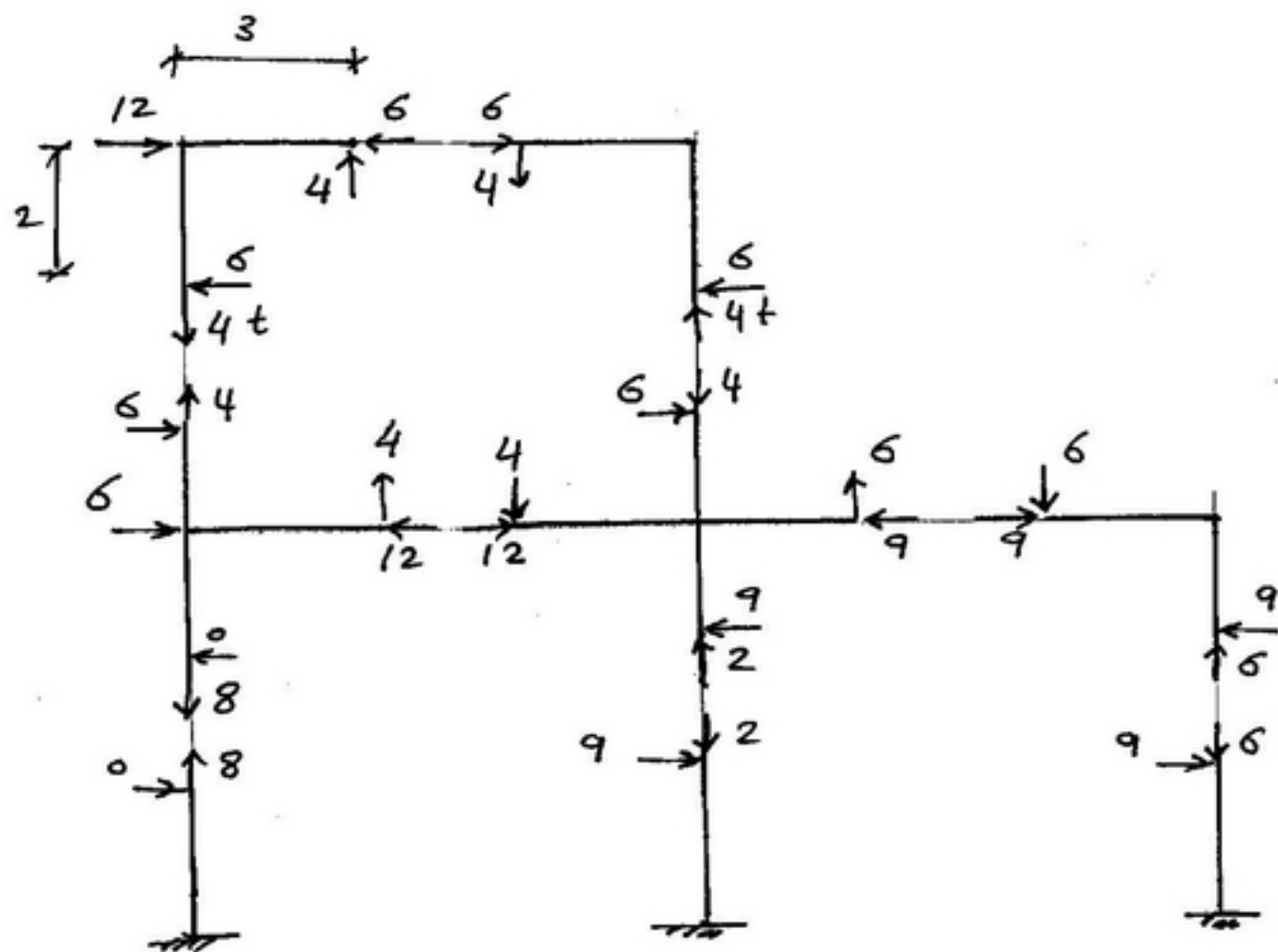
$$\therefore -12 \times (4 y_2) + 6 y_2 = -84$$

$$\therefore y_2 = 2 \text{ ton}$$

$$\therefore y_1 = 4 \times 2 = 8 \text{ ton}$$

$$y_3 = \frac{4 \times 2}{1.33} = 6 \text{ ton}$$

check  $\Rightarrow$  eqn. ②  $-8 + 2 + 6 = 0 \dots$  (o.k)



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طريقة: طريقة (Cantilever) أكثر دقة مع  
طريقة (portal frame) وذلك لتأخذ  
في الاعتبار مساحة مقطع الأعمدة.

Final Revision

3rd civil year

15

structural Analysis



- Revision - III  
- plates -

- Ex①: Final '2003'

for the shown circular plate calculate and draw  $M_r$ ,  $M_\theta$  and  $w$  diagrams, then sketch main steel reinforcement.

$$E = 200 \text{ t/cm}^2, t = 30 \text{ cm} \text{ \& } \mu = 0.0$$

Sol:

$$Q = \frac{2 * \pi (r^2 - l^2)}{2 \pi r}$$

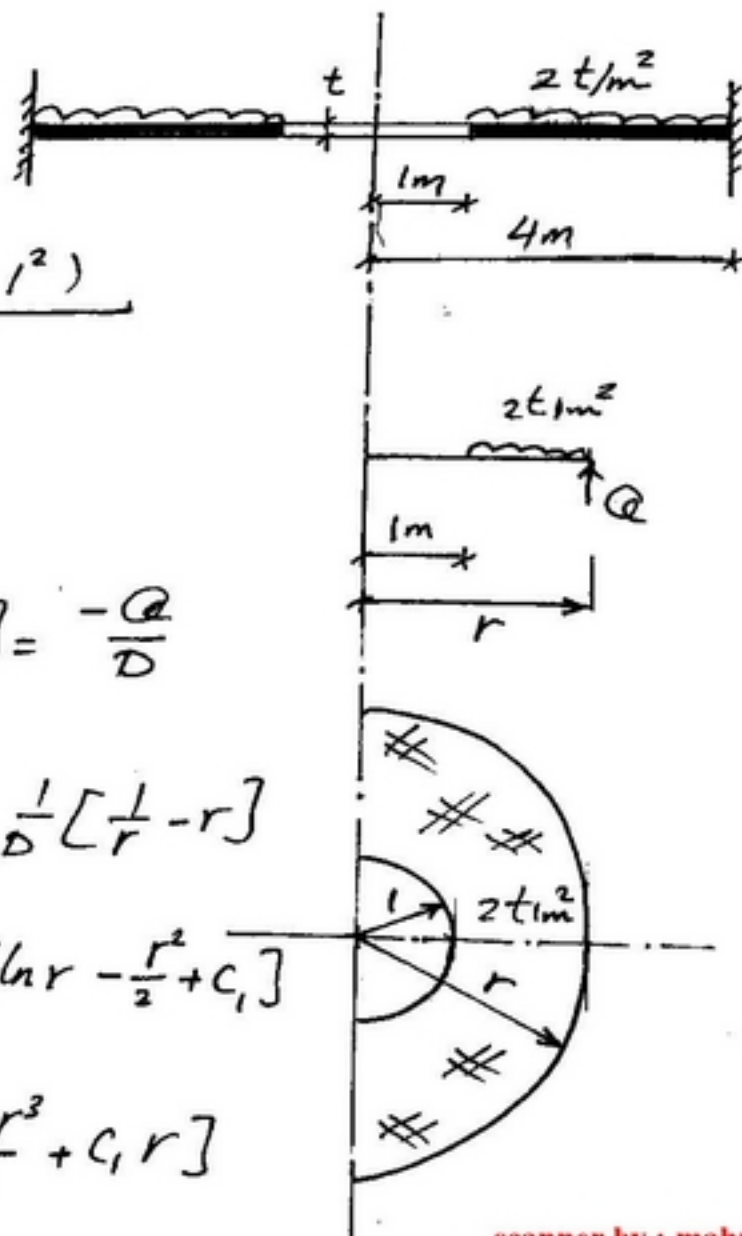
$$\therefore Q = (r - \frac{1}{r})$$

$$\frac{d}{dr} \left[ \frac{1}{r} \cdot \frac{d}{dr} (r\phi) \right] = -\frac{Q}{D}$$

$$\therefore \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (r\phi) \right] = \frac{1}{D} \left[ \frac{1}{r} - r \right]$$

$$\therefore \frac{1}{r} \frac{d}{dr} (r\phi) = \frac{1}{D} \left[ \ln r - \frac{r^2}{2} + C_1 \right]$$

$$\frac{d}{dr} (r\phi) = \frac{1}{D} \left[ r \ln r - \frac{r^3}{2} + C_1 r \right]$$



$$\therefore r\phi = \frac{1}{D} \left[ \frac{r^2}{2} \ln r - \frac{r^2}{4} - \frac{r^4}{8} + C_1 \frac{r^2}{2} + C_2 \right]$$

$$\therefore \phi = \frac{1}{D} \left[ \frac{1}{2} r \ln r - \frac{r}{4} - \frac{r^3}{8} + C_1 \frac{r}{2} + \frac{C_2}{r} \right]$$

$$M_r = D \left[ \frac{d\phi}{dr} + M \frac{\phi}{r} \right]$$

$$\Rightarrow \therefore M_r = \left[ \frac{1}{2} (1 + \ln r) - \frac{1}{4} - \frac{3r^2}{8} + \frac{C_1}{2} - \frac{C_2}{r^2} \right]$$

$$M_\theta = D \left[ M \frac{d\phi}{dr} + \frac{\phi}{r} \right]$$

$$\Rightarrow \therefore M_\theta = \left[ \frac{1}{2} \ln r - \frac{1}{4} - \frac{r^2}{8} + \frac{C_1}{2} + \frac{C_2}{r^2} \right]$$

B.Cs:

1- at  $r=4 \Rightarrow \phi=0$

$$\therefore 0 = \frac{1}{D} \left[ \frac{1}{2} \times 4 \ln 4 - \frac{4}{4} - \frac{4^3}{8} + C_1 \times \frac{4}{2} + \frac{C_2}{4} \right]$$

$$0 = \left[ 2.77 - 1 - 8 + 2C_1 + 0.25C_2 \right]$$

$$\therefore 2C_1 + 0.25C_2 = 6.23 \rightarrow \textcircled{1}$$

2- at  $r=1 \Rightarrow M_r=0$

$$\therefore 0 = \left[ \frac{1}{2} (1 + \ln 1) - \frac{1}{4} - \frac{3 \times 1^2}{8} + \frac{C_1}{2} - \frac{C_2}{1^2} \right]$$

$$\therefore 0.0 = [0.5 + 0.25 - 0.375 + 0.5C_1 - C_2]$$

$$\therefore 0.5C_1 - C_2 = -0.125 \rightarrow (2)$$

from (1), (2):

$$\therefore C_1 = 2.95$$

$$C_2 = 1.35$$

• for  $\omega$ :

$$\omega = - \int \Phi dr$$

$$\therefore \omega = -\frac{1}{D} \left[ \frac{1}{2} \left( \frac{r^2}{2} \ln r - \frac{r^2}{4} \right) - \frac{r^2}{8} - \frac{r^4}{32} + C_1 \frac{r^3}{6} + C_2 \ln r + C_3 \right]$$

$$\omega = -\frac{1}{D} \left[ \frac{r^2}{4} \ln r - \frac{r^2}{8} - \frac{r^2}{8} - \frac{r^4}{32} + C_1 \frac{r^3}{6} + C_2 \ln r + C_3 \right]$$

$$\therefore \omega = -\frac{1}{D} \left[ \frac{r^2}{4} \ln r - \frac{r^2}{4} - \frac{r^4}{32} + 0.49 r^3 + 1.35 \ln r + C_3 \right]$$

B.Cs:

$$\text{3-at } r=4 \Rightarrow \omega = 0.0$$

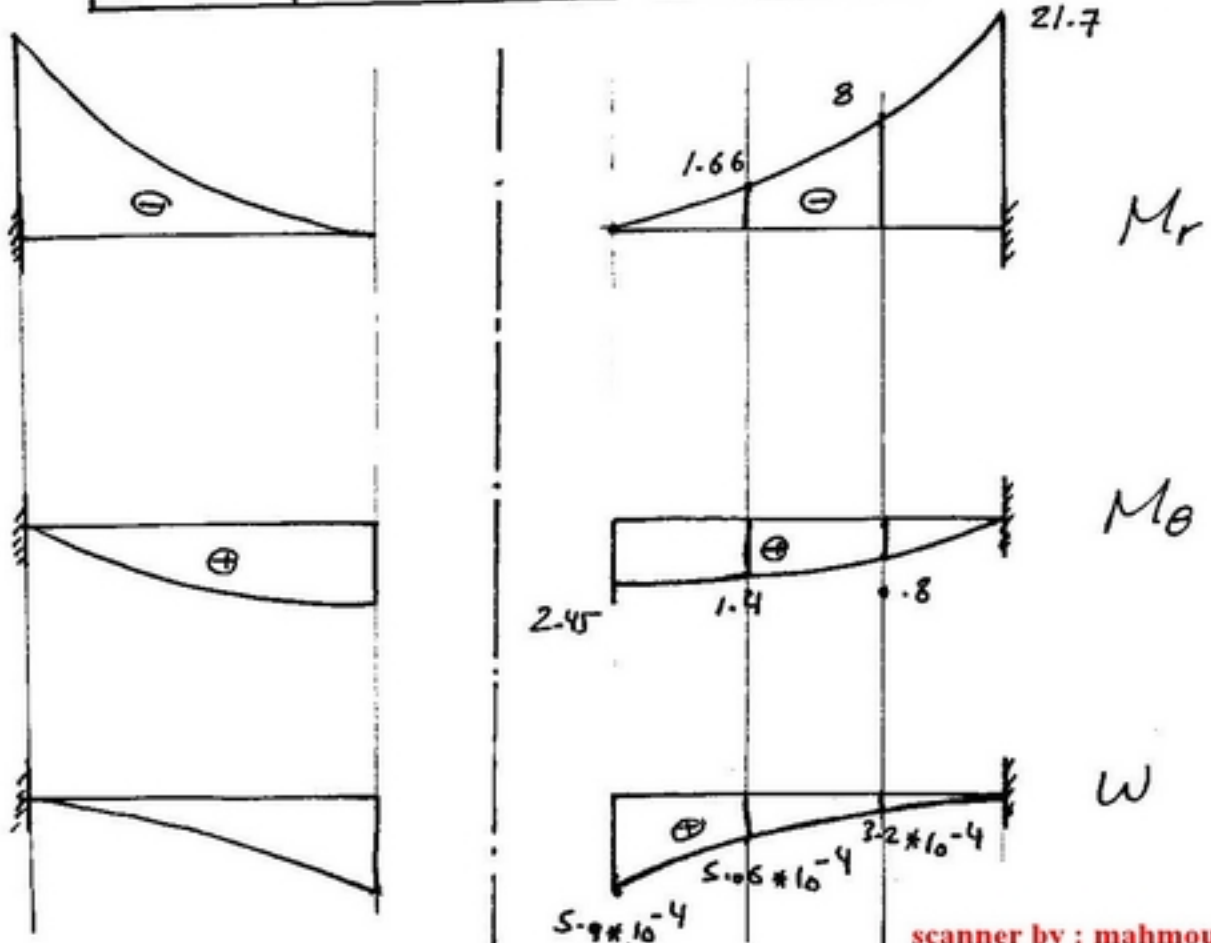
$$\therefore 0.0 = -\frac{1}{D} \left[ \frac{4^2}{4} \ln 4 - \frac{4^2}{4} - \frac{4^4}{32} + 0.49 \times 4^3 + 1.35 \ln 4 + C_3 \right]$$

$$\therefore C_3 = -26.8$$

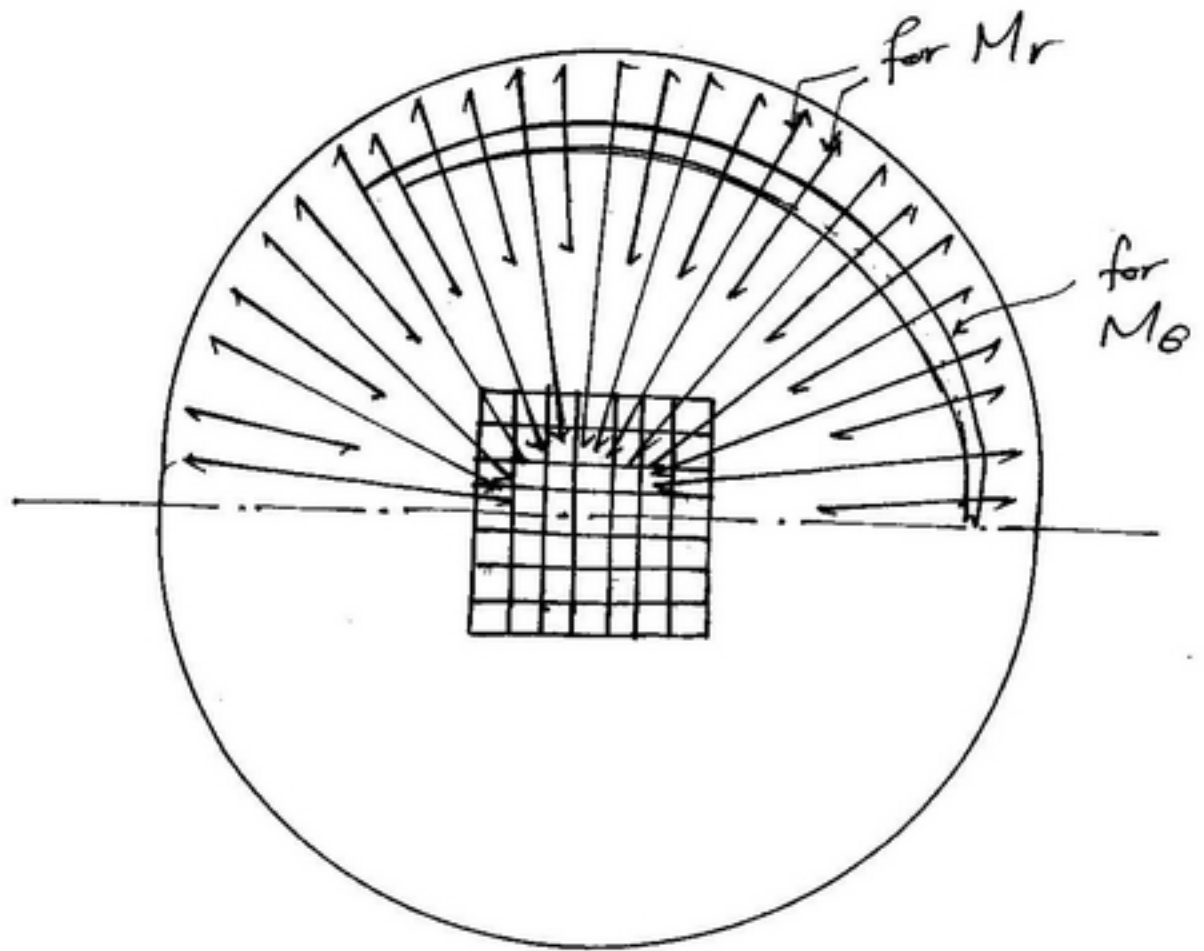
$$D = \frac{Et^3}{12(1-\mu^2)} = \frac{200 \times 30^3}{12} = 45000 \text{ t.cm}$$

$$\therefore w = \frac{-1}{45000} \left\{ \frac{r^2}{4} \ln r - \frac{r^2}{4} - \frac{r^4}{32} + 0.49 r^3 + 1.35 \ln r - 26.8 \right\}$$

| r          | 1                    | 2                     | 3                    | 4     |
|------------|----------------------|-----------------------|----------------------|-------|
| $M_r$      | 0                    | -1.66                 | -8                   | -21.7 |
| $M_\theta$ | 2.45                 | 1.4                   | 0.8                  | 0.0   |
| $w$        | $5.9 \times 10^{-4}$ | $5.06 \times 10^{-4}$ | $3.2 \times 10^{-4}$ | 0.0   |

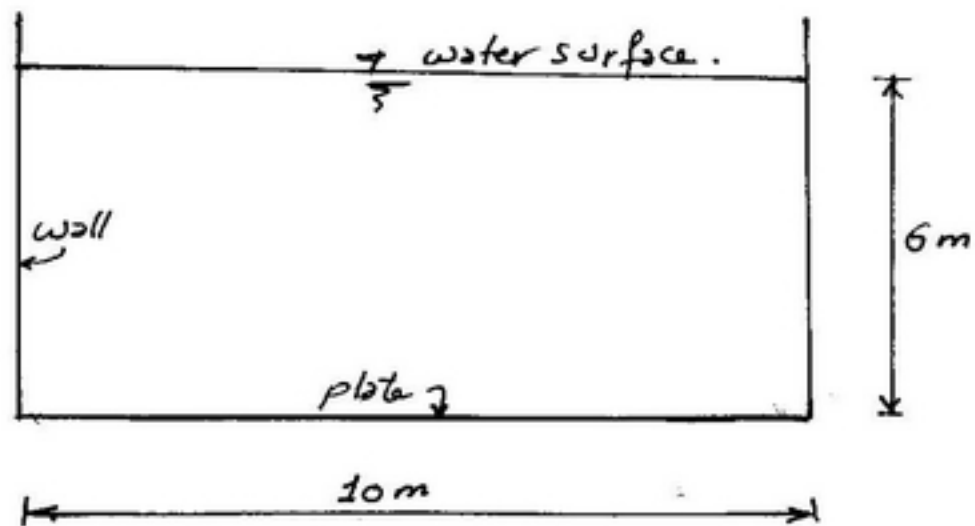


## Main steel reinforcement :



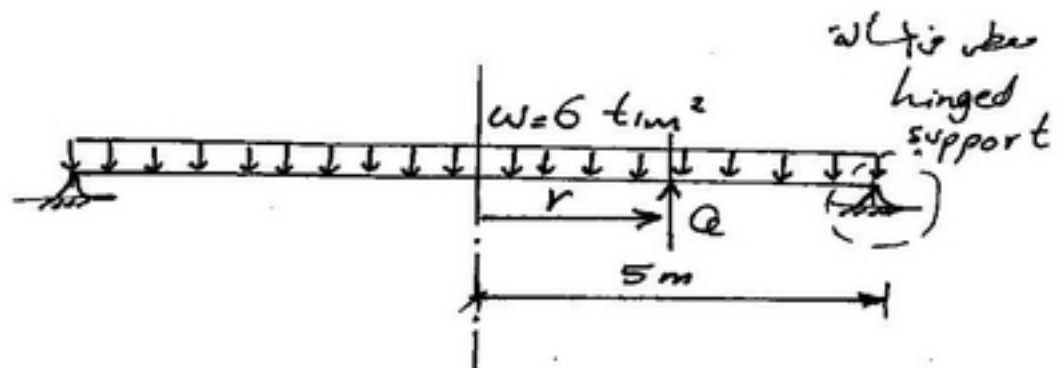
EX②: final (2001)

- Calculate the ring and tangential moments for the solid circular base of the shown water tank, considering hinged connection between the plate and supporting wall.



Sol.  $\sigma_w$  is

$$W = \sigma_w \cdot h_w = 1.0 \times 6 = 6 \text{ t/m}^2$$



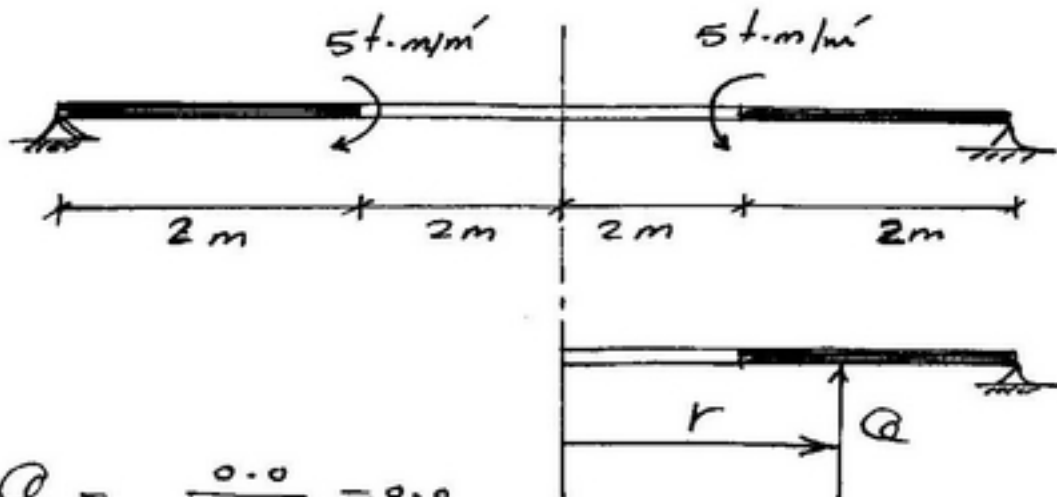
$$Q = \frac{6 \times \pi r^2}{2\pi r} = 3r$$

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Ex ③: final (2002):

- for the shown circular plate draw bending moment diagrams ( $M_r$ ,  $M_\theta$ )



Sol:

$$Q = \frac{0.0}{2\pi r} = 0.0$$

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (r\phi) \right] = -\frac{Q}{D}$$

$$\therefore \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (r\phi) \right] = 0.0$$

$$\therefore \left[ \frac{1}{r} \frac{d}{dr} (r\phi) \right] = C_1$$

$$\therefore \frac{d}{dr} (r\phi) = C_1 r$$

$$\therefore r\phi = C_1 \frac{r^2}{2} + C_2$$

$$\therefore \phi = \left[ C_1 \frac{r}{2} + \frac{C_2}{r} \right]$$

$$M_r = \left[ \frac{d\phi}{dr} + M \frac{\phi}{r} \right]$$

$$\Rightarrow \therefore M_r = \left[ \frac{C_1}{2} - \frac{C_2}{r^2} \right]$$

$$M_\theta = \left[ M \frac{d\phi}{dr} + \frac{\phi}{r} \right]$$

$$\Rightarrow \therefore M_\theta = \left[ \frac{C_1}{2} + \frac{C_2}{r^2} \right]$$

B.C.s:

$$1-\text{at } r=2 \Rightarrow M_r = -5 \text{ t.m/m}^2$$

$$\therefore -5 = \left[ \frac{C_1}{2} - \frac{C_2}{4} \right]$$

$$\therefore 2C_1 - C_2 = -20 \rightarrow \textcircled{1}$$

$$2-\text{at } r=4 \Rightarrow M_r = 0.0$$

$$\therefore 0.0 = \left[ \frac{C_1}{2} - \frac{C_2}{16} \right]$$

$$\therefore 8C_1 - C_2 = 0.0 \rightarrow \textcircled{2}$$

from  $\textcircled{1}, \textcircled{2}$ :

$$\therefore C_1 = 3.33$$

$$C_2 = 26.7$$

من المطلوبه قولي لة

$$\omega = - \int \phi dr$$

$$= - \left[ C_1 \frac{r^3}{6} + C_2 \ln r + C_3 \right]$$

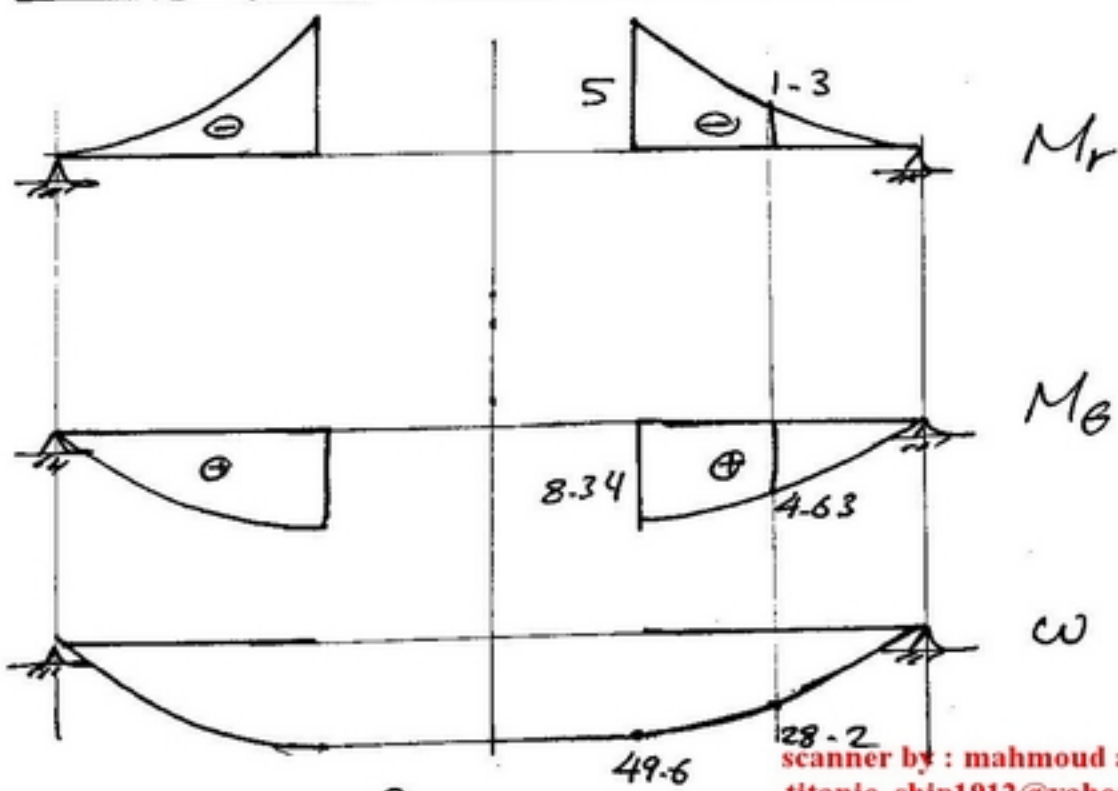
B.Cs :

at  $r = 4 \Rightarrow \omega = 0.0$

$$\therefore 0.0 = \left[ 3.33 * \frac{4^3}{6} + 26.7 \ln 4 + C_3 \right]$$

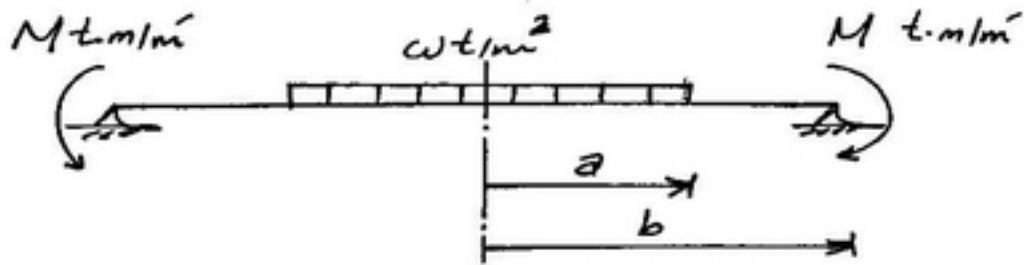
$$\therefore C_3 = -72.5$$

| r          | 2    | 3    | 4    |
|------------|------|------|------|
| $M_r$      | -5   | -1.3 | 0.0  |
| $M_\theta$ | 8.34 | 4.63 | 3.33 |
| $\omega$   | 49.6 | 28.2 | 0.0  |



EX④: for the given plates write the no. of intervals, B.C.s and the expression of the shear in each interval.

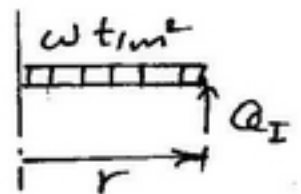
final (2002):



• no. of intervals = 2

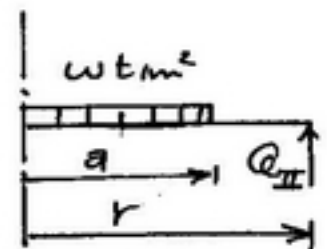
→ Interval ①  $0 \leq r \leq a$ :

$$Q_I = \frac{w * \pi r^2}{2 \pi r} = \frac{w r}{2}$$



→ Interval ②

$$Q_{II} = \frac{w * \pi a^2}{2 \pi r} = \frac{w a^2}{2r}$$



B.C.s:

1-at  $r = 0 \Rightarrow Q_I = 0$

2-at  $r = b \Rightarrow M_{rII} = -M$

3-at  $r = a \Rightarrow Q_I = Q_{II}$

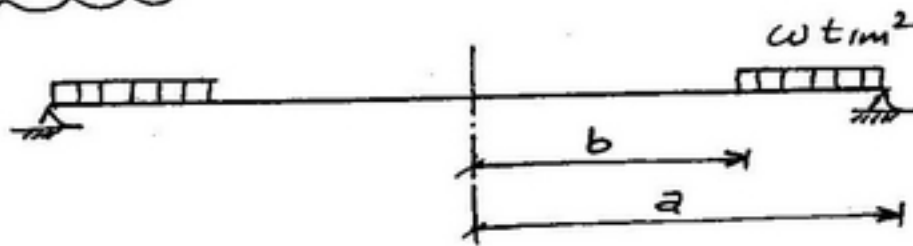
4-at  $r = a \Rightarrow M_{rI} = M_{rII}$

for  $w$ :

5-at  $r = b \Rightarrow w_{II} = 0$

6-at  $r = a \Rightarrow w_I = w_{II}$

final (2002):



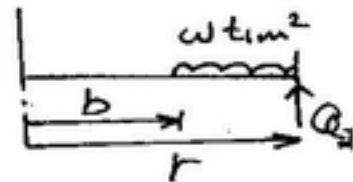
• no. of intervals = 2

⇒ Interval ①:  $0 \leq r \leq b$

$$Q_I = 0.0$$

⇒ Interval ②:  $b \leq r \leq a$

$$Q_{II} = \frac{\omega * \pi (r^2 - b^2)}{2\pi r}$$



B.C.s:

1-at  $r=0.0 \Rightarrow \Phi_I = 0.0$

2-at  $r=a \Rightarrow M_{rII} = 0.0$

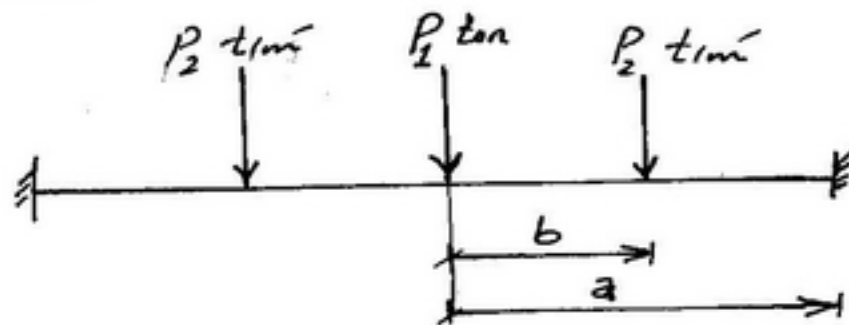
3-at  $r=b \Rightarrow M_{rI} = M_{rII}$

4-at  $r=b \Rightarrow \Phi_I = \Phi_{II}$

for ω: 5-at  $r=a \Rightarrow \omega_{II} = 0.0$

6-at  $r=b \Rightarrow \omega_I = \omega_{II}$

final (2002):



. no. of intervals = 2

⇒ Interval ①:  $0 \leq r \leq b$

$$Q_I = \frac{P_1}{2\pi r}$$

⇒ interval ②:  $b \leq r \leq a$

$$Q_{II} = \frac{P_1 + P_2 * 2\pi b}{2\pi r}$$

B.Cs:

1-at  $r=0 \Rightarrow \Phi_I = 0$

2-at  $r=a \Rightarrow \Phi_{II} = 0$

3-at  $r=b \Rightarrow \Phi_I = \Phi_{II}$

4-at  $r=b \Rightarrow M_{rI} = M_{rII}$

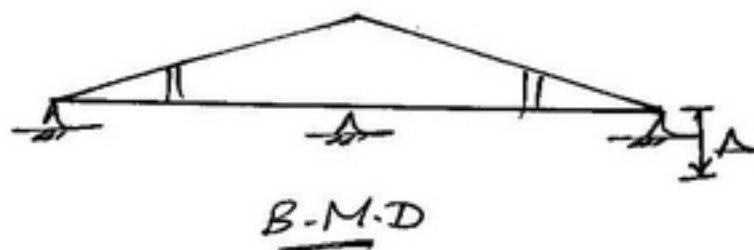
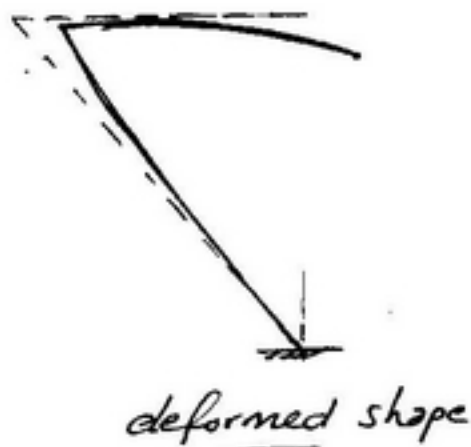
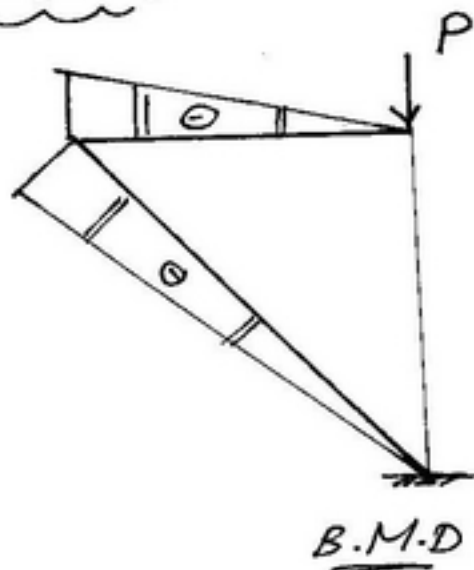
for  $\omega$ : 5-at  $r=a \Rightarrow \omega = 0$

6-at  $r=b \Rightarrow \omega_I = \omega_{II}$

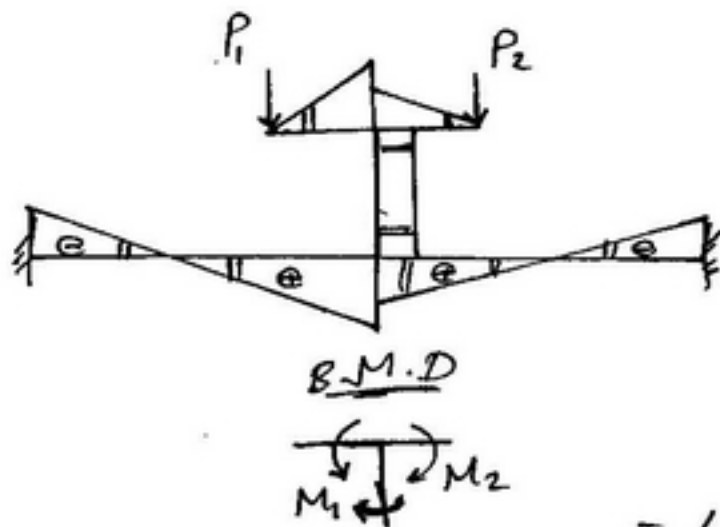


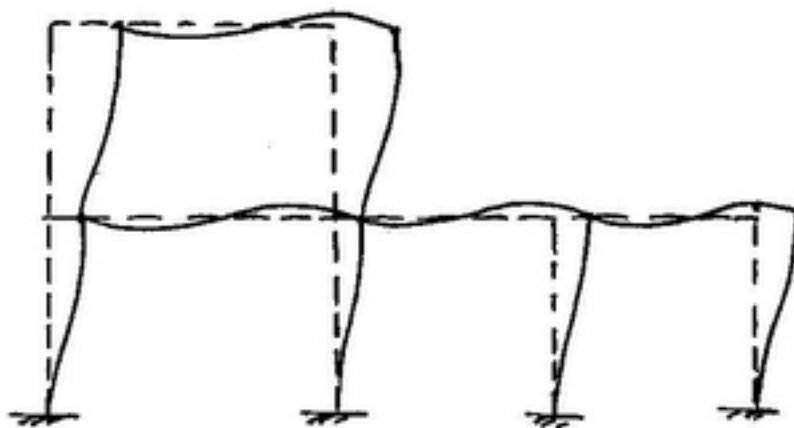
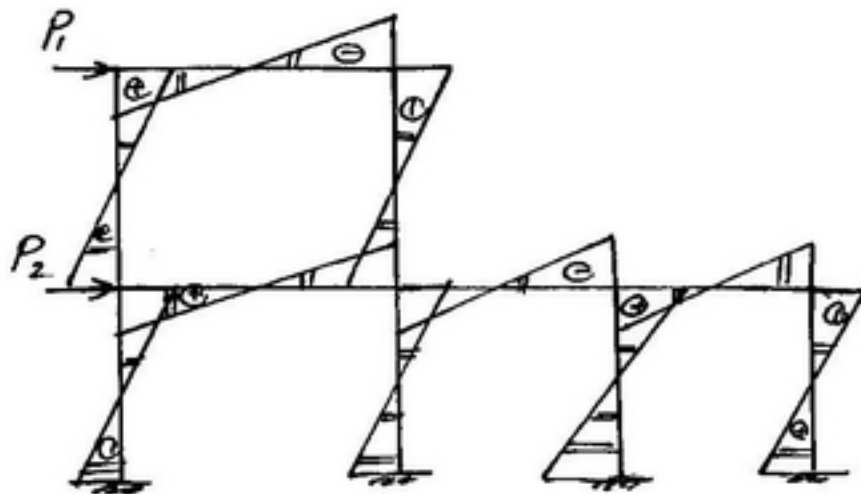
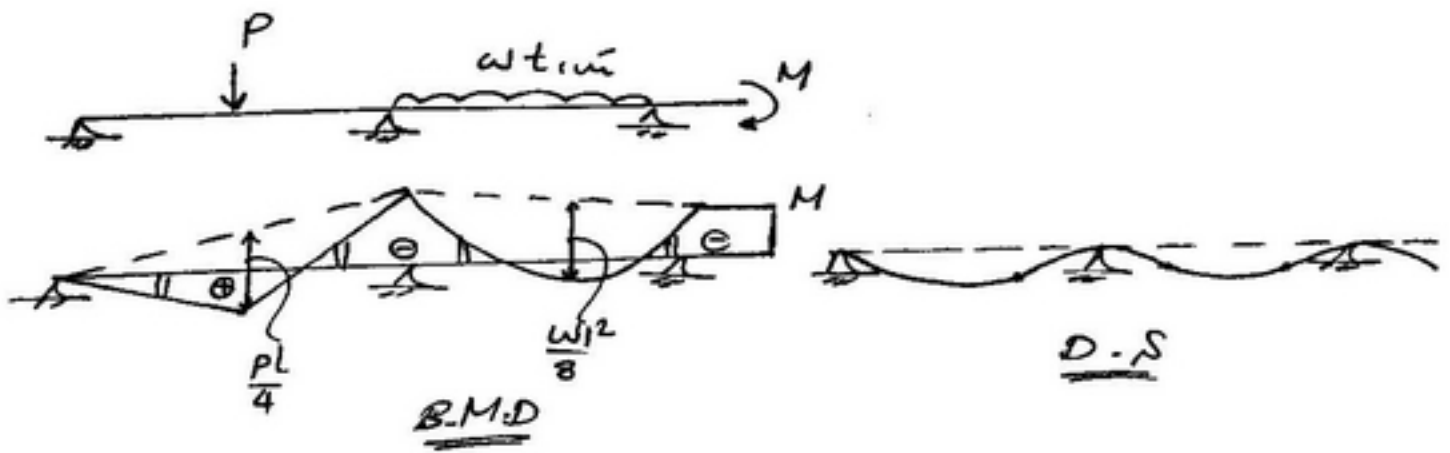
⇒ without calculations draw elastic curve and B.M.D for the given structures.

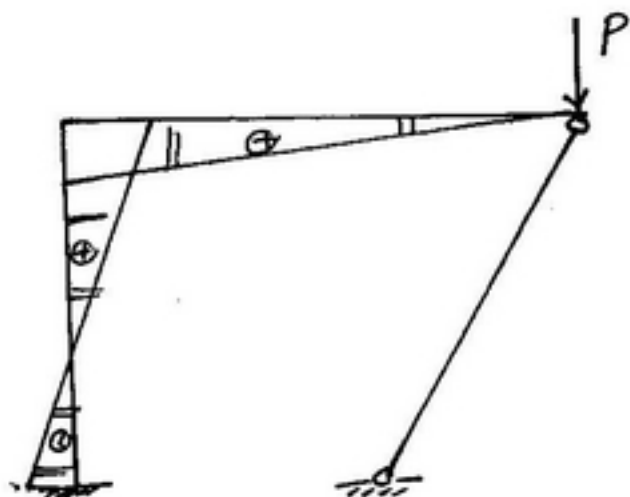
• final 2002:



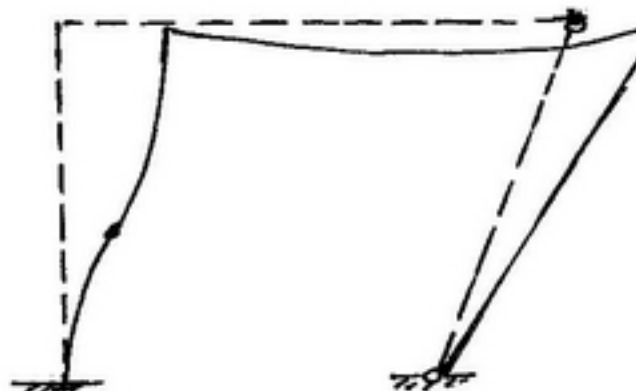
$$P_1 > P_2$$



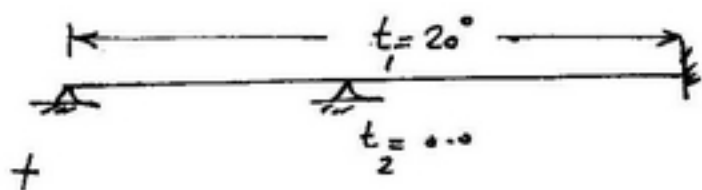




B.M.D



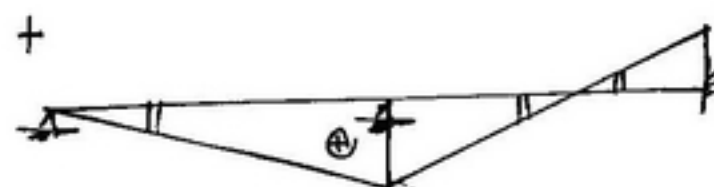
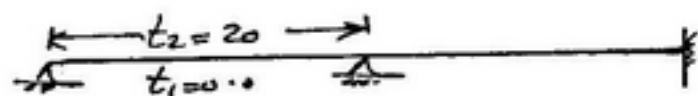
D.S



B.M.D



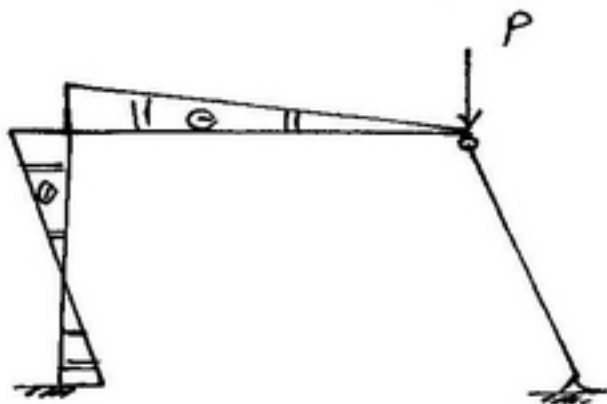
D.S



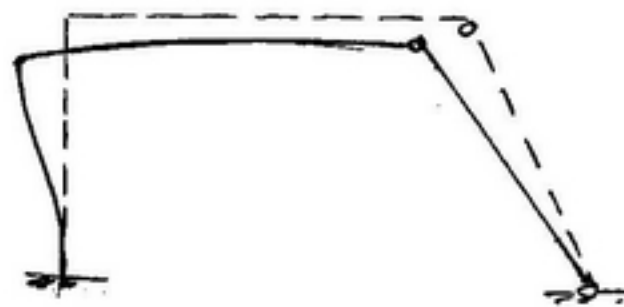
B.M.D



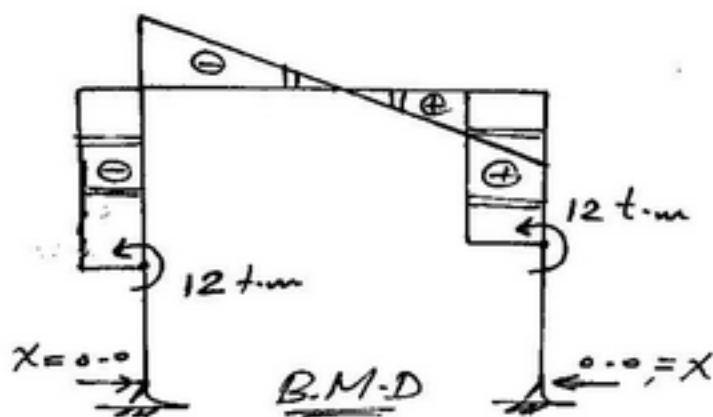
D.S



B.M.D



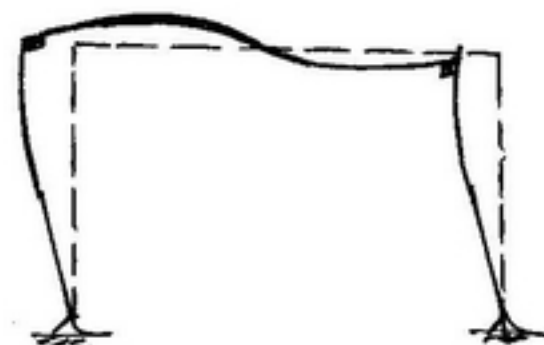
D-S



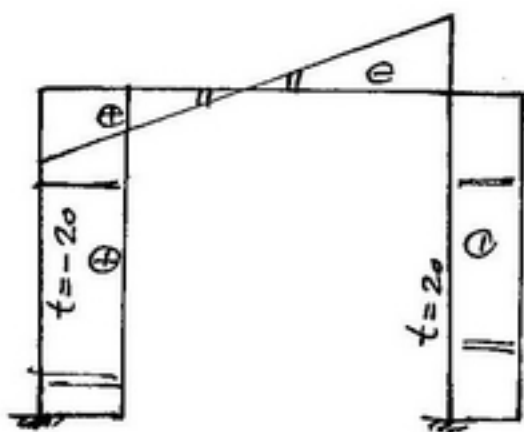
B.M.D

anti-symmetry

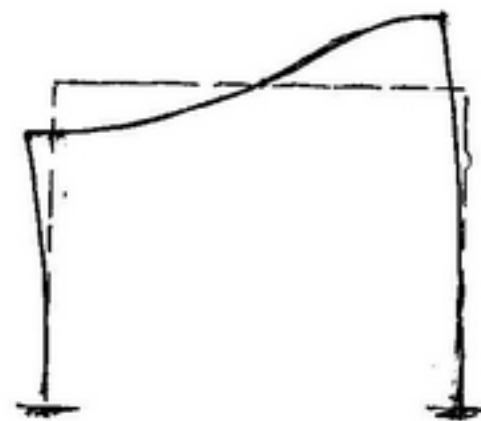
$\therefore N = 0.0$   
ف (ن)، ص 0



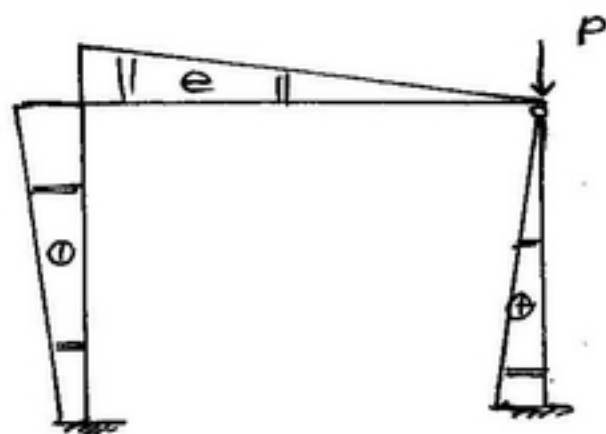
D-S



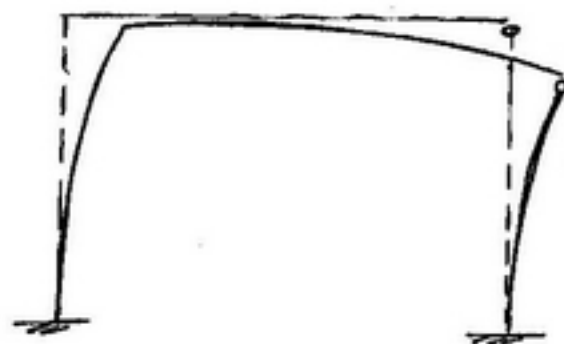
B.M.D



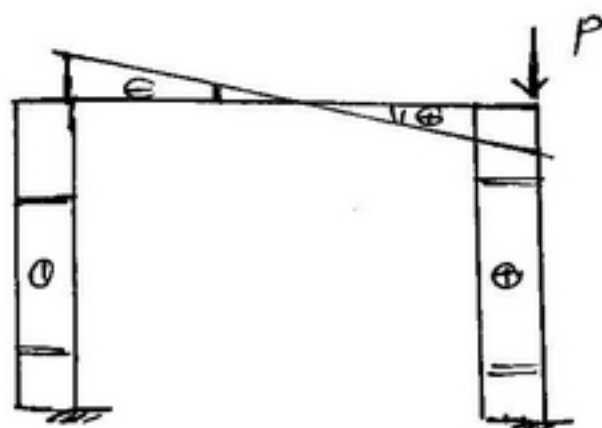
D-S



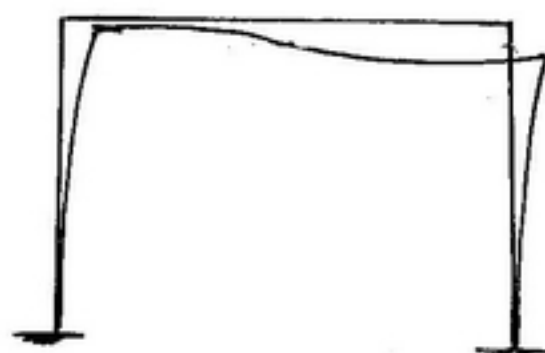
B.M.D



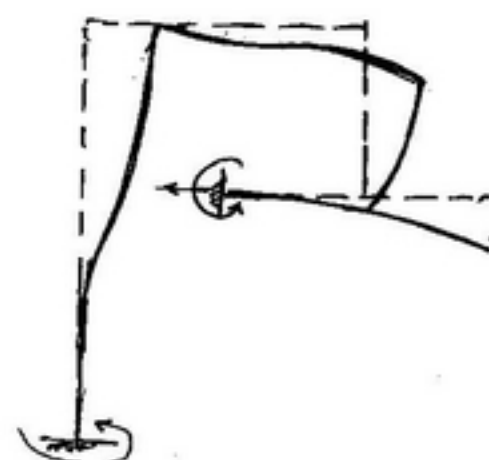
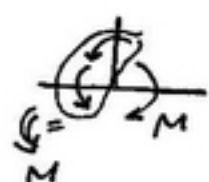
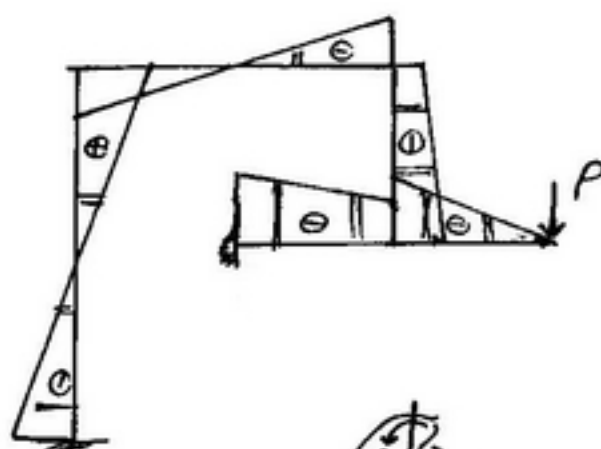
D.S



B.M.D



D.S



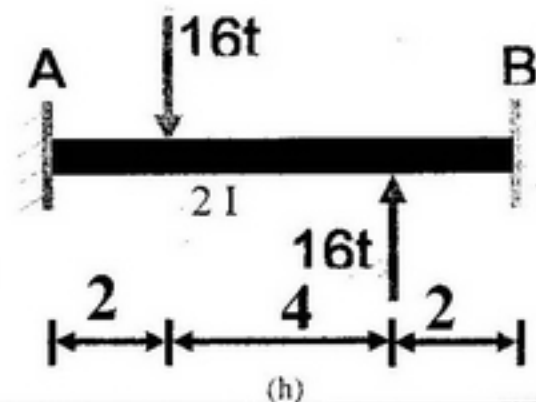
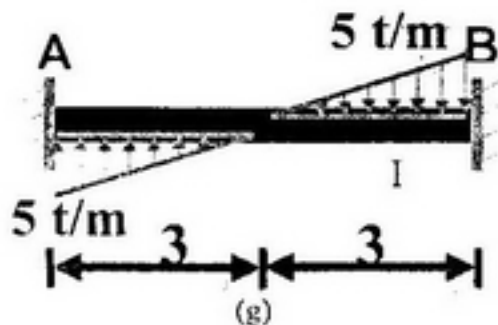
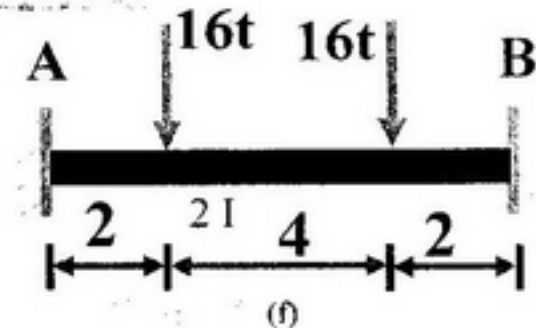
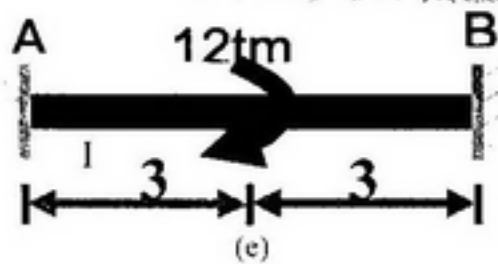
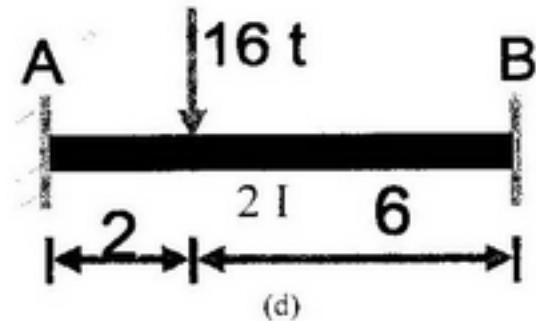
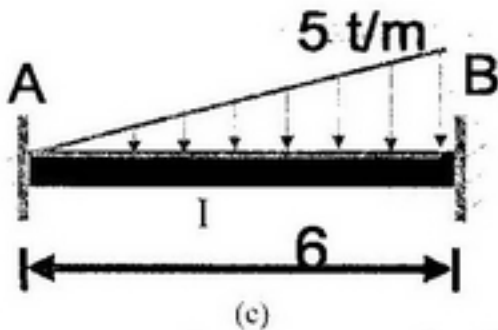
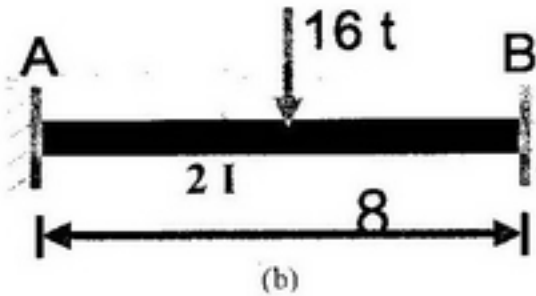
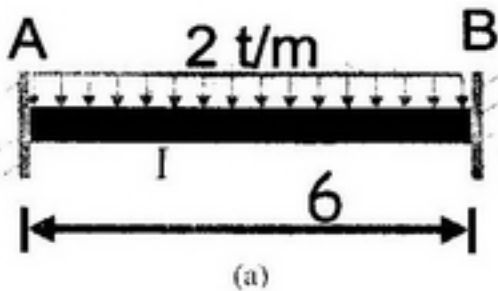






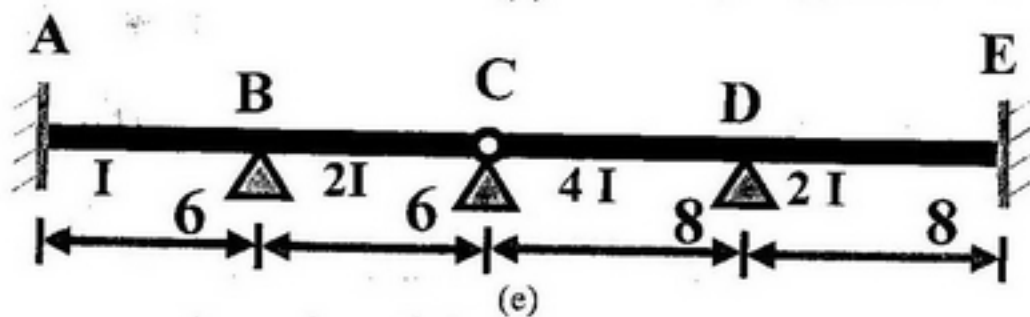
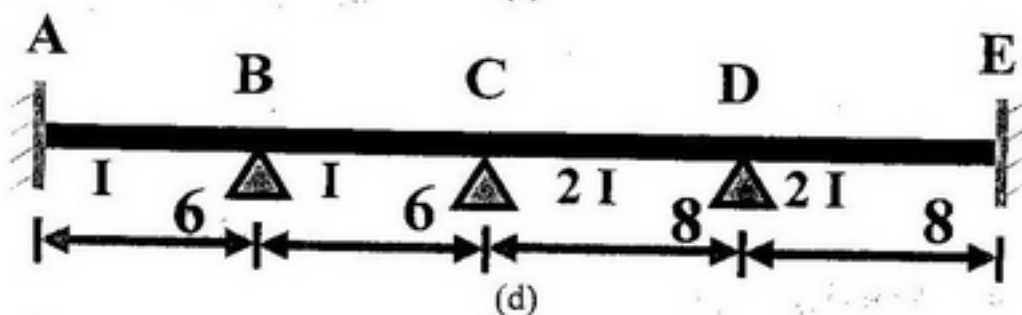
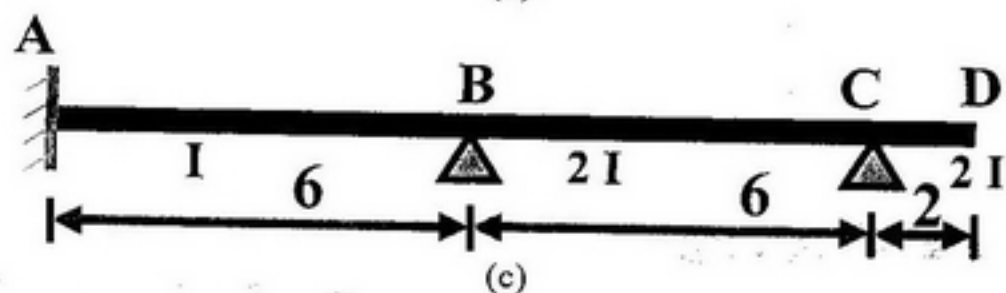
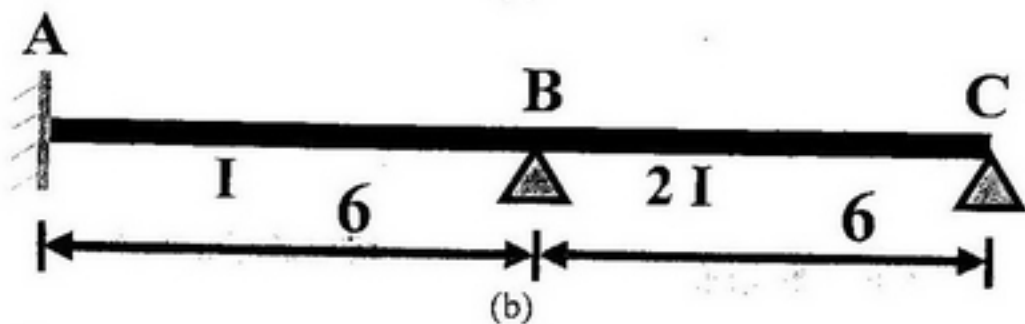
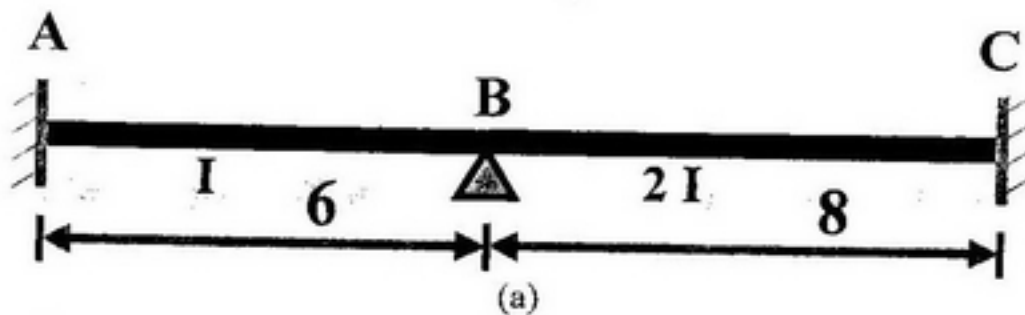
**Question No. (1) : [ 8 Degrees ] ( 7 min )**

Calculate the fixed end moments for both ends of the following beams :



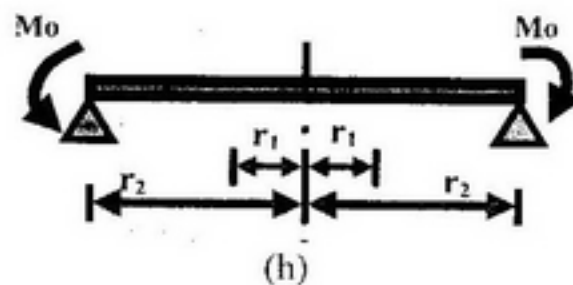
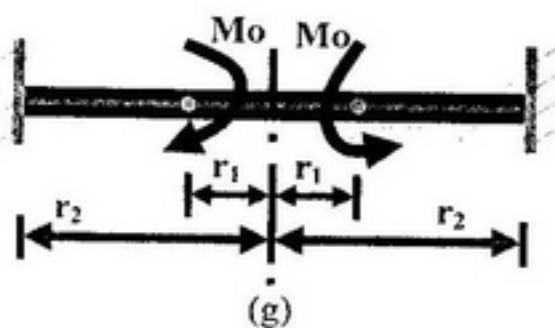
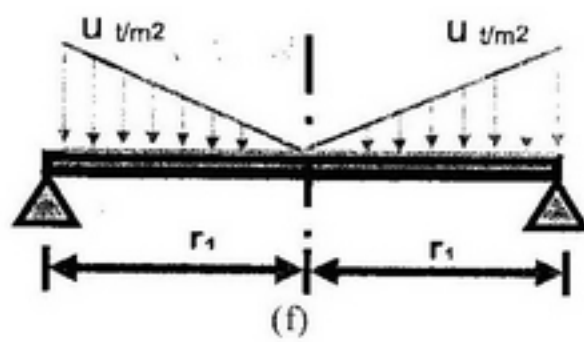
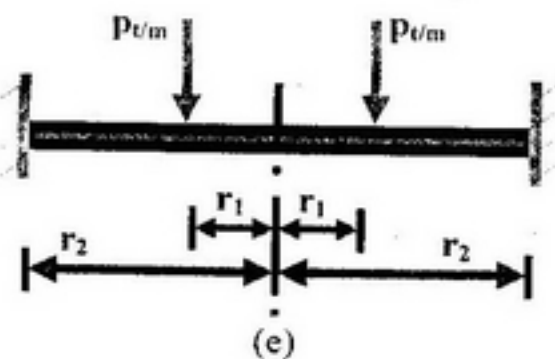
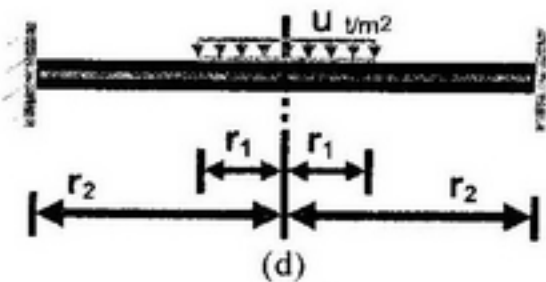
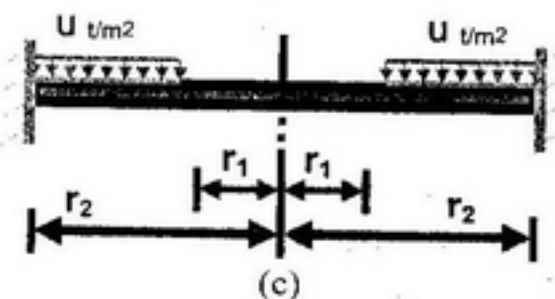
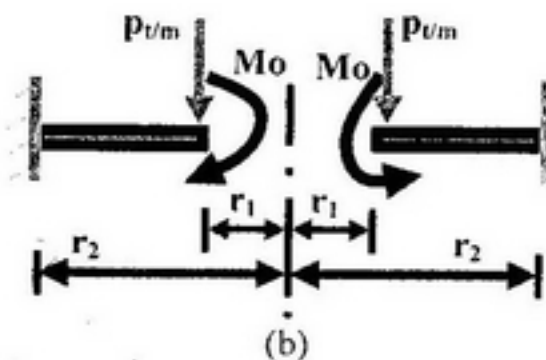
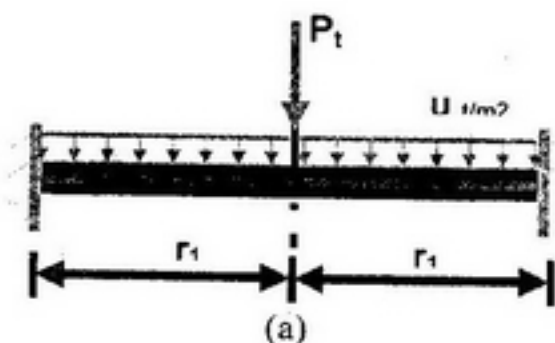
**Question No. (2) : [ 10 Degrees ] (7 min)**

Calculate the relative stiffness and the distribution factors for the following beams :



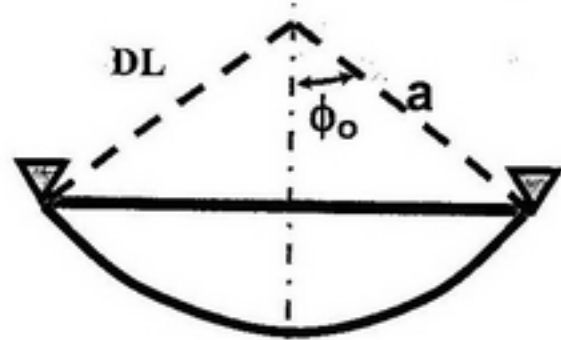
**Question No. (3) : [ 20 Degrees ] (16 min)**

State the number of intervals and its limits (  $.. < r < ..$  ) and calculate the shear for each interval and write the boundary conditions for the following plates :

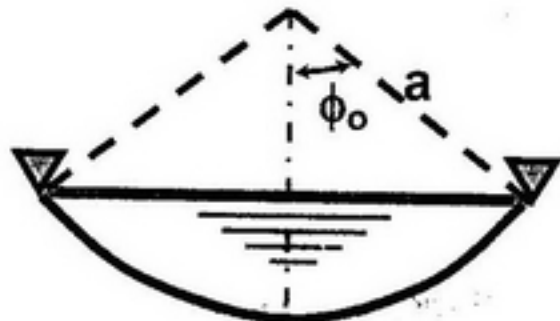


**Question No. (4) : [ 20 Degrees ] (17 min)**

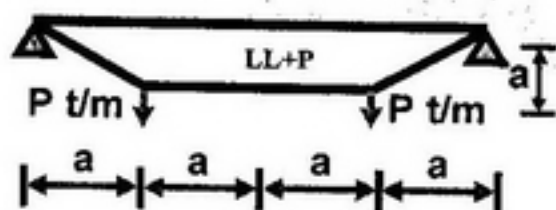
calculate the external load components ( $P_r$ ,  $P_\phi$ ,  $P_s$ ) and its limits ( $.. < \phi < ..$ ) and write the boundary conditions for the following shells due to the shown loads :



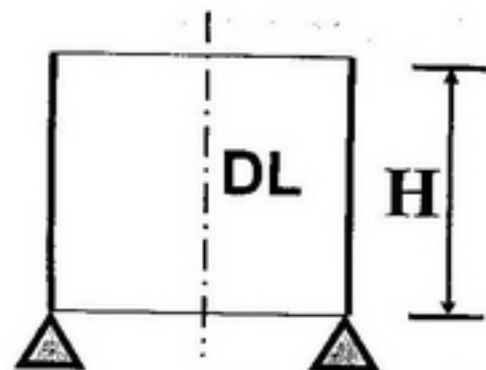
(a)



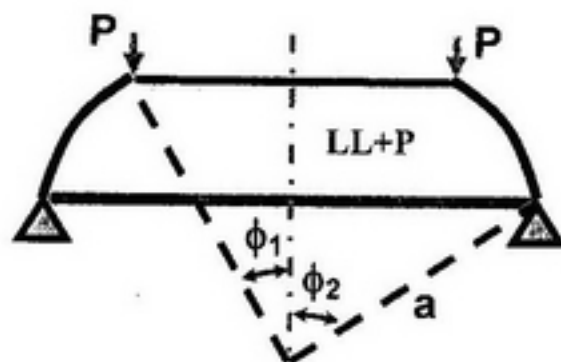
(c)



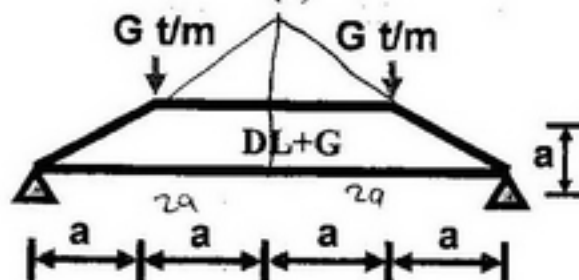
(e)



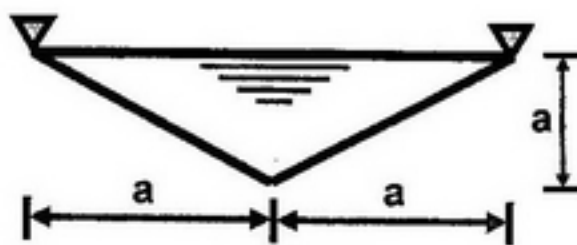
(g)



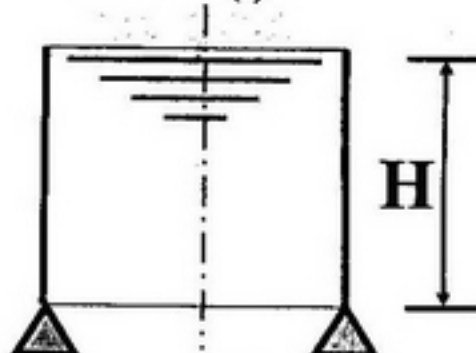
(b)



(d)



(f)

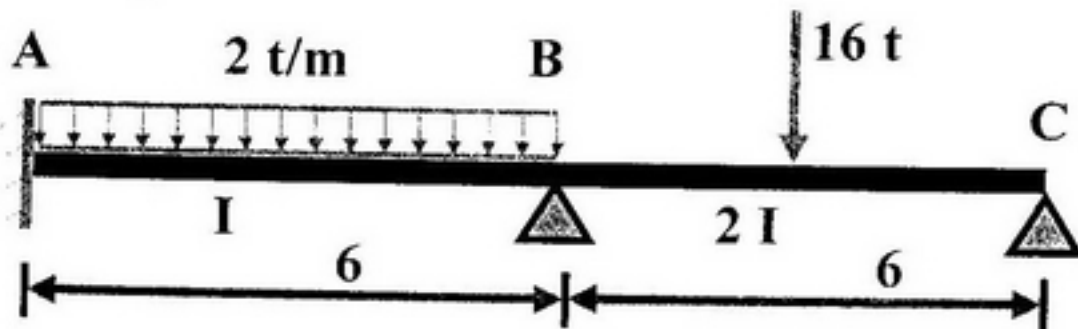


(h)



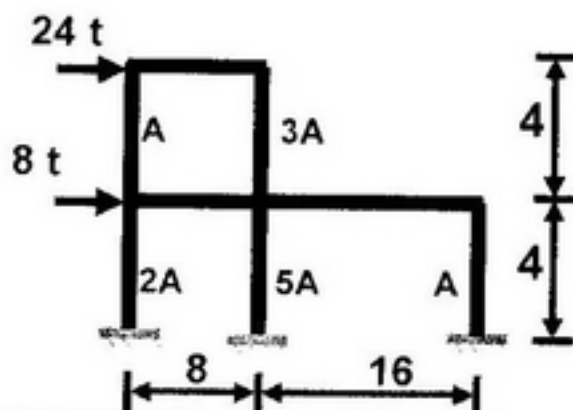
**Question No. (5) : [ 8 Degrees ] (7 min)**

Using the **moment distribution method**, draw the B.M.D and S.F.D for the shown beam due to the given loads :



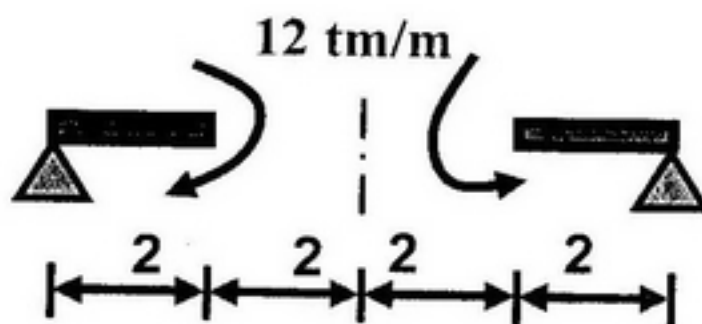
**Question No. (6): [ 10 Degrees ] (10 min)**

Using *Portal Frame* method, draw N.F, S.F, B.M.Ds for the shown Frame.



**Question No. (7): [ 12 Degrees ] (10 min)**

Draw Bending moment diagrams for the shown plate where  $\mu = 0$ .



With best wishes

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